

# Abstraction in Reinforcement Learning

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# Systematized Problem Solving

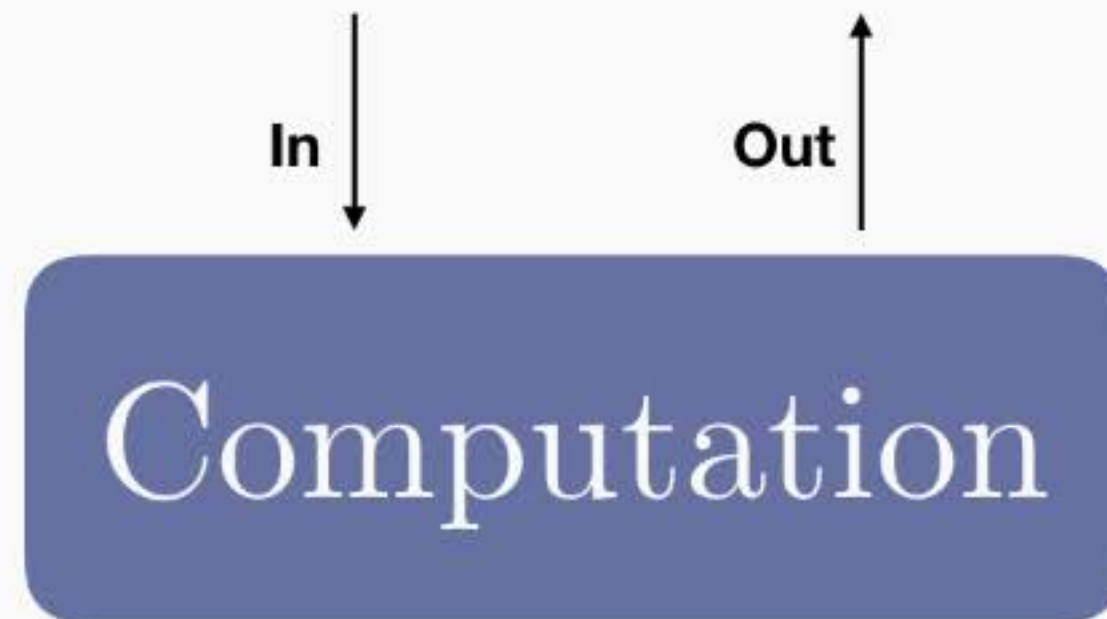
Computation

# Systematized Problem Solving

