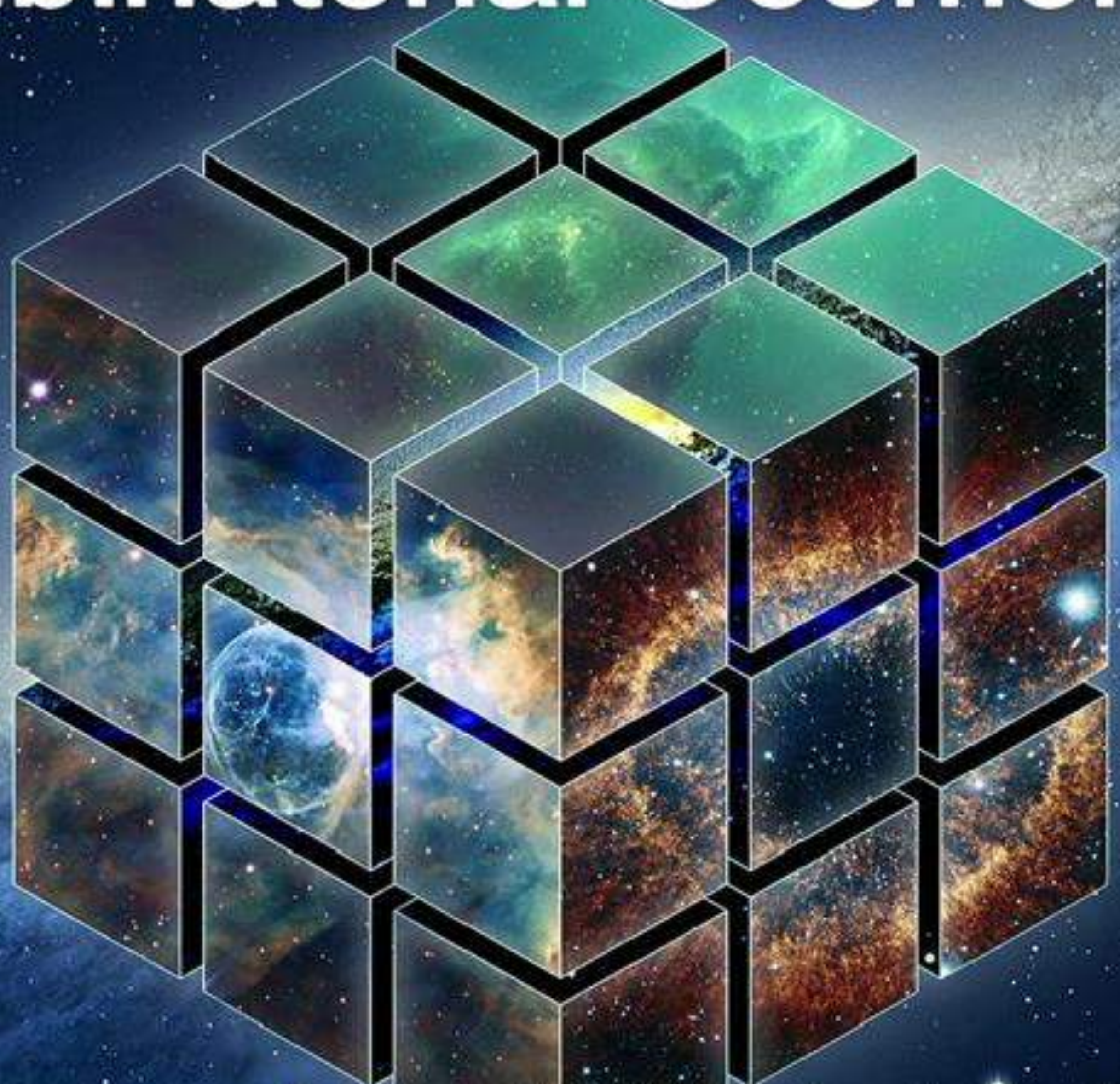
# Combinatorial Cosmology



**Liam McAllister**

**Cornell**

Physics Meets ML, Microsoft Research, April 26, 2019

# Summary

This talk is about realistic solutions of string theory.

I will explain why the solutions allowed by cosmological measurements are fully specified by finite lists of integers: quantized parameters.

To understand what string theory predicts, we need to connect these integers to observables.

I will formulate this task as a computational problem, and as a target for ML.

# Plan

I. Invitation to quantum gravity

II. Quantized parameters and the string landscape

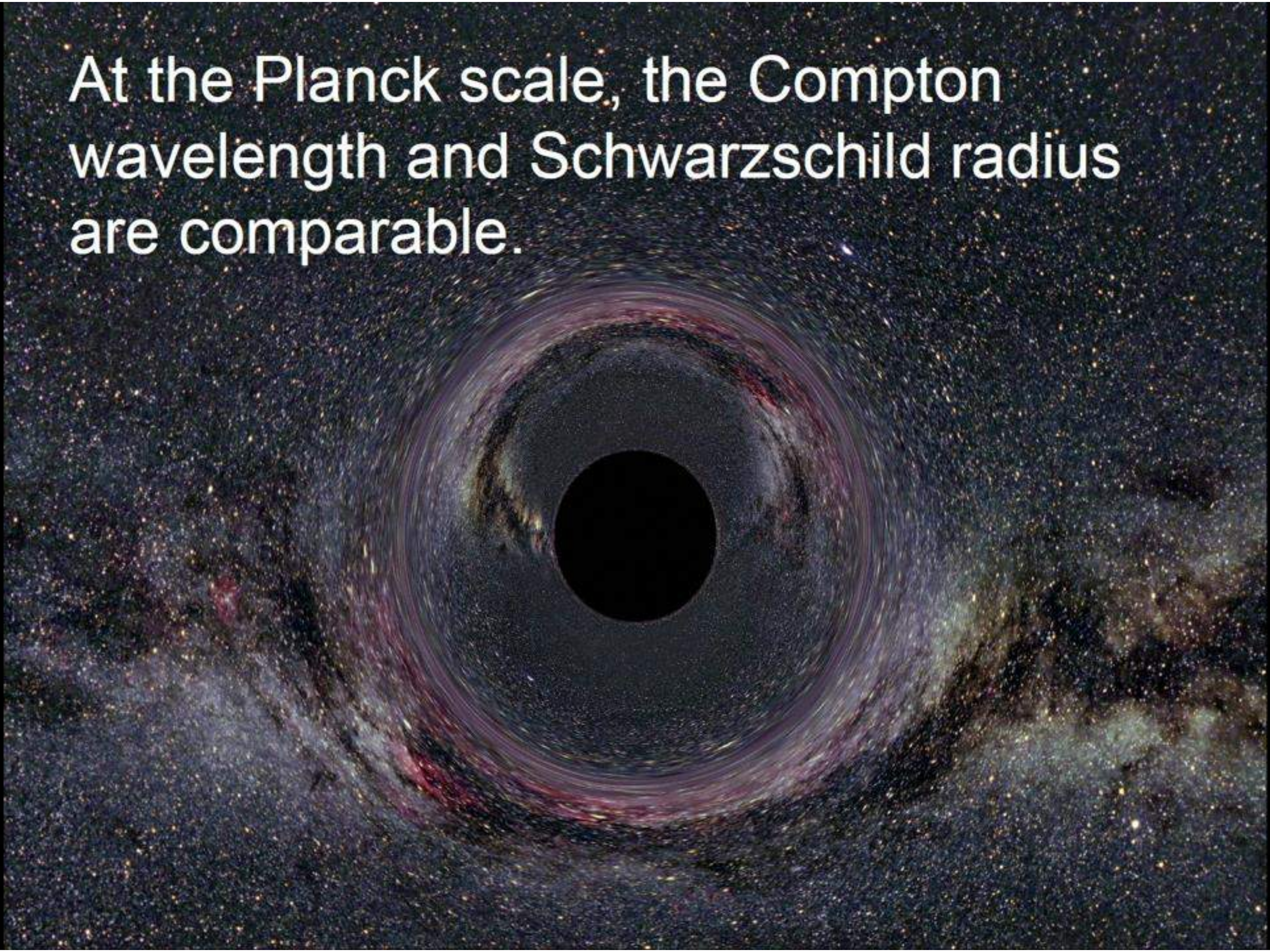
III. Example: toric hypersurfaces

IV. Targets for ML

# Quantum gravity and the Planck scale

$$M_p = \sqrt{\frac{\hbar c}{G_N}} \approx 22\mu\text{g} \qquad E_p \approx 1.2 \times 10^{18}\ \text{GeV} \approx 500\ \text{kWh}$$

$$t_p \approx 5.4 \times 10^{-44}\ \text{s} \qquad \ell_p \approx 1.6 \times 10^{-35}\ \text{m}$$


At the Planck scale, the Compton wavelength and Schwarzschild radius are comparable.

# When does quantum gravity matter?

In extreme conditions:

- Black hole singularities
- Planckian-energy collisions
- Cosmological singularities
- Cosmological inflation

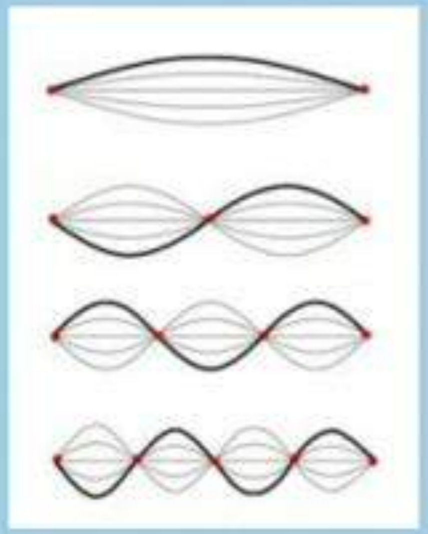
# Why is quantum gravity interesting?

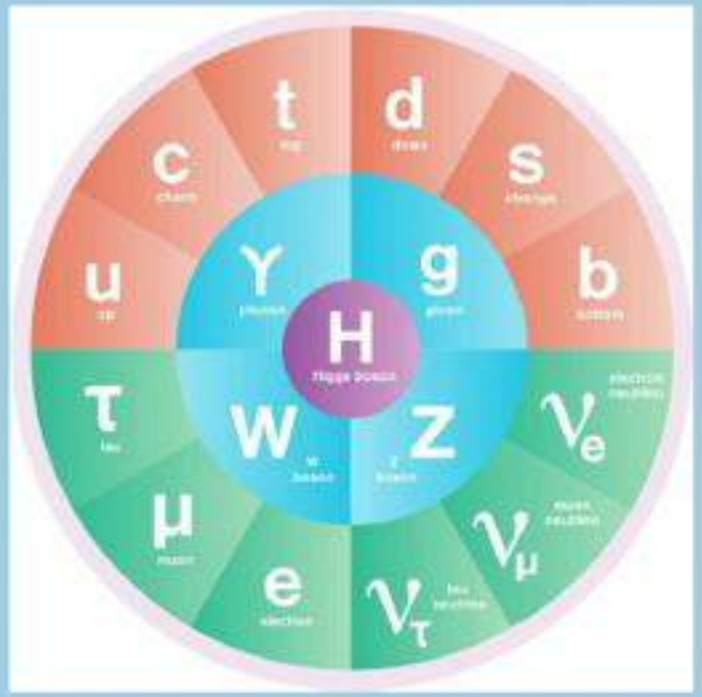
- By understanding more extreme conditions, we can learn more fundamental laws.
- Required to understand the origin of the universe, and to interpret certain observations in cosmology.
- Key to understanding the nature and origin of gravity.
- Best hope of understanding nature of physical law itself.

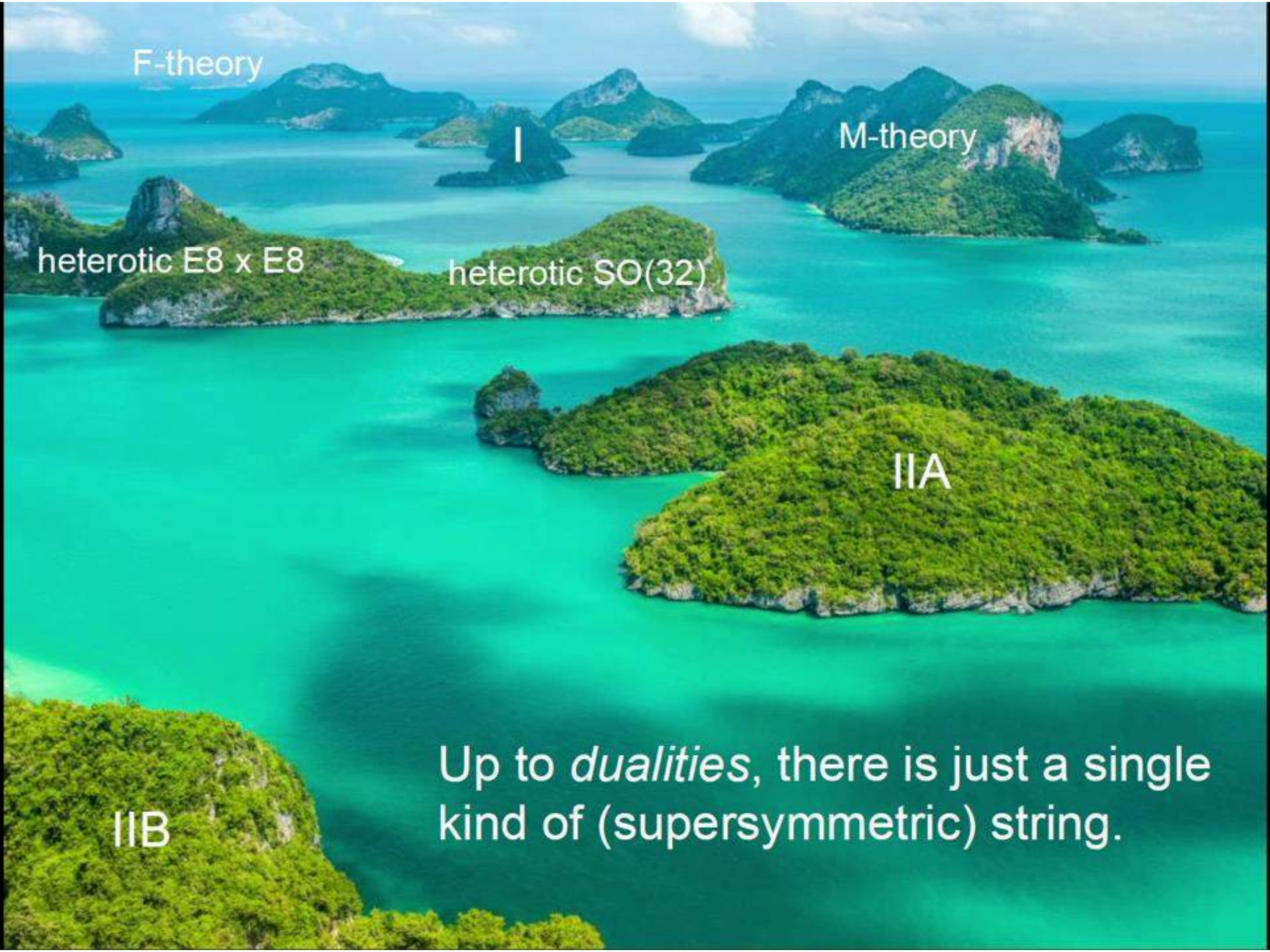
# String theory

A finite theory of quantum gravity.

Fundamental constituents are strings, rather than point particles.








Up to *dualities*, there is just a single kind of (supersymmetric) string.

# Compactification

String theory naturally exists in 10 dimensions.

Its fundamental solution is 10d Minkowski space.

0123 456789


4d spacetime 6d Euclidean space

# Compactification

Einstein equations in vacuum: $\mathbf{Ricci} = 0$

x

4d Minkowski space

6d compact space

0123

456789

Einstein equations in vacuum: $\mathbf{Ricci} = 0$

$$R_{\mu\nu} = 0 \qquad \text{and} \qquad R_{mn} = 0$$

$$\mu, \nu \in \{0, 1, 2, 3\} \qquad m, n \in \{4, 5, 6, 7, 8, 9\}$$

x

4d Minkowski space | 6d compact space

0123 | 456789

Einstein equations in vacuum: $\mathbf{Ricci} = 0$

$$R_{\mu\nu} = 0 \qquad \text{and} \qquad R_{mn} = 0$$

Vacuum solution of string theory =

Compact Ricci-flat six-manifold

x

4d Minkowski space

6d compact space

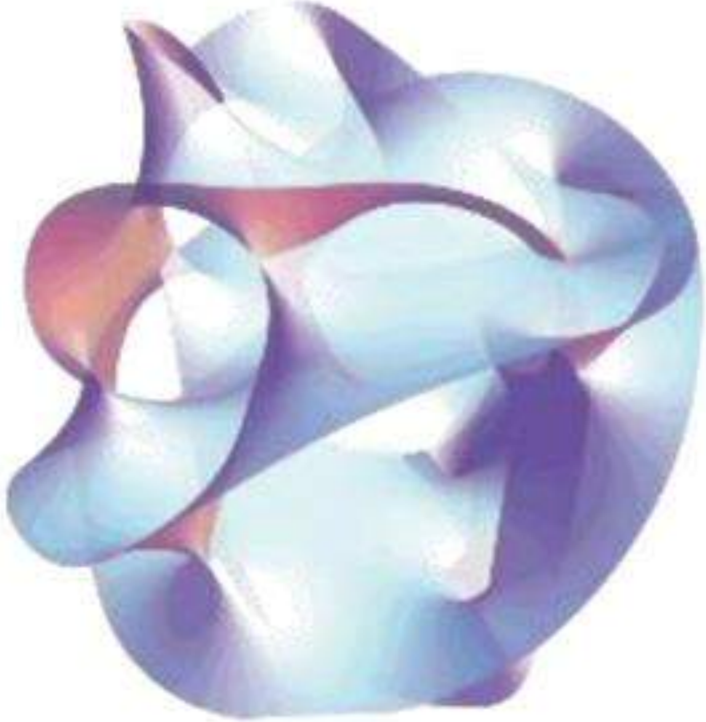
0123

456789

# String theory is unique

- The theory has no fundamental dimensionless parameters.
- The parameters we see at low energy are determined by the geometry of the internal space.
  - i.e. by the *solution* to Einstein's equations.

So all parameters in nature are the expectation values of fields, principally those that dictate the internal geometry (size and shape).



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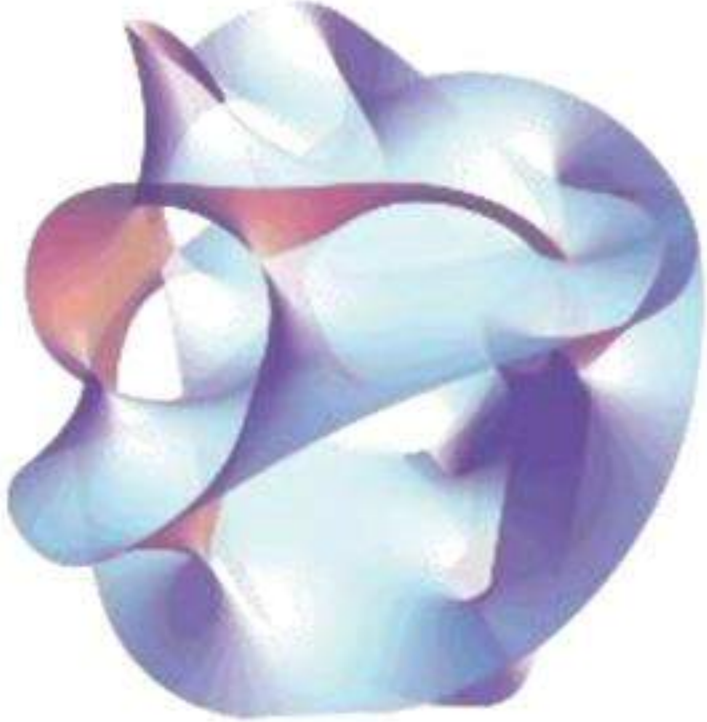

$$\phi(\vec{x}, t) = \mathrm{Vol}_6(\vec{x}, t)$$

- *Continuous* parameters correspond to massless scalar fields (moduli).
- These are catastrophic for cosmology.


Cosmological effects of light scalars:

- Dissociation of light elements after nucleosynthesis
- Overproduction of dark matter
- Overclosure of the universe
- Fifth forces

- The only cosmologically viable solutions of string theory are those without continuous deformations (moduli).
- We call these solutions ‘isolated vacua’, or just ‘vacua’.
- Their spin-zero fields are all massive,

$$m^2 \geq m^2_{\rm min} > 0$$


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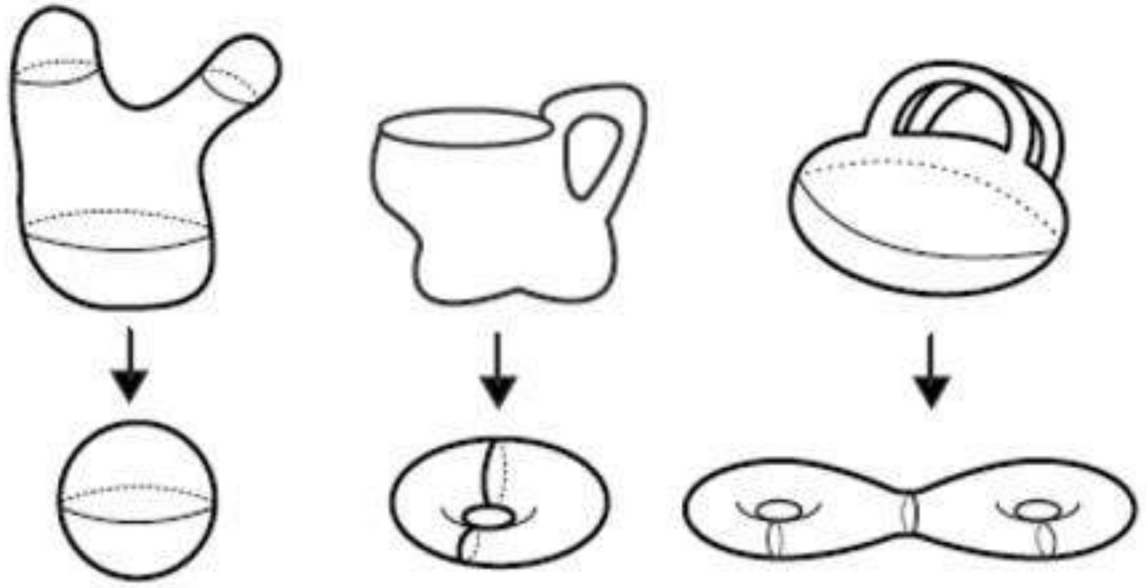
- The only cosmologically viable solutions of string theory are those without continuous deformations (moduli).
- There are no fundamental parameters in the theory, and we demand there be no continuous parameters in the solution.
- All that remains are discrete parameters in the solution.
- When you have discarded the continuous data describing a geometry, what remains is topology.

A cosmologically viable solution of string theory is specified by the topological invariants of the internal six-manifold X.

Heuristically, these invariants count holes, handles, etc., in X.



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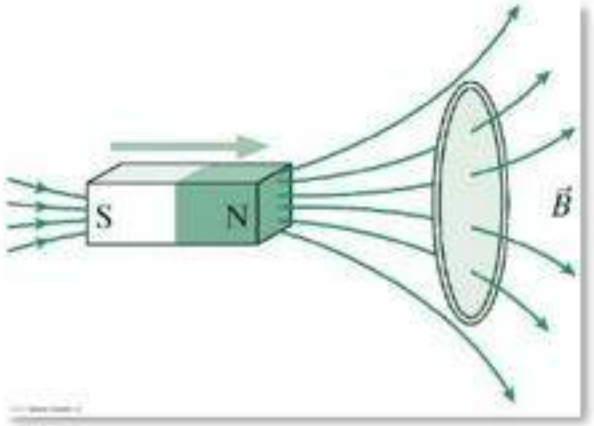
More precisely, they involve:

- Topology of X: Hodge numbers, intersection numbers
- Topology of D-branes on submanifolds S in X: wrapping numbers, bundles on S
- Number of units of flux threading cycles of X



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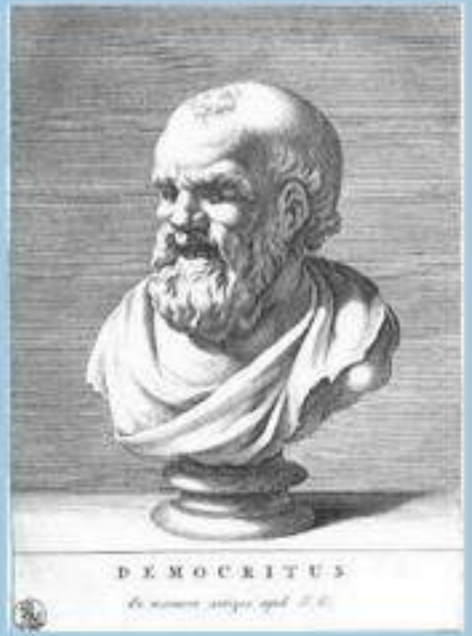
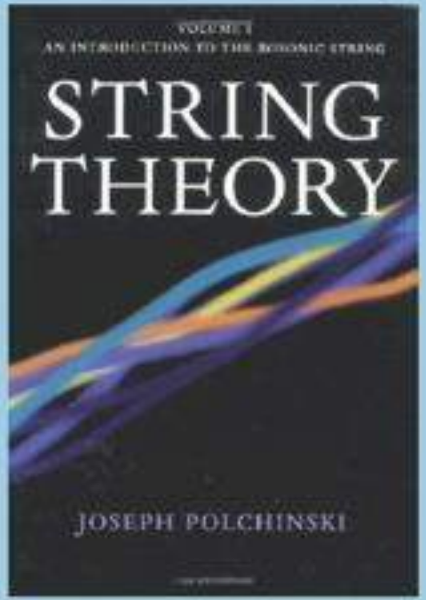
For each solution, the invariants amount to a finite number N of integers.