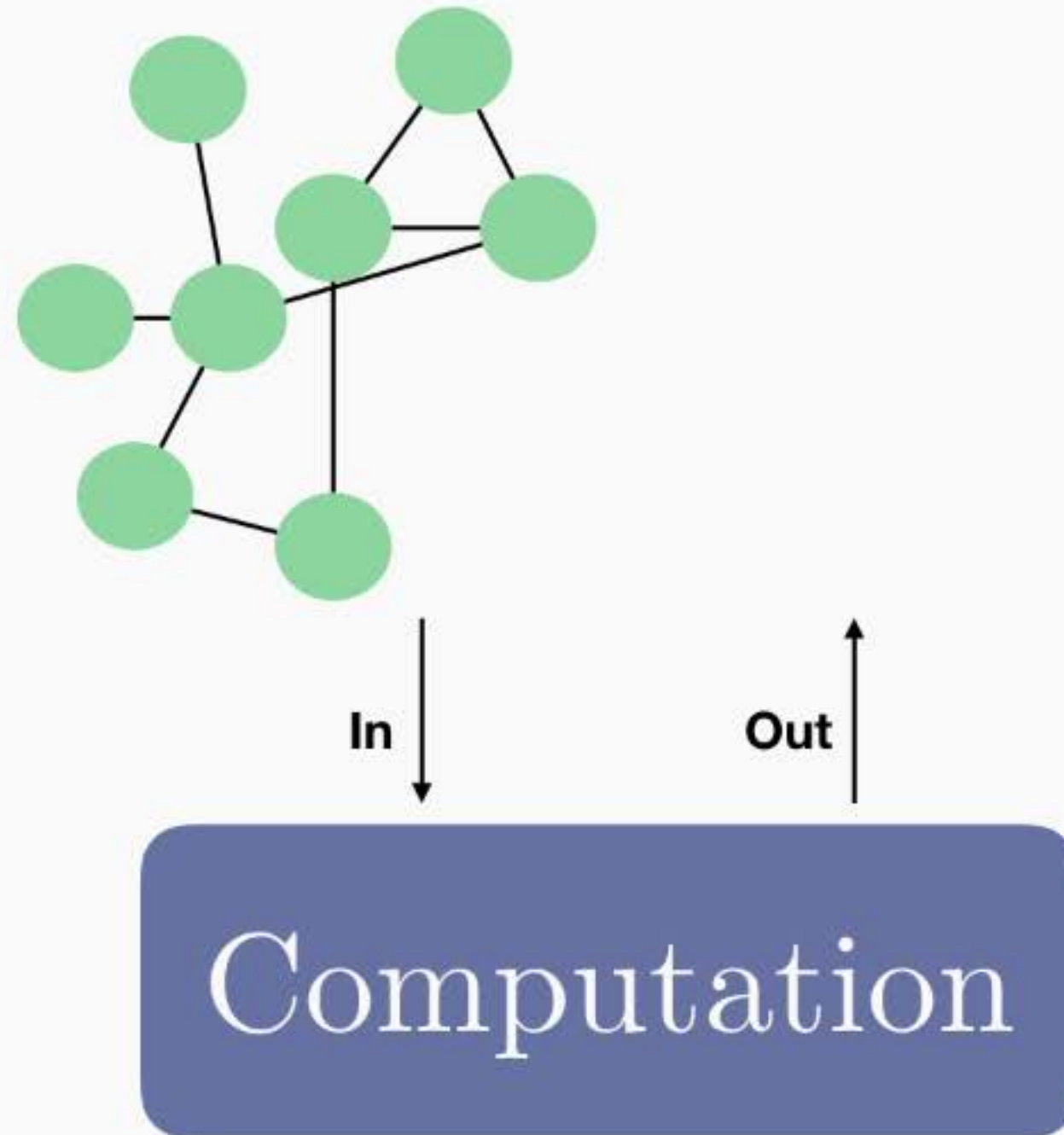


Computation

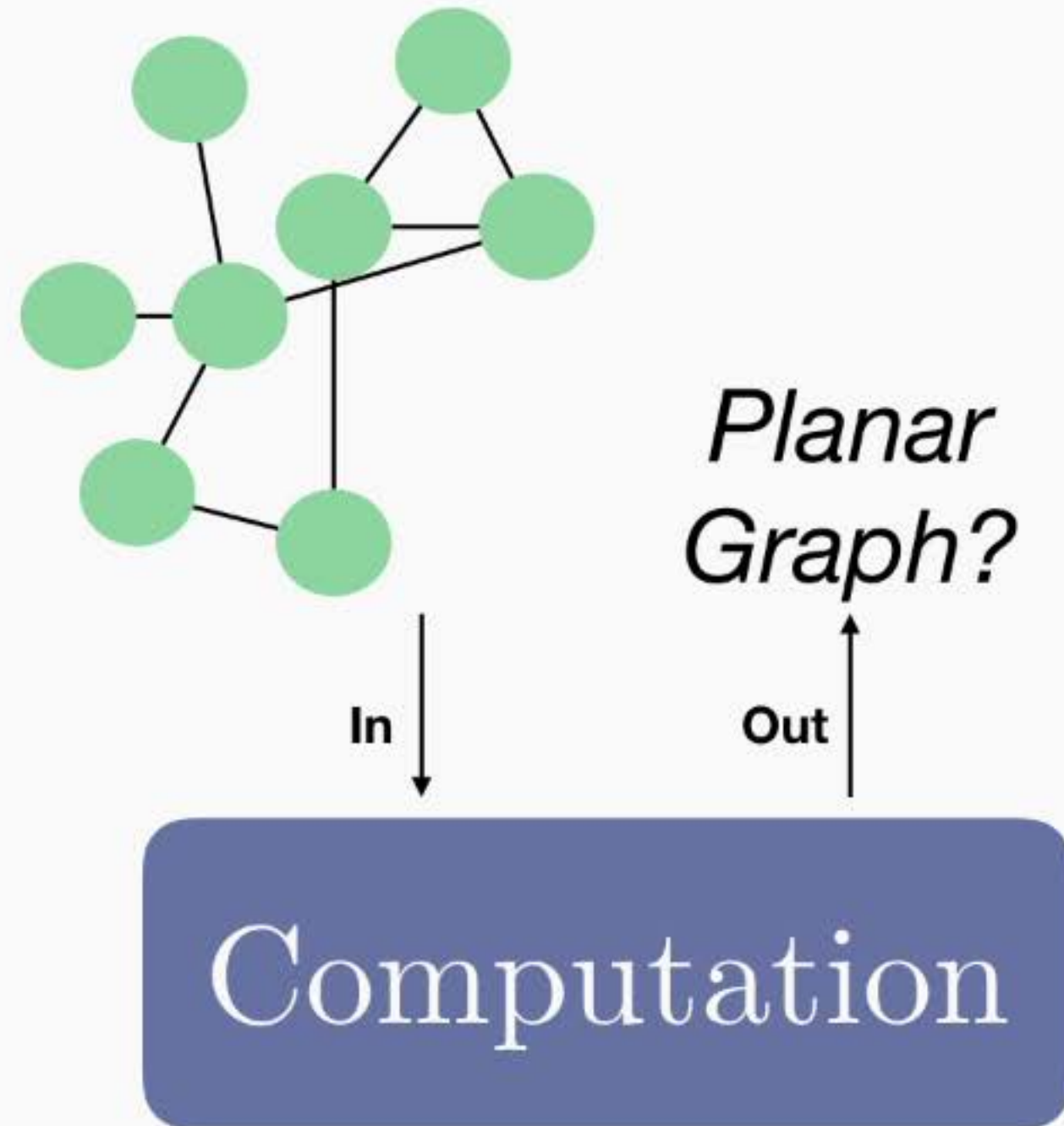
# Systematized Problem Solving



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# Systematized Problem Solving