Matter is quantized

Force carriers are quantized




Parameters are quantized

$$\alpha = \frac{1}{137}$$

Cosmologically viable quantum gravity theories are specified by finitely many integers, the fundamental parameters in Nature.

$$\vec{p} \in \mathbb{Z}^N \qquad N \sim \mathcal{O}(100 - 1000)$$

Not all $\vec{p} \in \mathbb{Z}^N$ are consistent/allowed.

There is no compelling evidence for any infinite family of vacua of string theory with $m_{\min}$ fixed. I assume the number of viable vacua is finite.

The set of such vacua is the string landscape.

# Size of the landscape

- In known 'islands', number of cycles in internal space, hence N, is large:
  - Type IIB string theory on Calabi-Yau threefolds:

    $N \sim \mathcal{O}(100-500)$ e.g. $\dim H_4(CY_3, \mathbb{R}) \leq 491$

    $\mathcal{N}_{\mathrm{vac}} \sim 10^N \sim 10^{500}$ Denef, Douglas
  - F-theory on Calabi-Yau fourfolds

    $N \lesssim \mathcal{O}(10^6)$ e.g. $\chi(CY_4) \leq 1{,}820{,}448$ Klemm, Lian, Roan, Yau

    $\mathcal{N}_{\mathrm{vac}} \sim 10^{272{,}000}$ Taylor, Wang

# String theory and reality

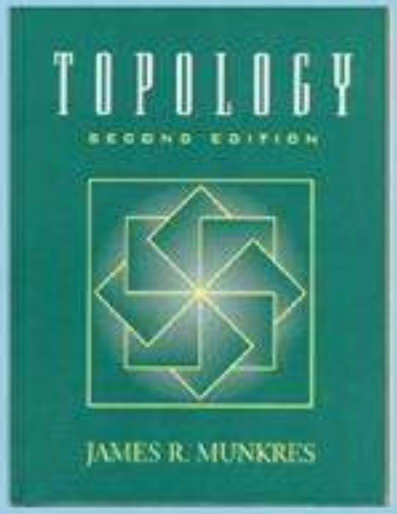
- Goal: connect solutions of string theory to the observed universe.
- To achieve this, must know:
  - What values can the quantized parameters take?
  - Given an allowed set of values, what would be observed in the resulting universe?
- Examples of observables:
  - Matter content, gauge group, vacuum energy, masses and couplings of visible and dark sector particles

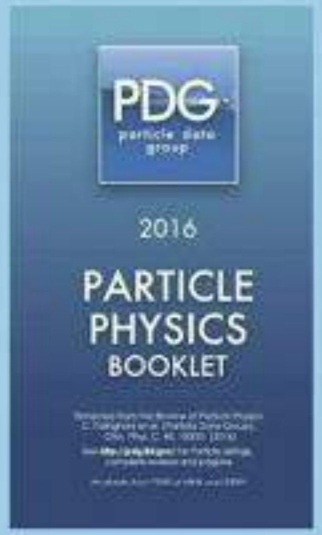
# Parameters to Observables

$$\mathcal{P} \to \mathcal{O}$$

$$\mathcal{P} = \left\{ \vec{p} \in \mathbb{Z}^N \middle| \, \vec{p} \text{ allowed in string theory} \right\}$$ N: number of discrete parameters

$$\mathcal{O} = \left\{ \vec{\mathbf{o}} \in \mathbb{Q}^M \, \middle| \, \vec{o} \text{ possible values of M measurable parameters} \right\}$$



# Parameters, Effective Theory, Observables

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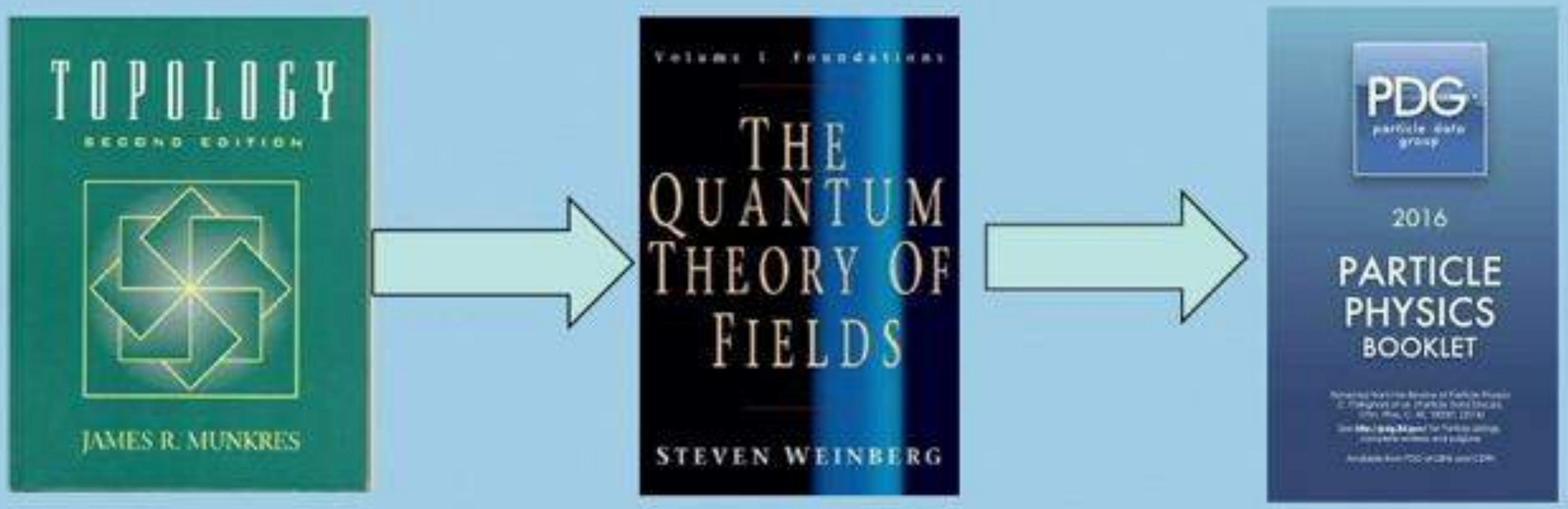


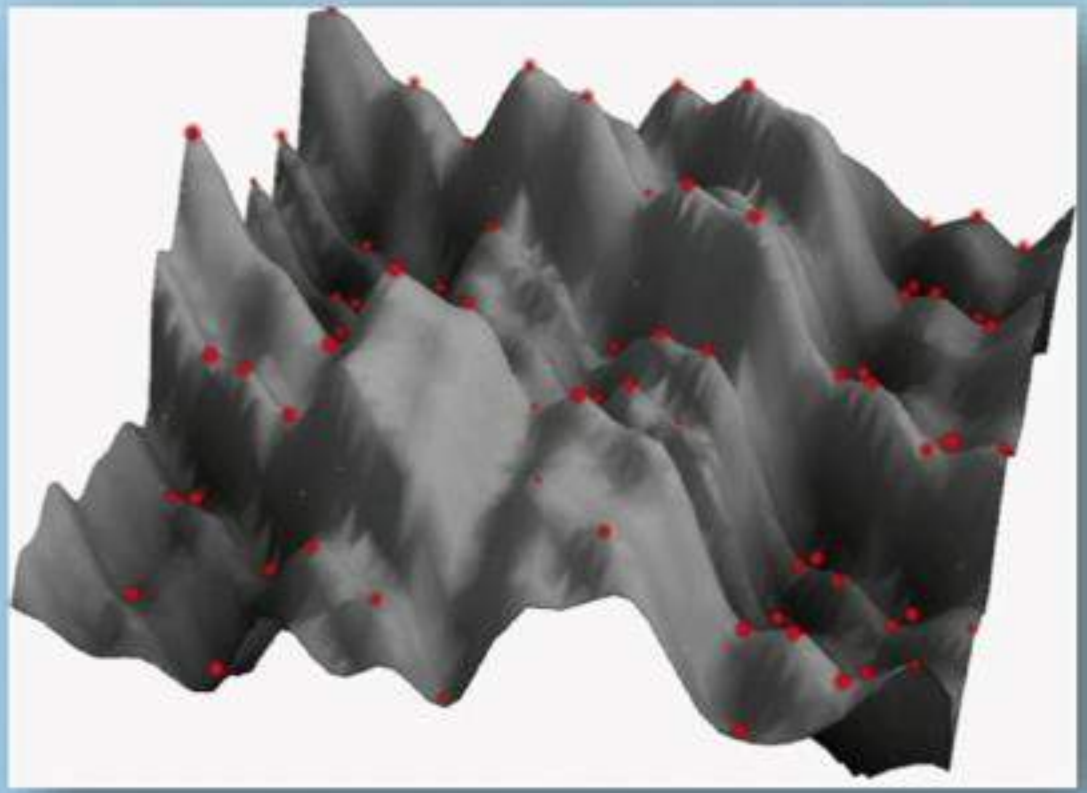
# Parameters, Effective Theory, Observables

$$\mathcal{E} = \Big\{ \text{data of a quantum field theory } + \text{ gravity} \Big\}$$

$$\supset \text{Riemannian manifold } \mathcal{M} \text{ ("field space"), metric } g;$$

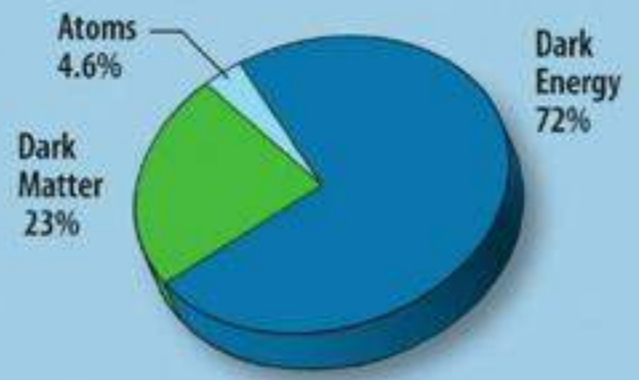
$$\text{function } V : \mathcal{M} \to \mathbb{R} \text{ ("potential")}$$

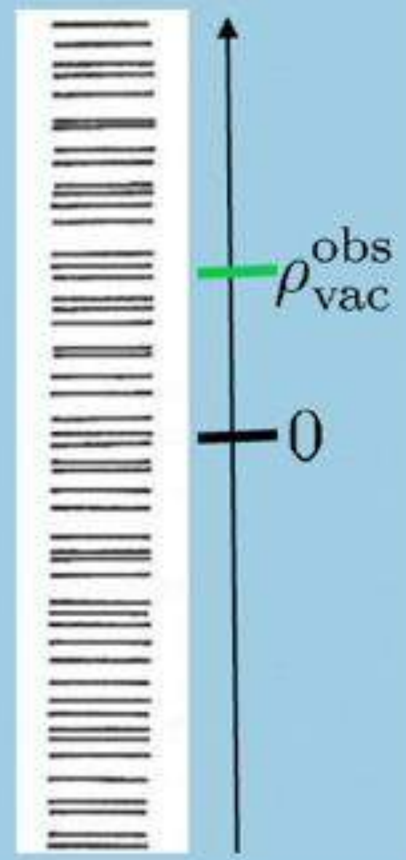
vacua : local minima of $V$ $\qquad \nabla V = 0, \quad \nabla\nabla V > 0$