*Planar  
Graph?*

In

Out

Computation

# Systematized Problem Solving



*Motor  
Controls*

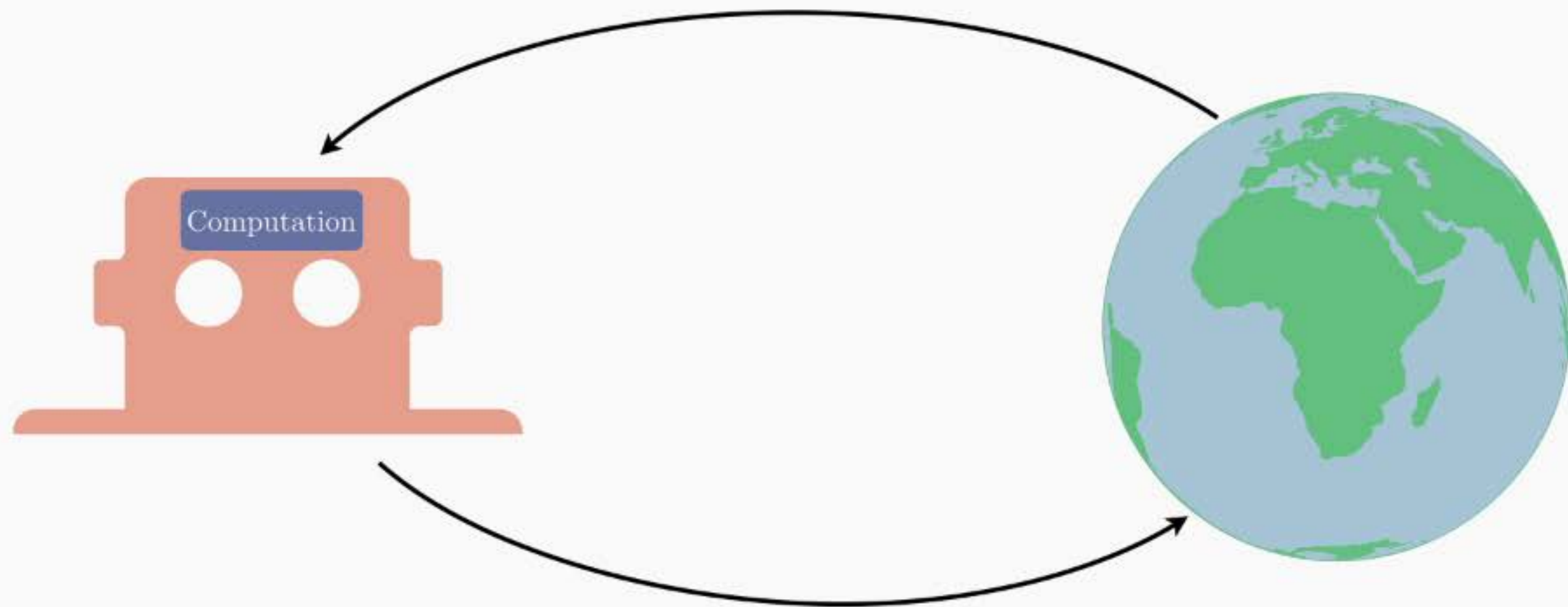
In

Out

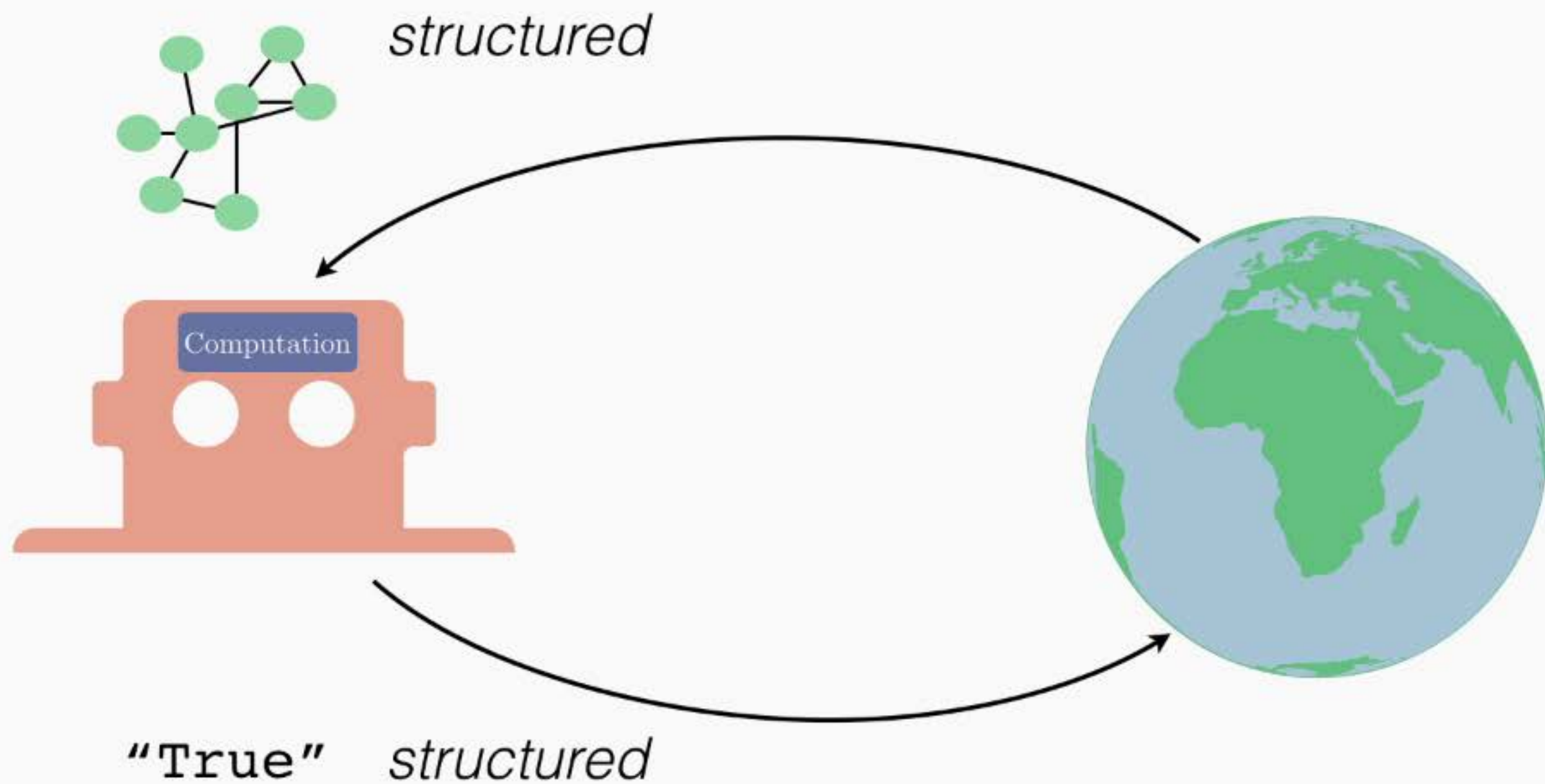
Computation



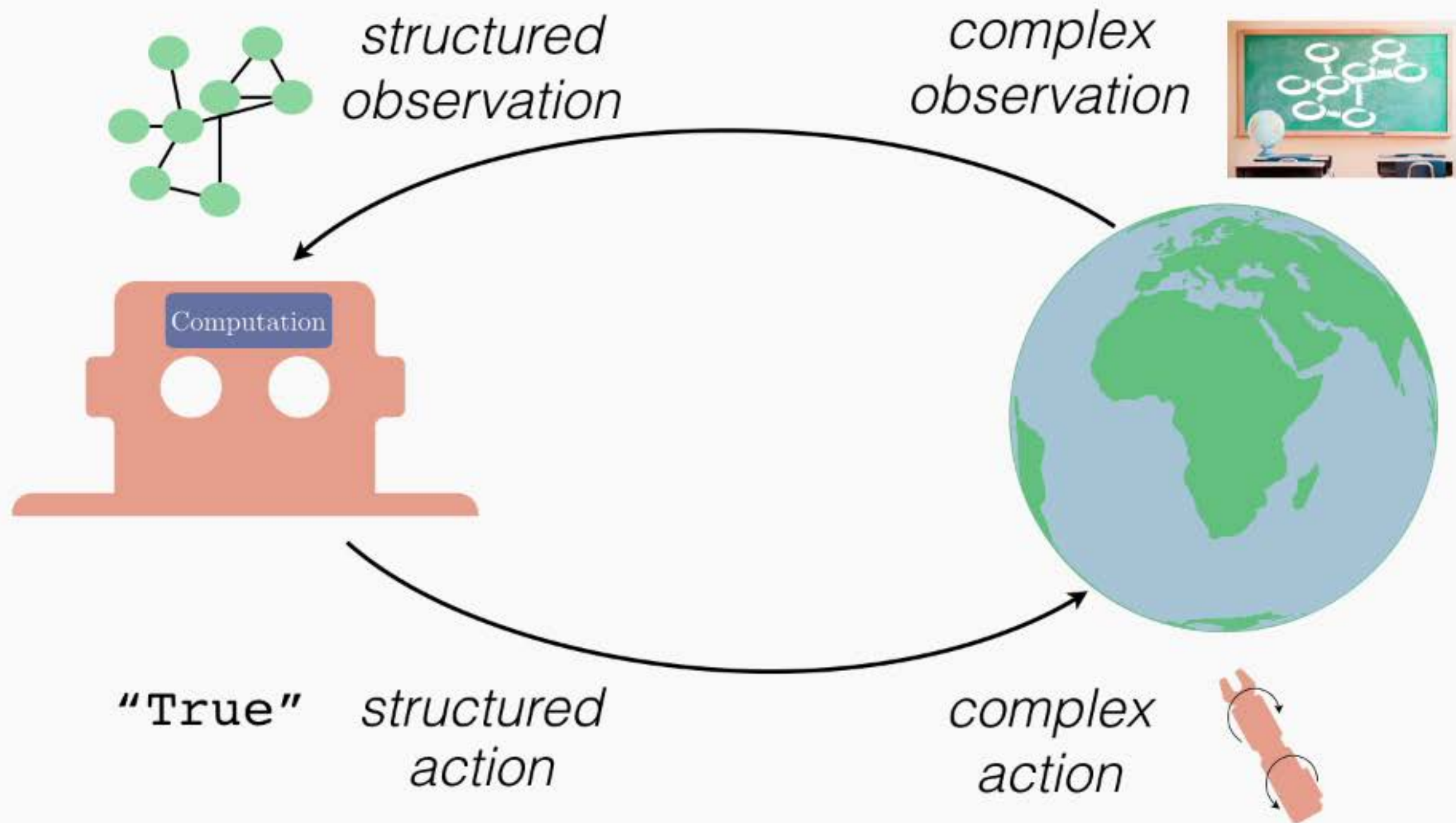
# RL as Learning to Solve Problems



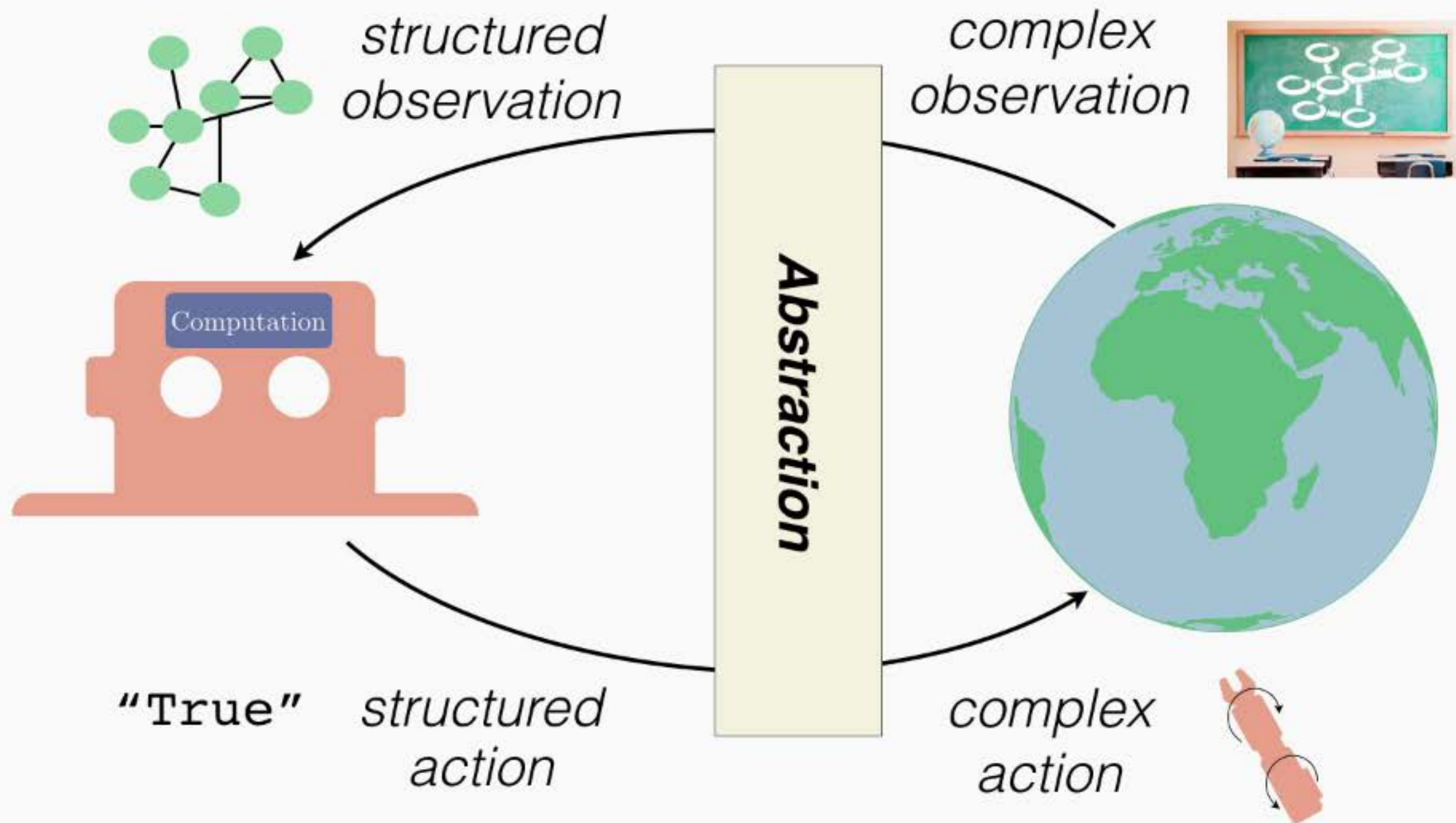
# RL as Learning to Solve Problems



# RL as Learning to Solve Problems



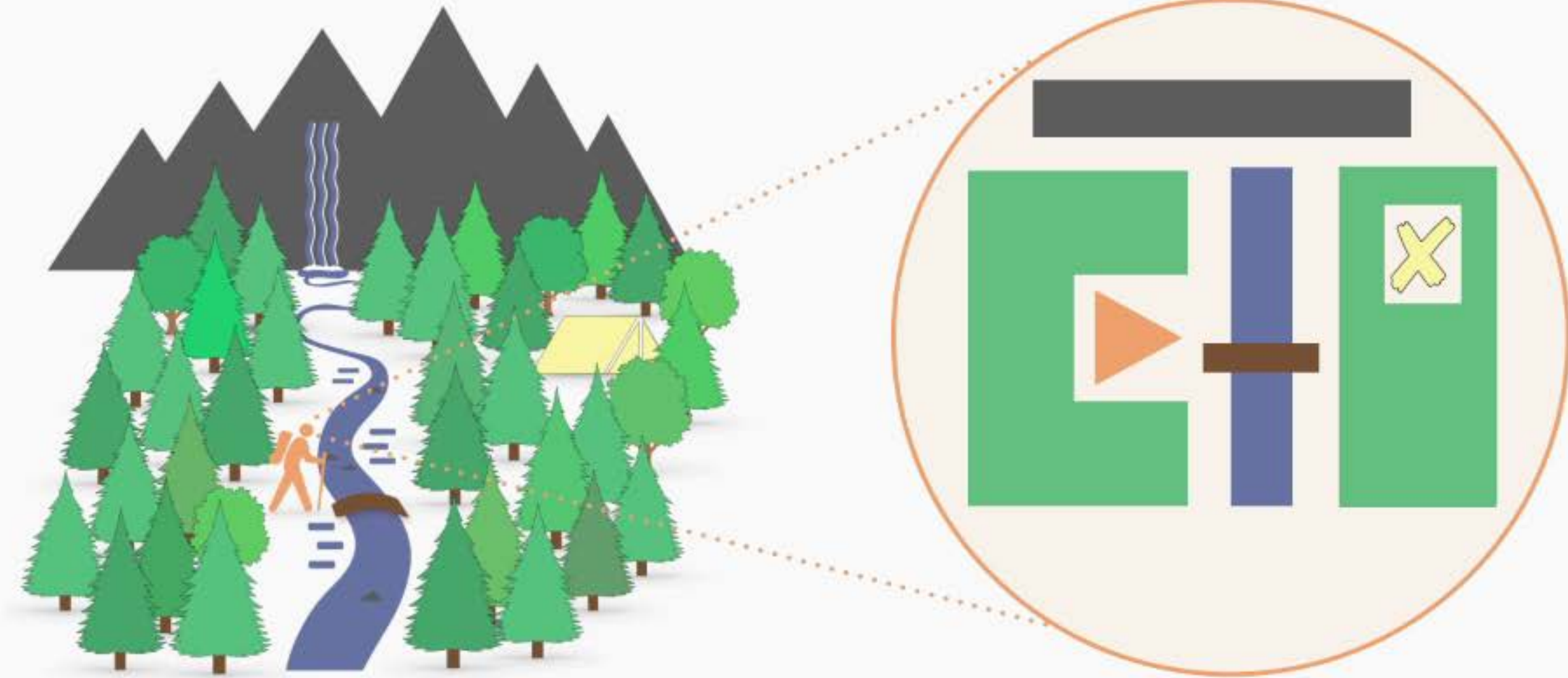
# RL as Learning to Solve Problems



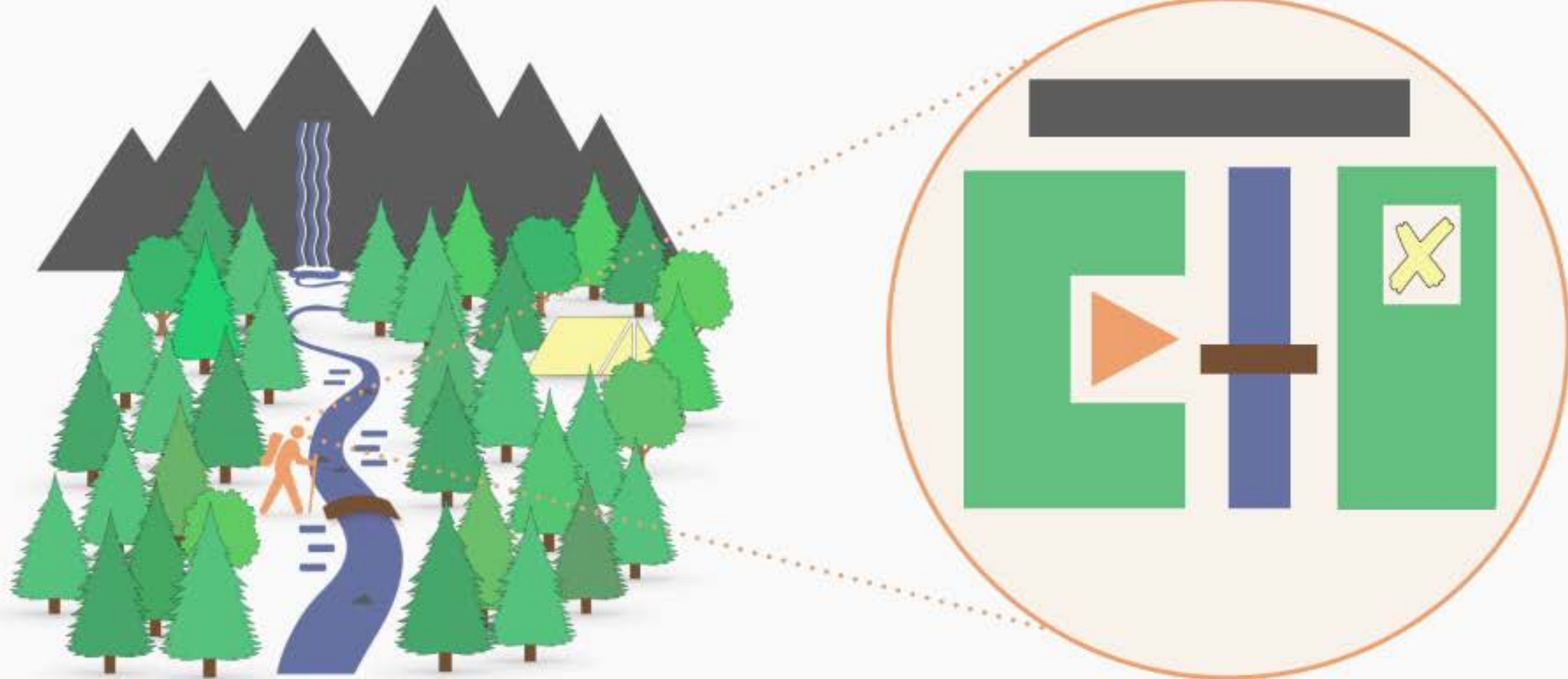
# Abstraction



# Abstraction

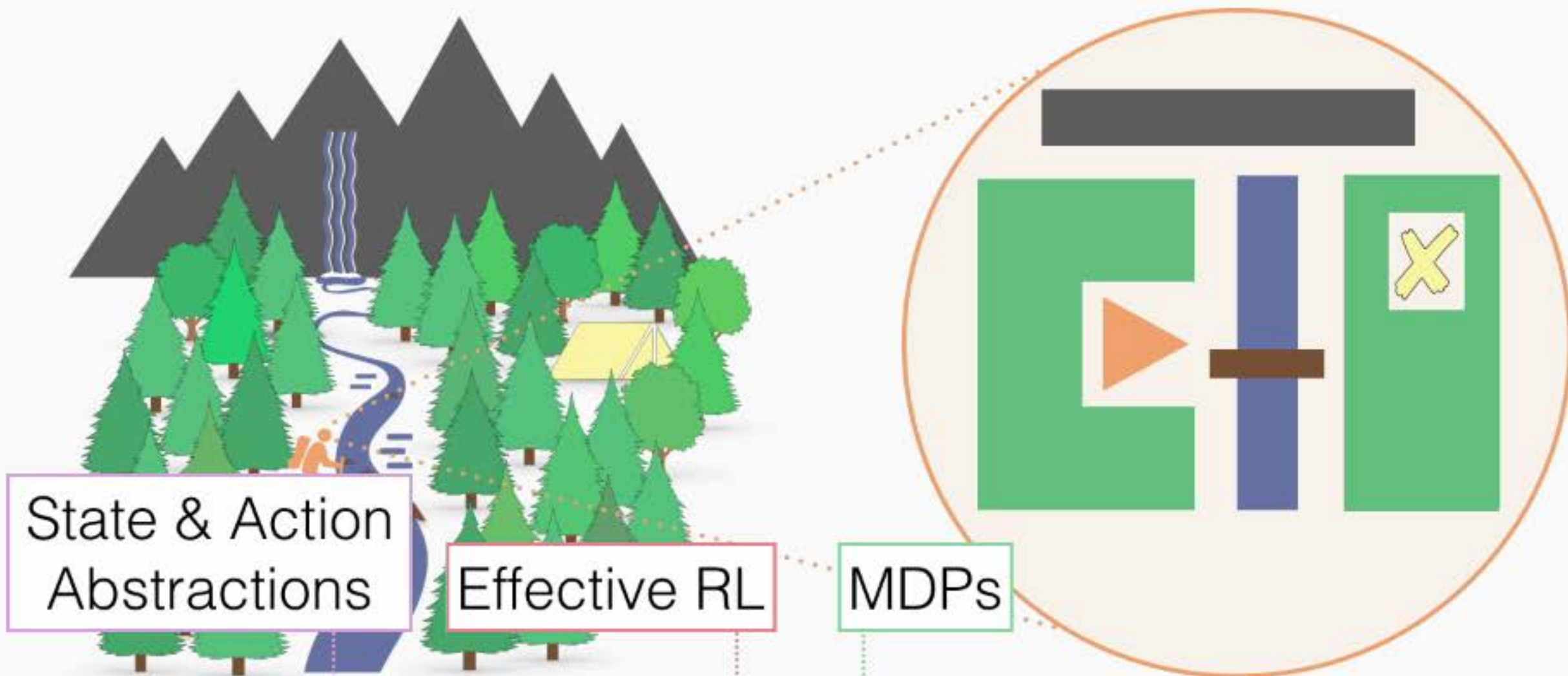


# Abstraction



**Question:** *How do intelligent agents come up with the right abstract understanding of the worlds they inhabit?