# Observable: dark energy

- Dark energy density $\rho_{\rm vac}^{\rm obs} \approx 10^{-26}\ \frac{\rm kg}{\rm m^3} \approx 10^{-123} M_{\rm pl}^4$
- No continuous parameter to adjust to match observations.
- If vacua of string theory have $\rho_{\rm vac} \in (-M_{\rm pl}^4, M_{\rm pl}^4)$ sufficiently uniform,



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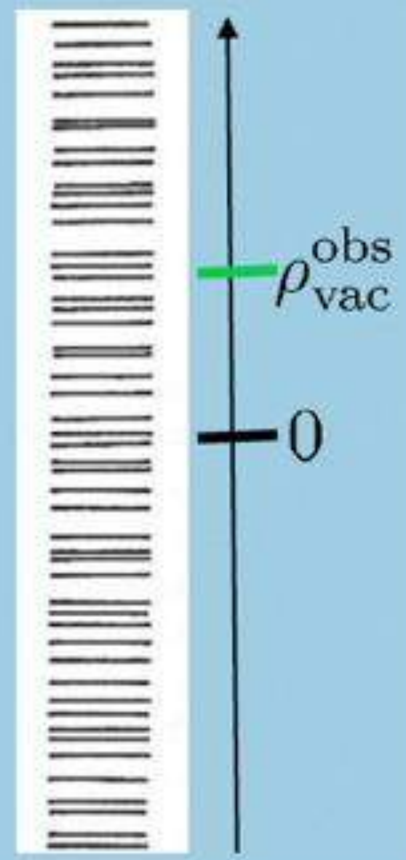

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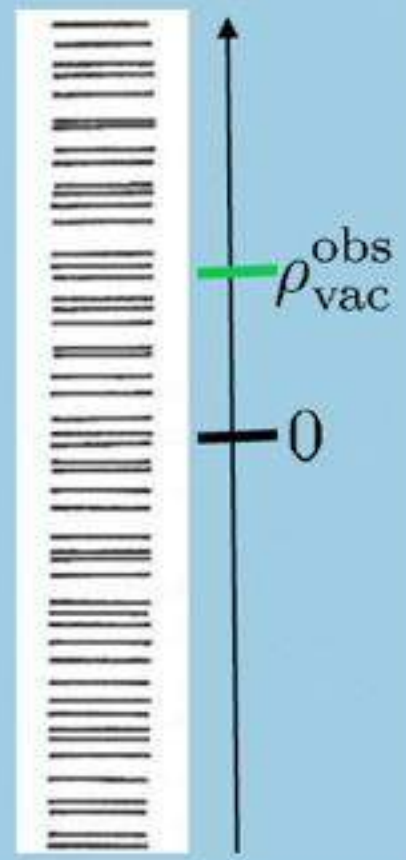
- 'Anthropic solution to cosmological constant problem'.

Banks; Abbott; Brown and Teitelboim; Linde; Weinberg; Bousso and Polchinski

# Plan

I. Invitation to quantum gravity

II. Quantized parameters and the string landscape

III. Example: toric hypersurfaces

IV. Targets for ML

Mehmet Demirtas

Cody Long

Mike Stillman

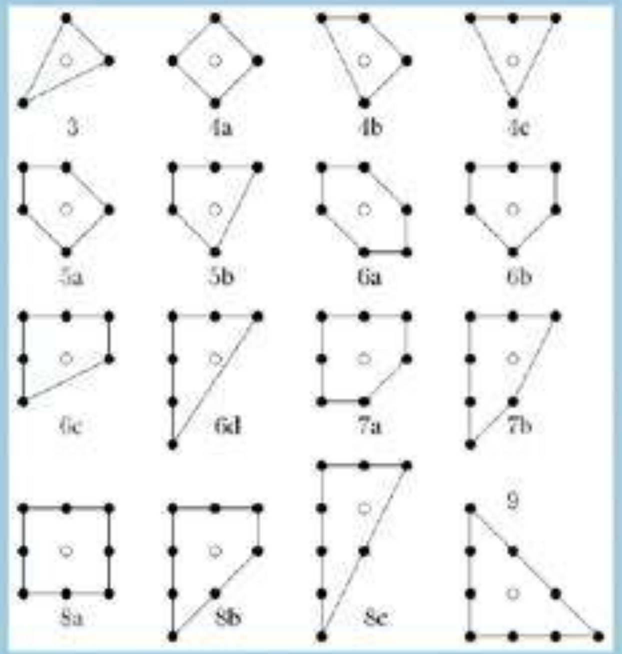
Andres Rios Tascon

# Ricci-flat six-manifolds

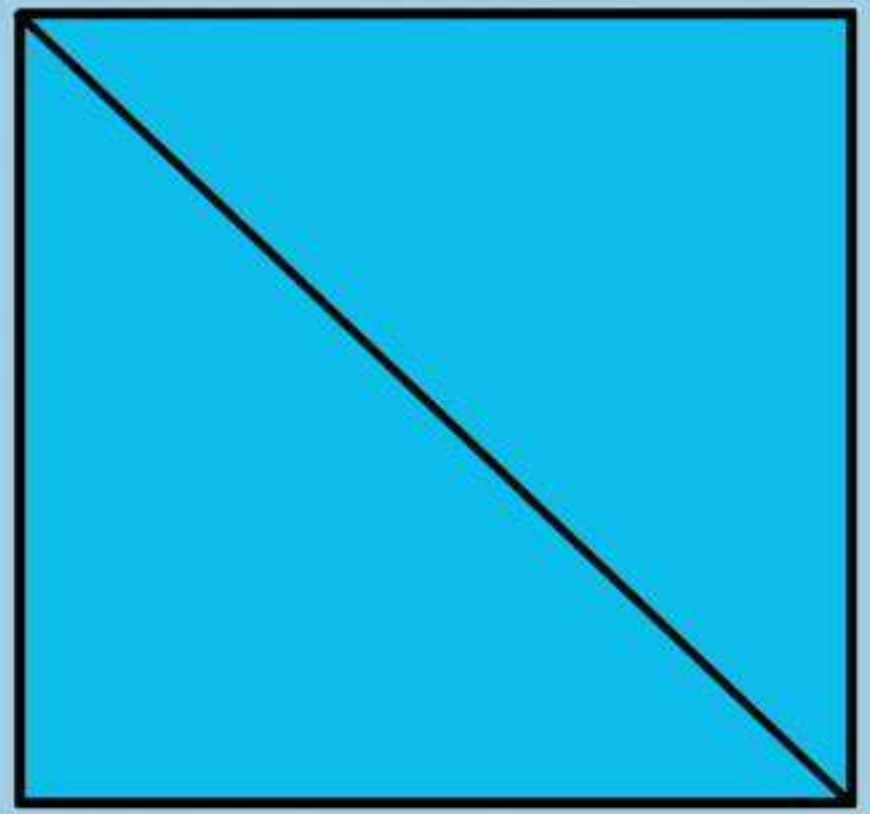
Finding Ricci-flat 6-manifolds directly is challenging.
In the case of Calabi-Yau threefold ($CY_3$) hypersurfaces,
the problem is combinatorial and can be automated.

- Toric varieties are algebraic varieties that can be assembled by gluing copies of algebraic tori, $(\mathbb{C}_*)^n$, according to instructions encoded in a lattice polytope.
- Vast class of TV encoded in 4d reflexive polytopes $\Delta \subset \mathbb{Z}^4$.
- 4d reflexive polytopes have been classified.
- There are 473,800,776 of them.
- Each 'fine regular star triangulation' (FRST) of such a $\Delta$ yields a TV in which the generic anticanonical hypersurface is a smooth $CY_3$.


Kreuzer

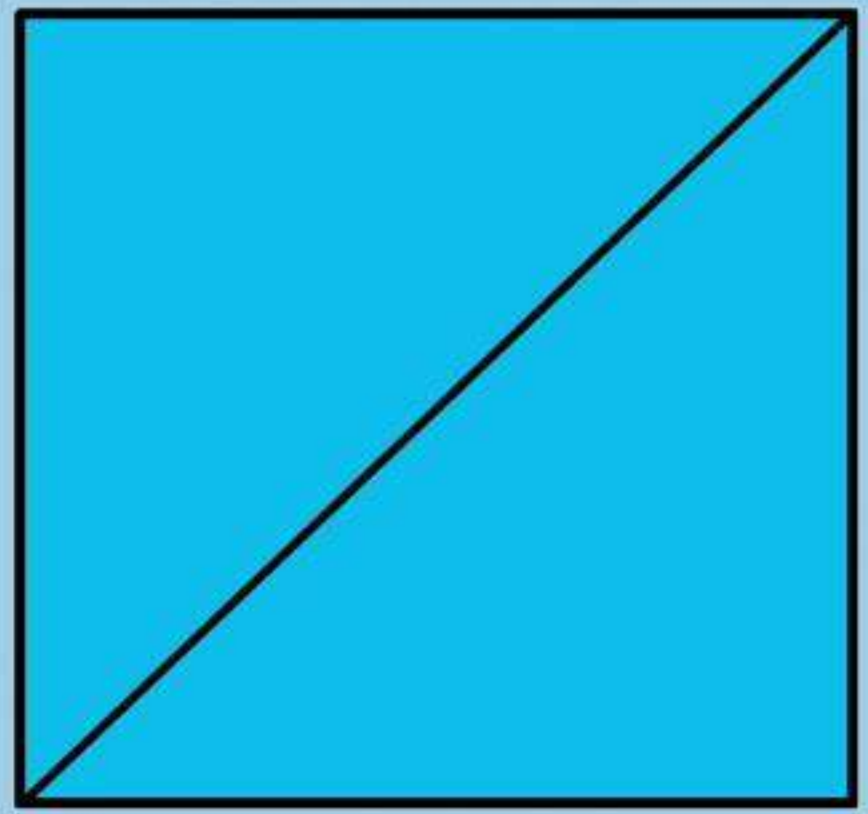


# Triangulation



Triangulation of polytope = division into simplices.

# Triangulation



Triangulation of polytope = division into simplices.

Fine: uses all the points.
Star: all simplices contain origin.
Regular: descends from 1 higher dimension.

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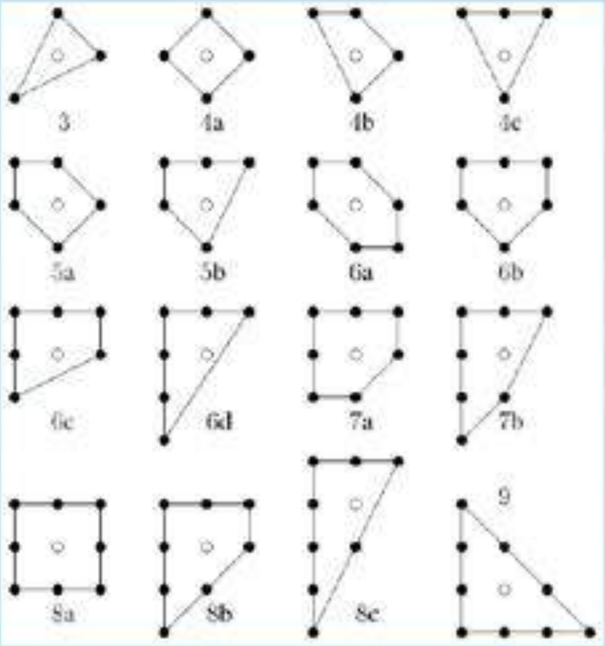
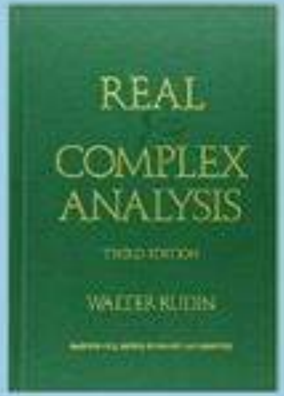


Task of triangulating polytopes is combinatorial, and amenable to automation.




analysis

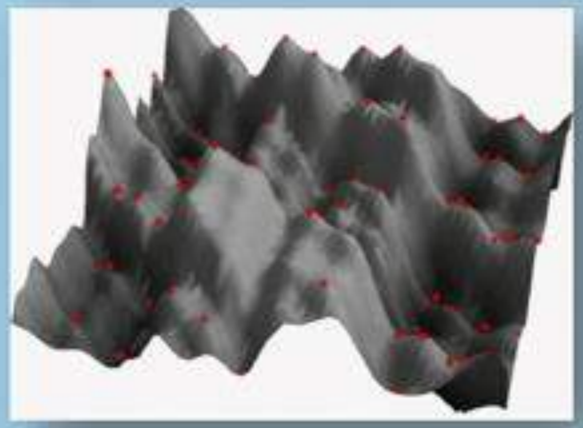
algebraic geometry

combinatorics

# Example of discrete parameters

1. Choose a 4d reflexive polytope $\Delta \subset \mathbb{Z}^4$ 473,800,776
2. Choose a fine star regular triangulation of $\Delta$, determining a toric variety V, and a $CY_3$ hypersurface $X \subset V$.
3. Choose quantized fluxes, $\vec{f}, \vec{h} \in H_3(X, \mathbb{Z}) \cong \mathbb{Z}^{2h^{2,1}(X)}$
4. Choose D7-branes $\{\vec{w}_A\} \in H_4(X, \mathbb{Z}) \cong \mathbb{Z}^{h^{1,1}(X)}$
5. For each D7-brane, choose bundle $\vec{\mathcal{F}}_A \in H^2(w_A, \mathbb{Z})$

One can then compute the potential, V.



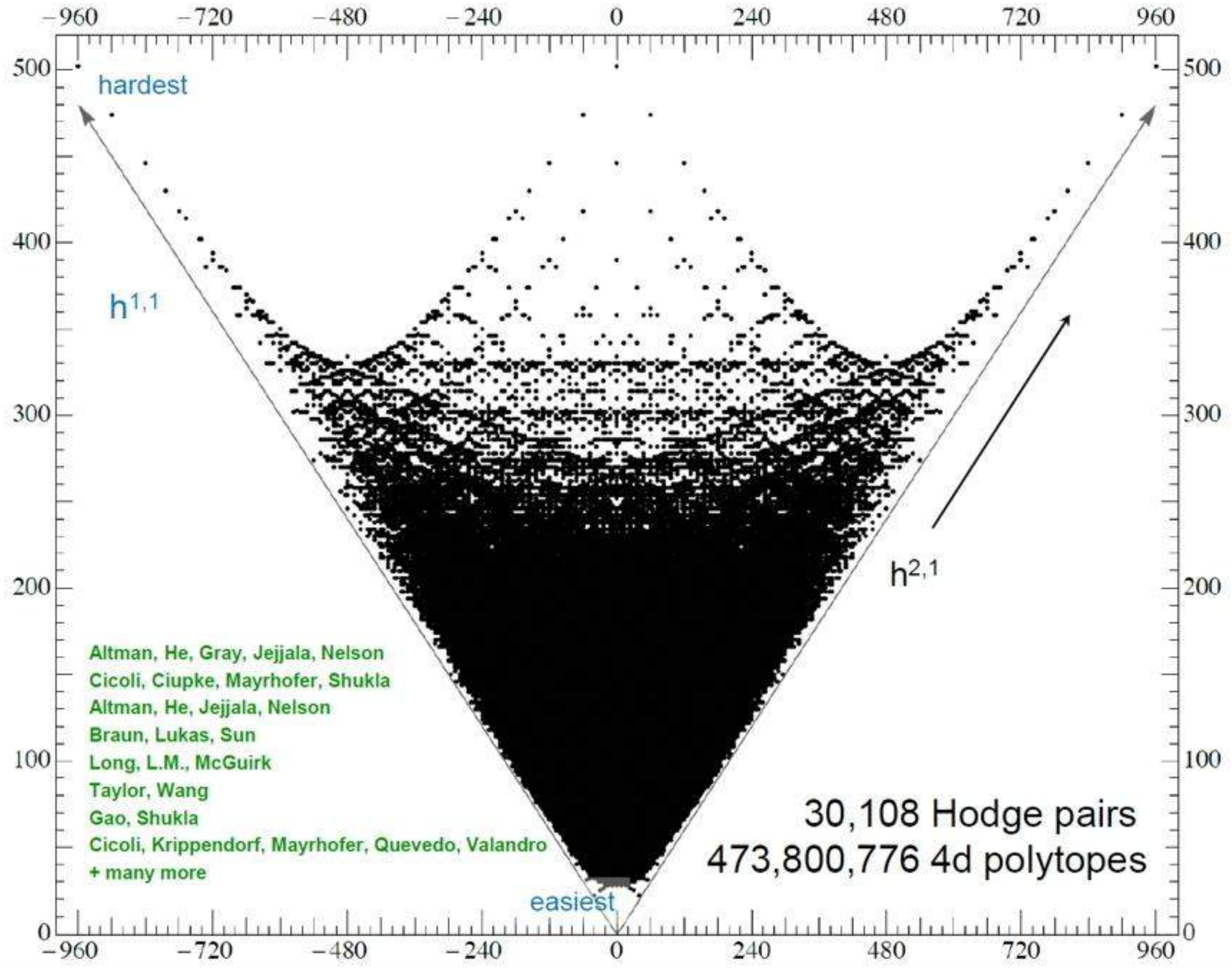
# Computational task

1. Triangulate 4d polytope — A. Braun
2. Examine fine regular star triangulation to obtain
   a. Toric variety V
   b. Intersection numbers of $X \subset V$ — Oda, Park; Berglund, Katz, Klemm

Number of relevant points = $h^{1,1}(X) + 4$. ('favorable' case)

Number of intersection numbers ~ $(h^{1,1})^3$

$$1 \leq h^{1,1}(X) \leq 491$$


hardest
h1,1
h2,1
easiest
Altman, He, Gray, Jejjala, Nelson
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+ many more
30,108 Hodge pairs
473,800,776 4d polytopes
−960 −720 −480 −240 0 240 480 720 960
0 100 200 300 400 500

# Number of FRSTs

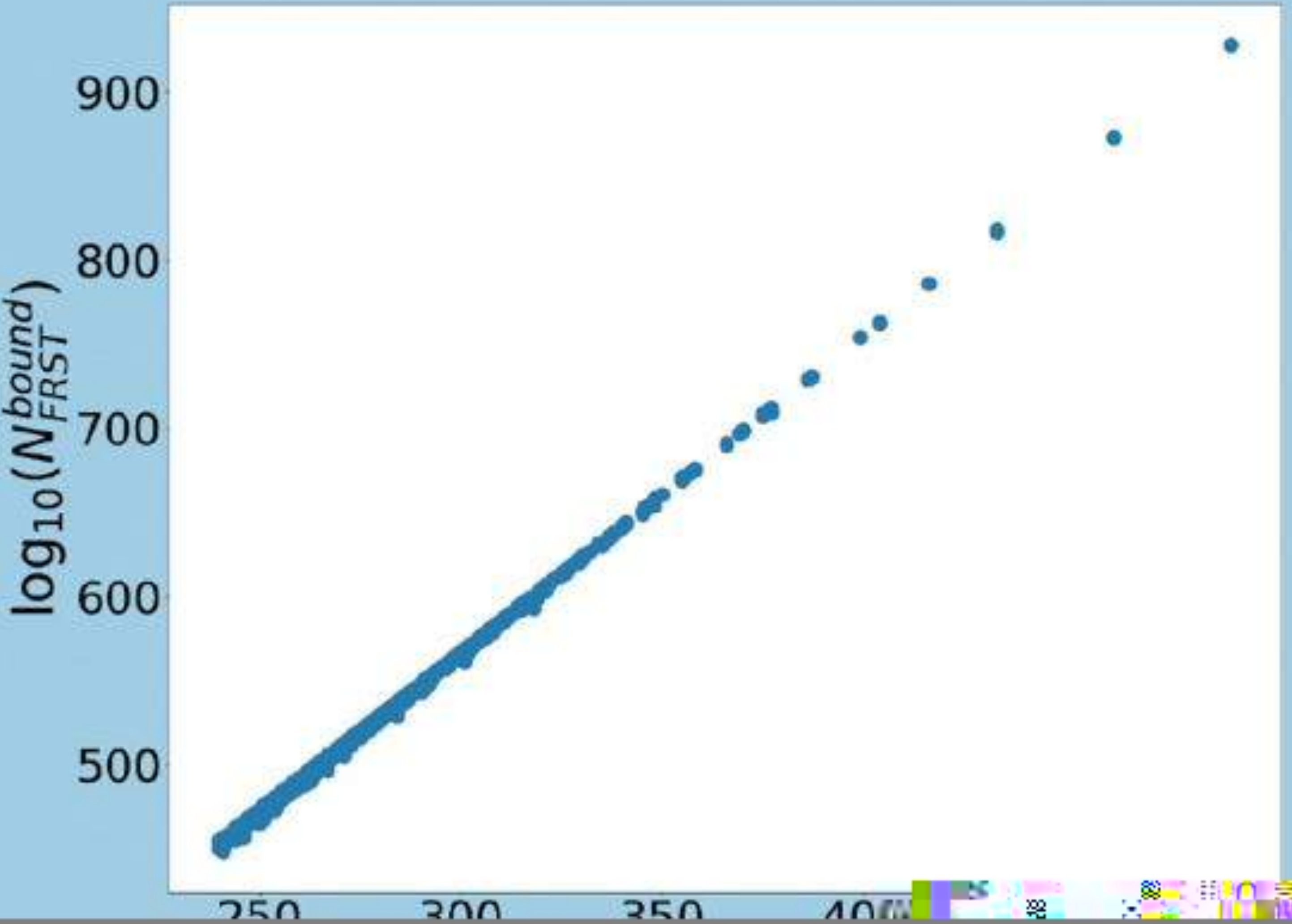
Number of triangulations/polytope grows combinatorially.

$$N_{\mathrm{FRST}} \leq \binom{4V-1}{h^{1,1}+3}$$

V: volume of polytope

$$N_{\mathrm{FRST}} \leq \binom{14{,}111}{494} \approx 1.5 \times 10^{928}$$

$$N_{\mathrm{FRST}} \leq \binom{13{,}271}{465} \approx 2.7 \times 10^{863}$$



# Why count solutions?

- Suppose string theory has a large but finite number $\mathscr{N}$ of isolated solutions that fulfill a set of criteria C.
  - e.g., three large dimensions, Standard Model gauge group, …