*

# Abstraction



State & Action Abstractions

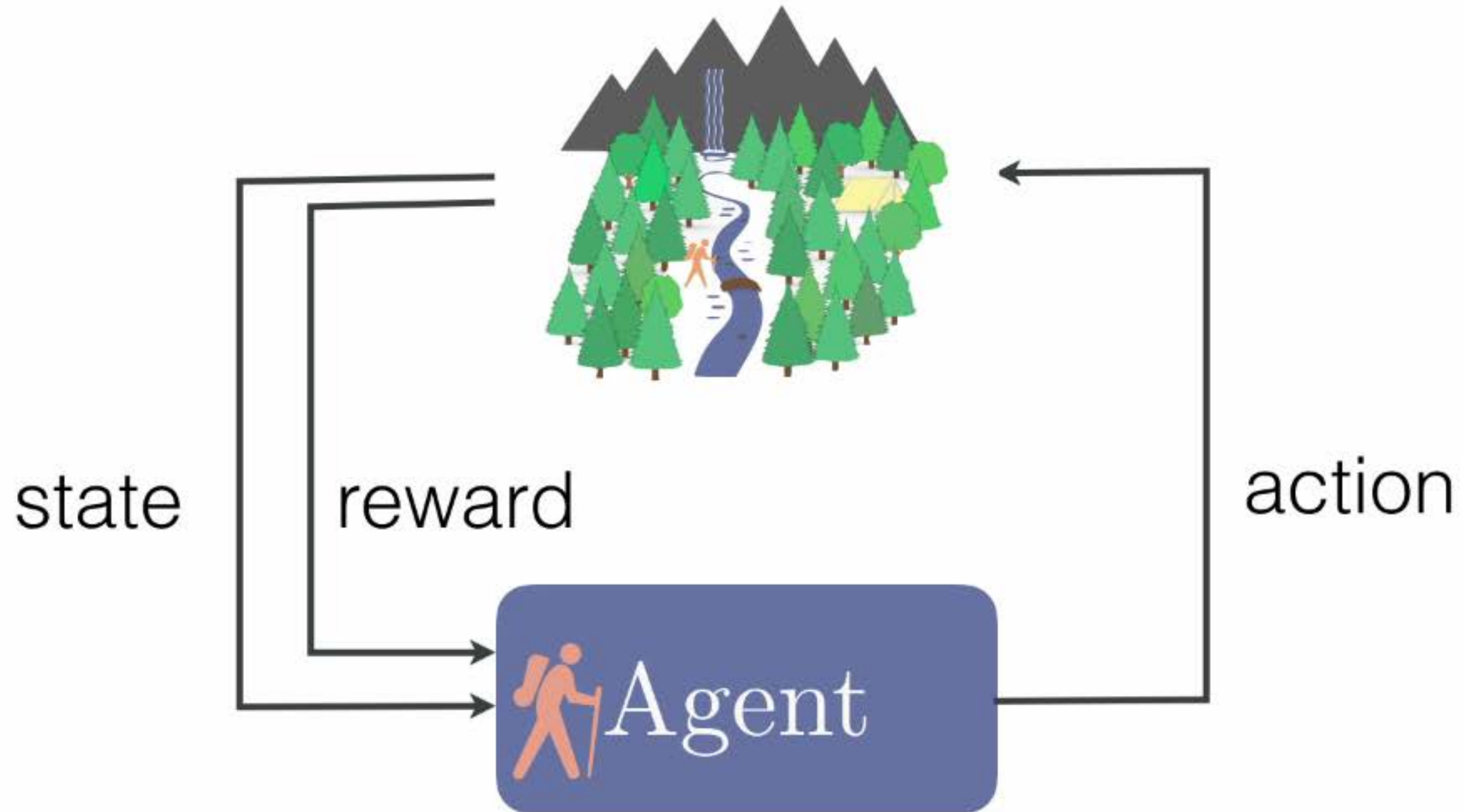
Effective RL

MDPs

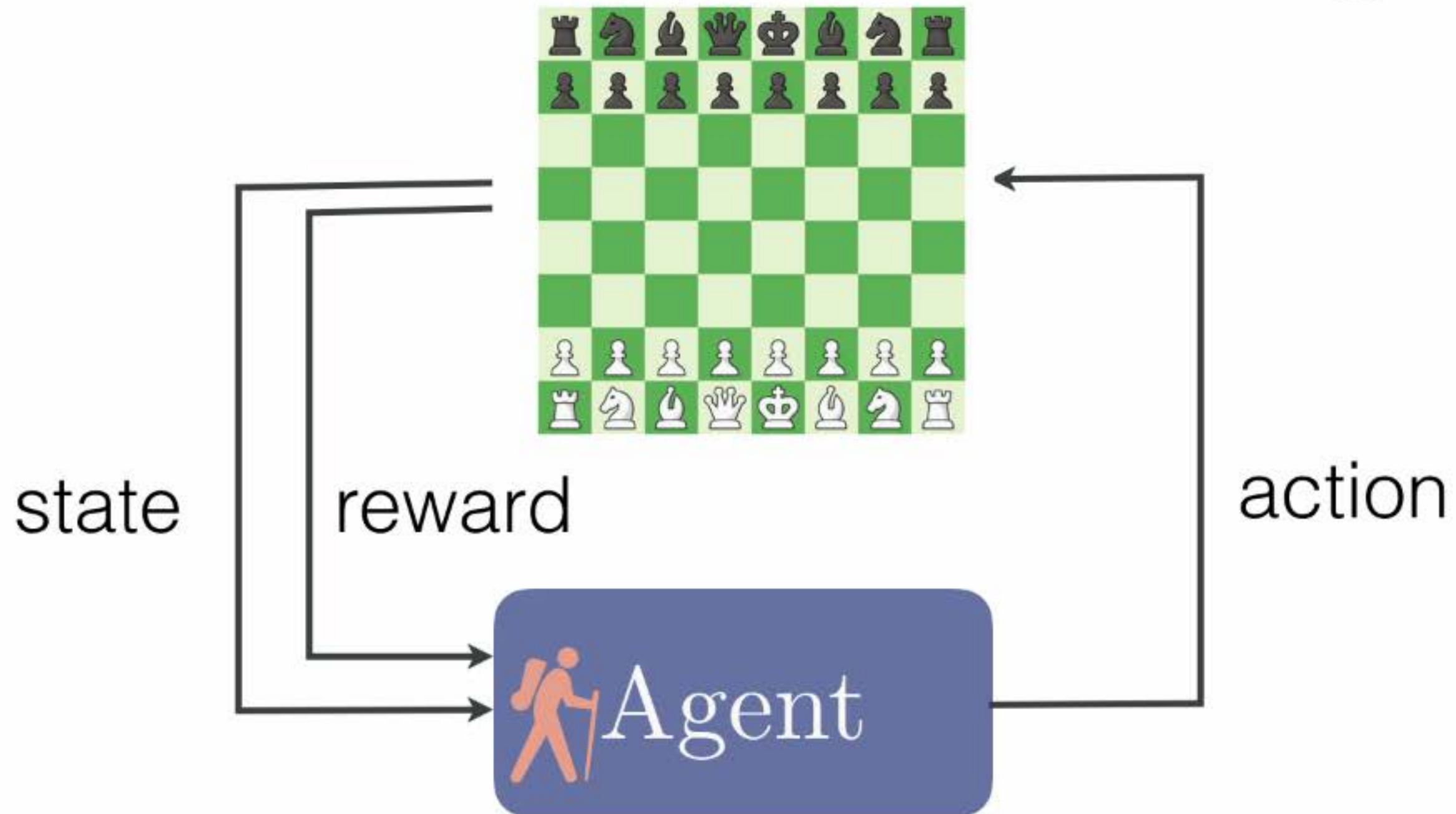
**Question:** How do intelligent agents come up with the right abstract understanding of the worlds they inhabit?