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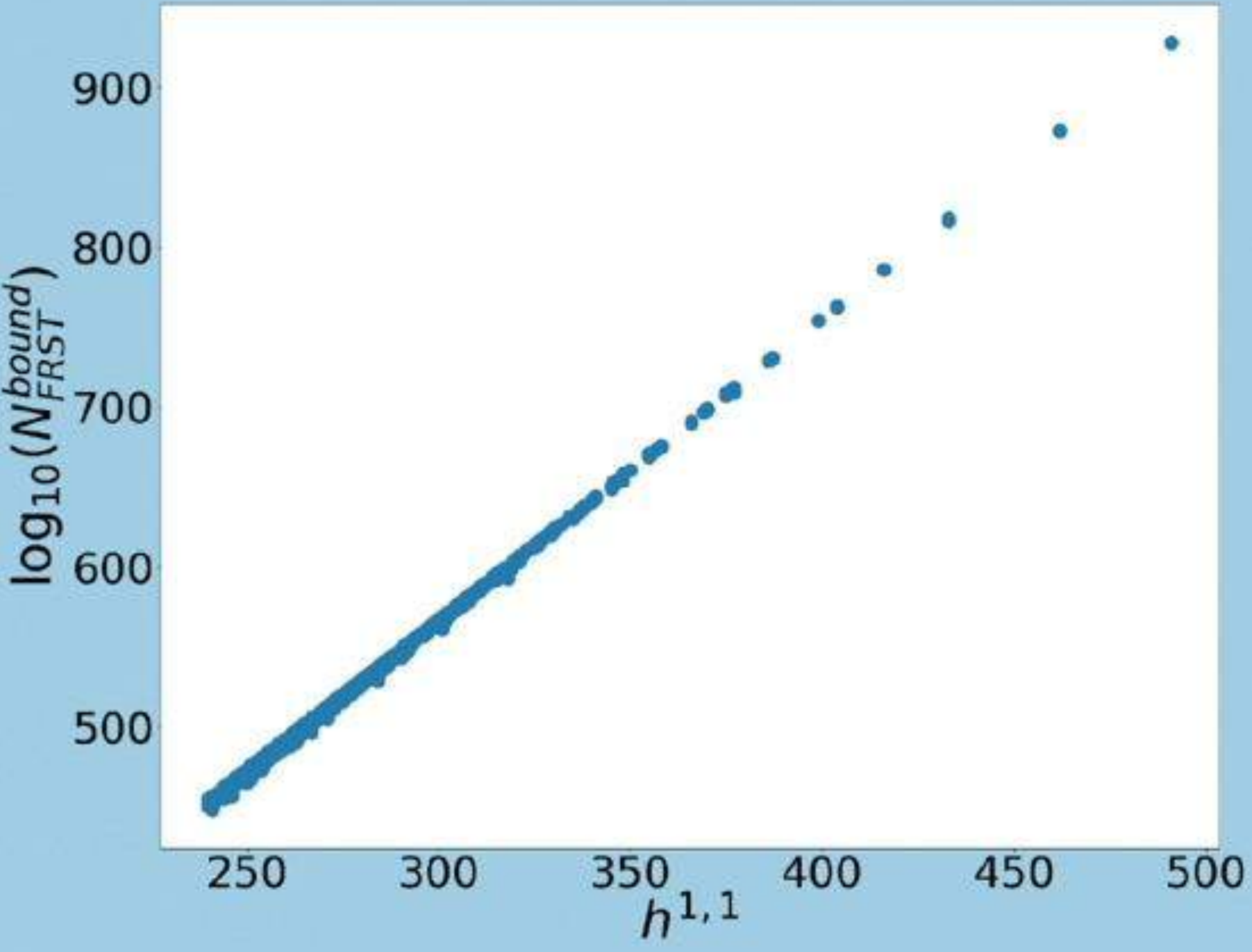
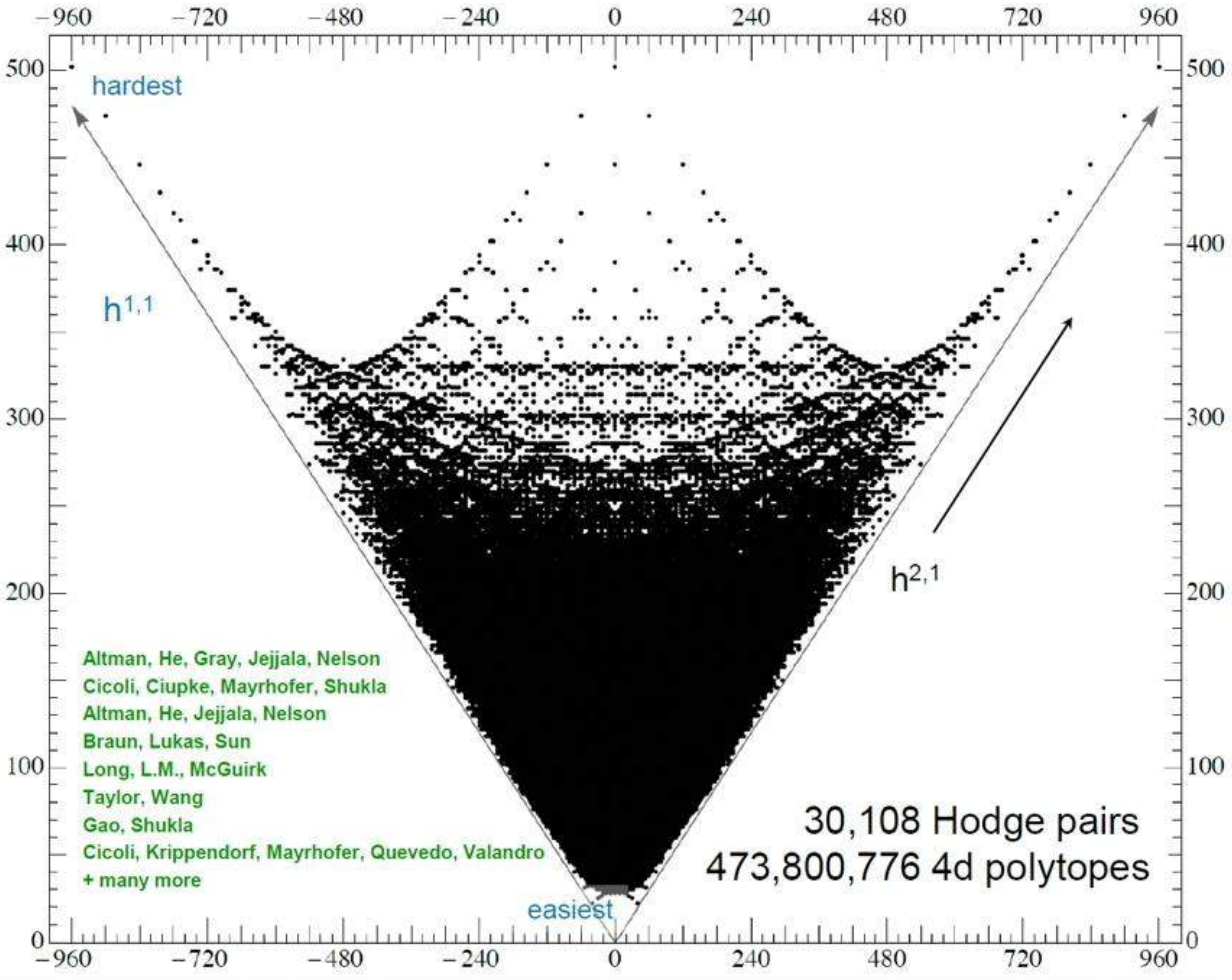



cf. Altman, Carifio, Halverson, Nelson

Demirtas, Rios Tascon, L.M.



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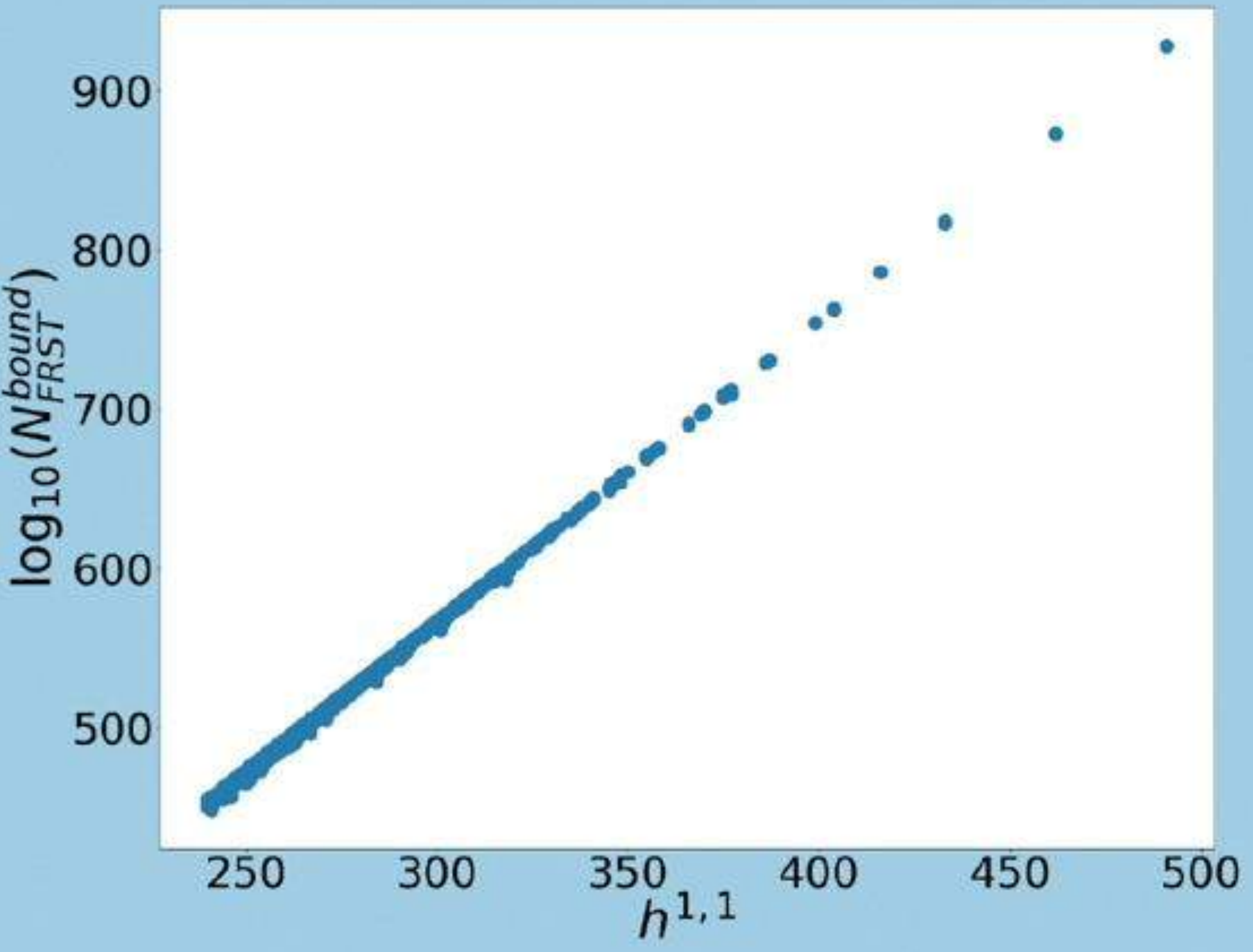


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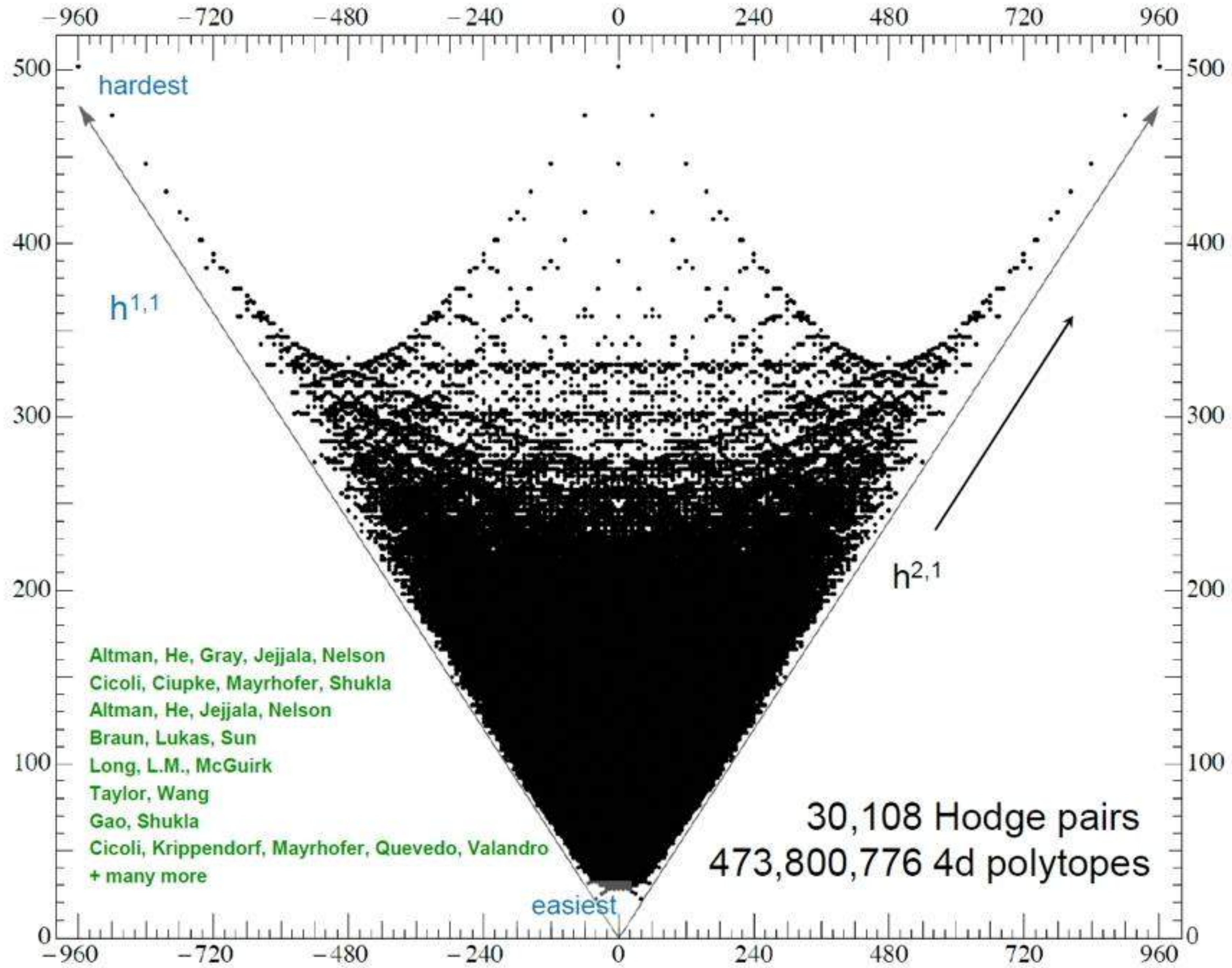
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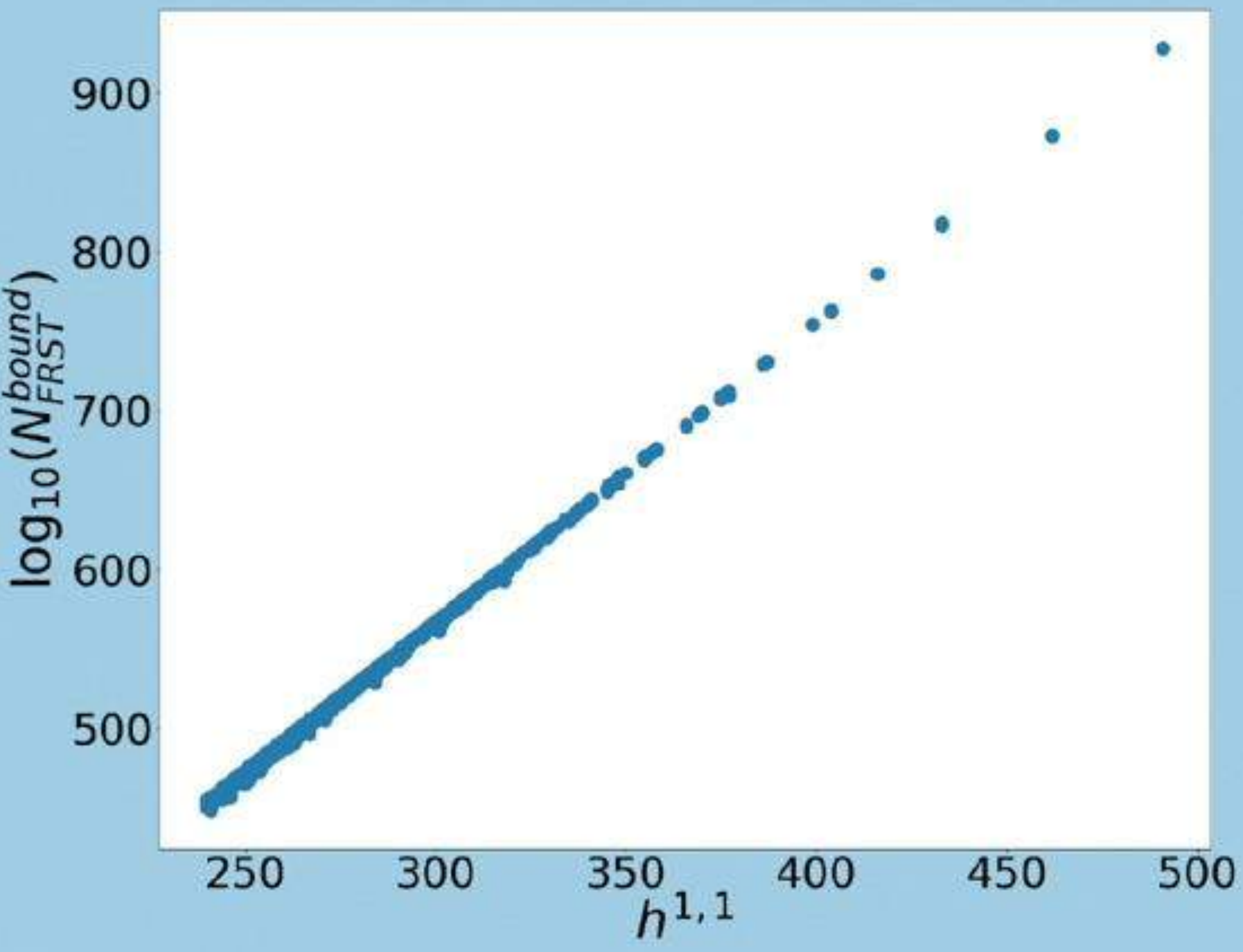


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  - e.g., three large dimensions, Standard Model gauge group, …
- If an overwhelming fraction of these solutions also have some property P,
  - e.g., more than one species of dark matter particle

  then a reasonable null hypothesis is that P holds in our universe.
- This ‘counting measure’ prediction excludes selection effects during cosmic (pre)history.
- Observing P would be a successful prediction.
- Observing not-P would mean our universe involves fine-tuning, and/or dynamical selection, and/or is not described by solutions of string theory obeying C.

# Number of FRSTs

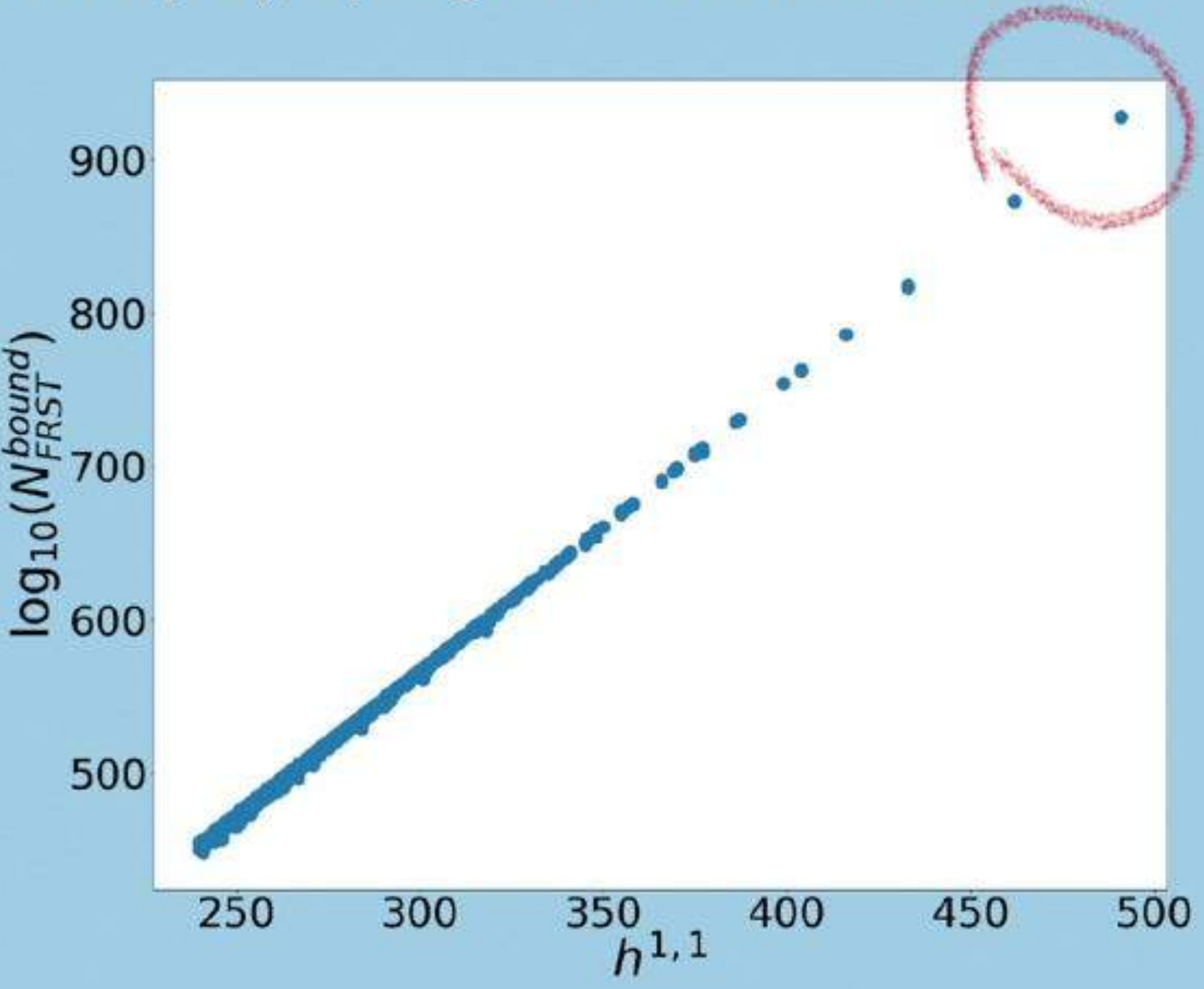
Number of triangulations/polytope grows combinatorially.

$$N_{\text{FRST}} \leq \binom{4V-1}{h^{1,1}+3}$$

V: volume of polytope

$$N_{\text{FRST}} \leq \binom{14{,}111}{494} \approx 1.5 \times 10^{928}$$

$$N_{\text{FRST}} \leq \binom{13{,}271}{465} \approx 2.7 \times 10^{863}$$




cf. Altman, Carifio, Halverson, Nelson

Demirtas, Rios Tascon, L.M.

# Why count solutions?

- Suppose string theory has a large but finite number $\mathscr{N}$ of isolated solutions that fulfill a set of criteria C.
  - e.g., three large dimensions, Standard Model gauge group, …
- If an overwhelming fraction of these solutions also have some property P,
  - e.g., more than one species of dark matter particle

  then a reasonable null hypothesis is that P holds in our universe.
- This ‘counting measure’ prediction excludes selection effects during cosmic (pre)history.
- Observing P would be a successful prediction.
- Observing not-P would mean our universe involves fine-tuning, and/or dynamical selection, and/or is not described by solutions of string theory obeying C.

# Number of FRSTs

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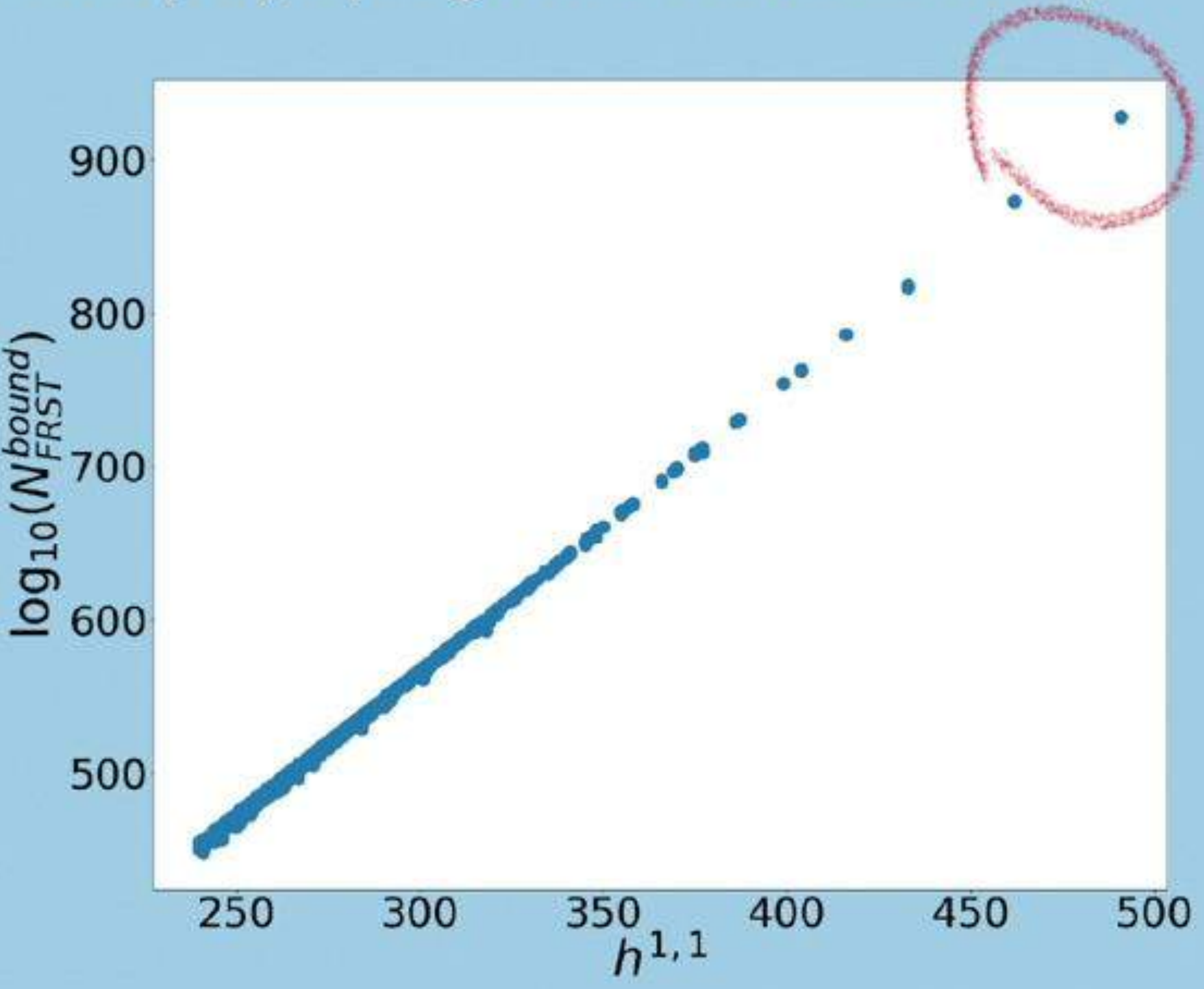



cf. Altman, Carifio, Halverson, Nelson

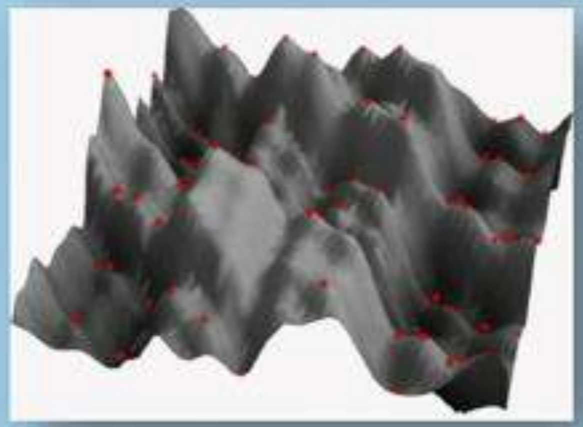
Demirtas, Rios Tascon, L.M.

# Example of discrete parameters

1. Choose a 4d reflexive polytope $\Delta \subset \mathbb{Z}^4$ 473,800,776
2. Choose a fine star regular triangulation of $\Delta$, determining a toric variety V, and a $CY_3$ hypersurface $X \subset V$.
3. Choose quantized fluxes, $\vec{f}, \vec{h} \in H_3(X, \mathbb{Z}) \cong \mathbb{Z}^{2h^{2,1}(X)}$

4. Choose D7-branes $\{\vec{w}_A\} \in H_4(X, \mathbb{Z}) \cong \mathbb{Z}^{h^{1,1}(X)}$
5. For each D7-brane, choose bundle $\vec{\mathcal{F}}_A \in H^2(w_A, \mathbb{Z})$

One can then compute the potential, V.



# $h^{1,1} = 491$ Polytope

$$\begin{pmatrix} -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 2 & -1 \\ -1 & -1 & 6 & -1 & -1 \\ -1 & 83 & -1 & -1 & -1 \end{pmatrix}$$

# $h^{1,1} = 491$ Polytope

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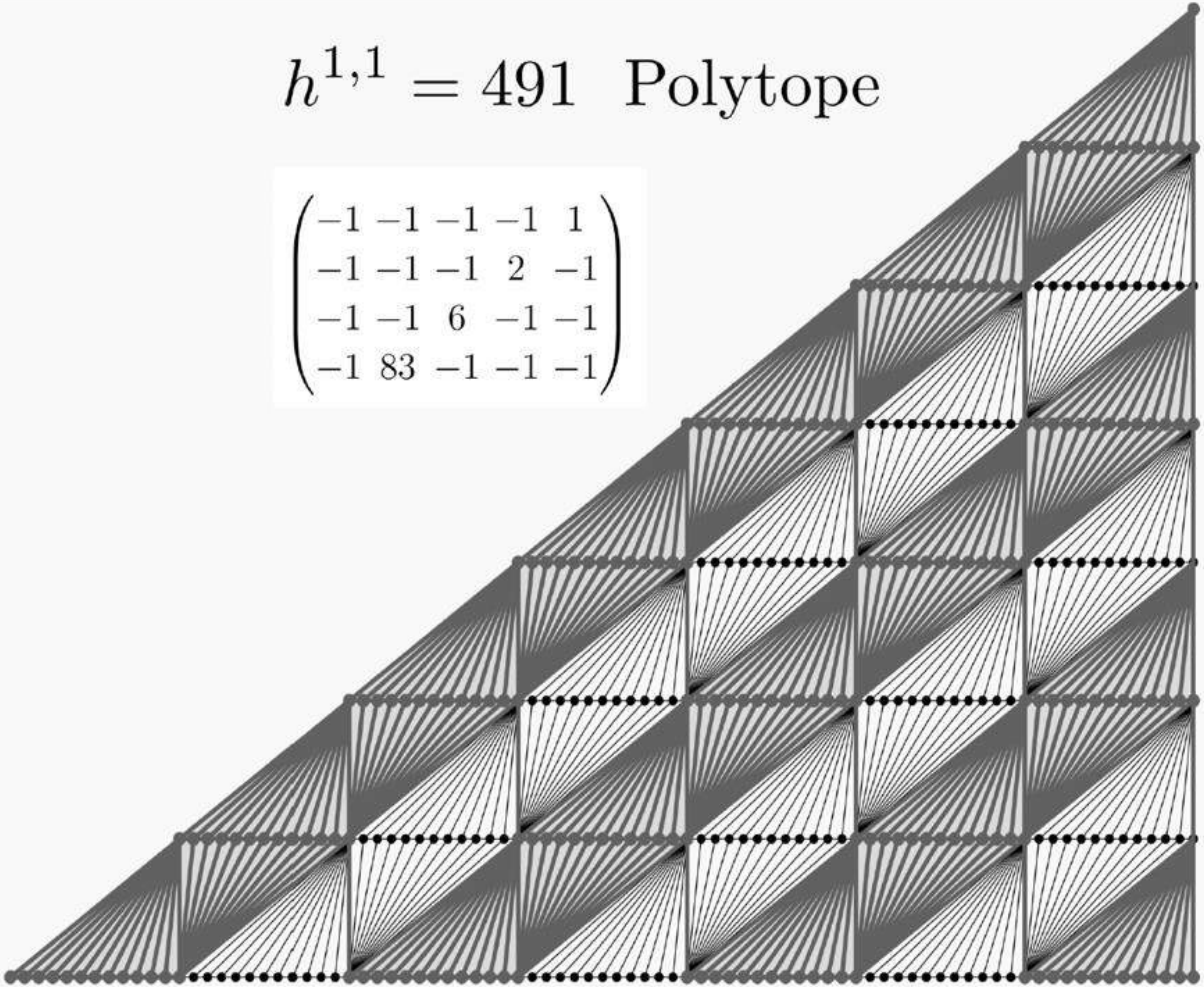


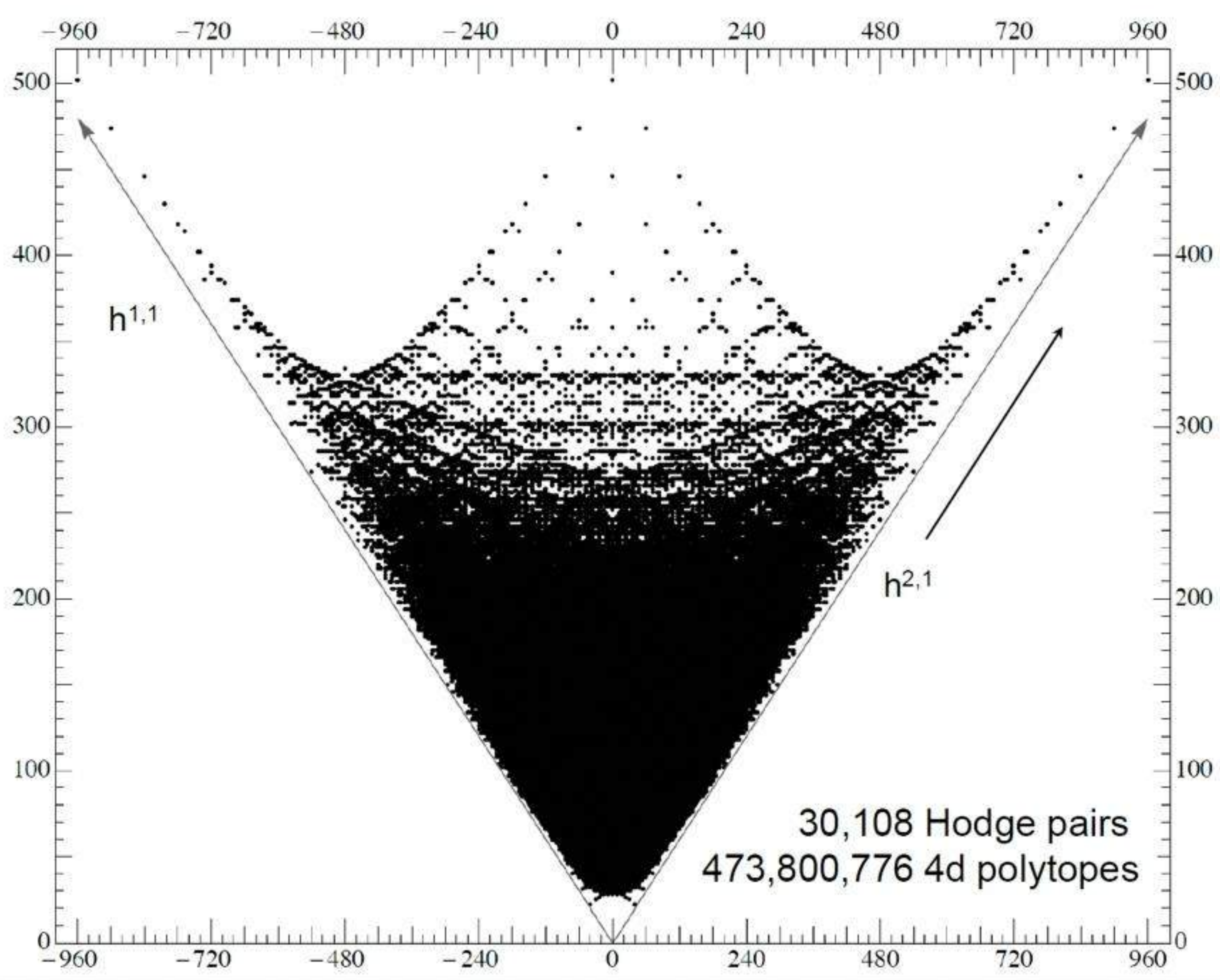
# Plan

I. Invitation to quantum gravity

II. Quantized parameters and the string landscape

III. Example: toric hypersurfaces

IV. Targets for ML



−960
−720
−480
−240
0
240
480
720
960
500
400
300
200
100
0
h^{1,1}
h^{2,1}
30,108 Hodge pairs
473,800,776 4d polytopes

# Target 1: Count Vacua

- Estimate or bound the number of triangulations of a polytope, when this is so large that direct enumeration is impossible.

# Target 2: Find Desirable Vacua

- Given a polytope, find triangulations with a desired feature
  - e.g., top eigenvalue of metric > threshold
  - e.g., given pattern of intersections of cycles
- 'With' a feature F: $P(F) \gg P_{\mathrm{median}}(F)$
- Most features verifiable in ~seconds.

# Target 3: Predictions from Polytopes

- Given a polytope, predict features that hold for all triangulations.
- Or, for all triangulations in a new class that one identifies.

$$473{,}800{,}776 \text{ polytopes vs. } 10^{928}(??) \text{ triangulations}$$

# Summary of task

- We face a finite but extremely large set, the ($CY_3$ hypersurface subset of the) string landscape.
- We wish to evaluate various fitness functions on this set, and find local or global maxima.
- Milliseconds to seconds per evaluation.
- There are many correlations in this data, but there are not enough graduate students to extract them.
- Would be extremely helpful to find and exploit patterns via ML!
- Immense array of related problems, here and in other regions of the landscape.

Thanks!