# Reinforcement Learning



# Reinforcement Learning



# Reinforcement Learning

Central Formalism: ***Markov Decision Process*** (MDP):

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# Reinforcement Learning

Central Formalism: *Markov Decision Process (MDP)*:

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$\mathcal{S}$

A set of states.



...

# Reinforcement Learning

Central Formalism: *Markov Decision Process (MDP)*:

---

$S$

A set of states.

$A$

A set of actions.





# Reinforcement Learning

Central Formalism: *Markov Decision Process (MDP)*:

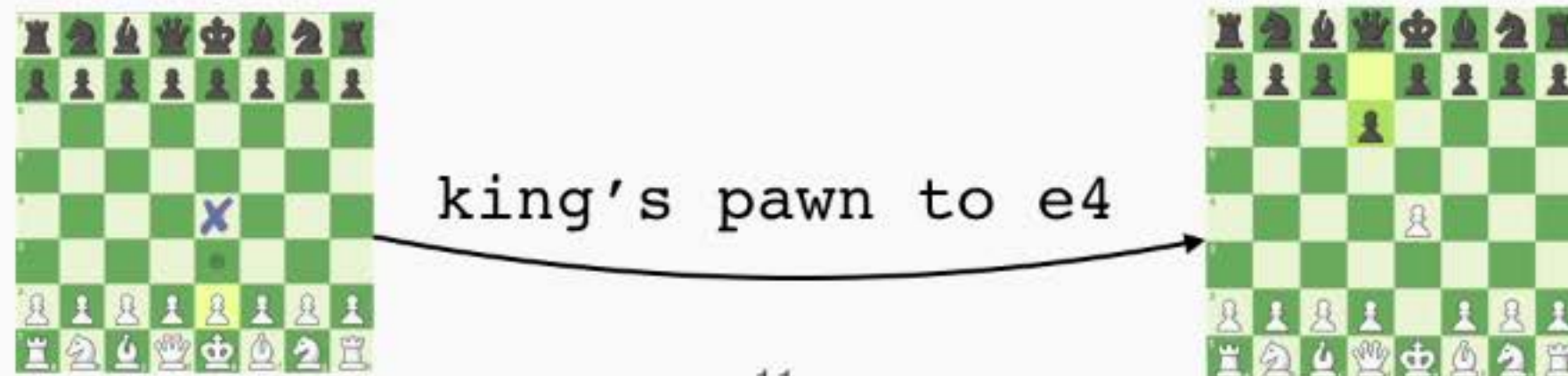
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$\mathcal{S}$  A set of states.

$\mathcal{A}$  A set of actions.

$\mathcal{R} : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$  A reward function.

$\mathcal{T} : \mathcal{S} \times \mathcal{A} \rightarrow \text{Pr}(\mathcal{S})$  A transition function.



# Reinforcement Learning

Central Formalism: *Markov Decision Process (MDP)*:

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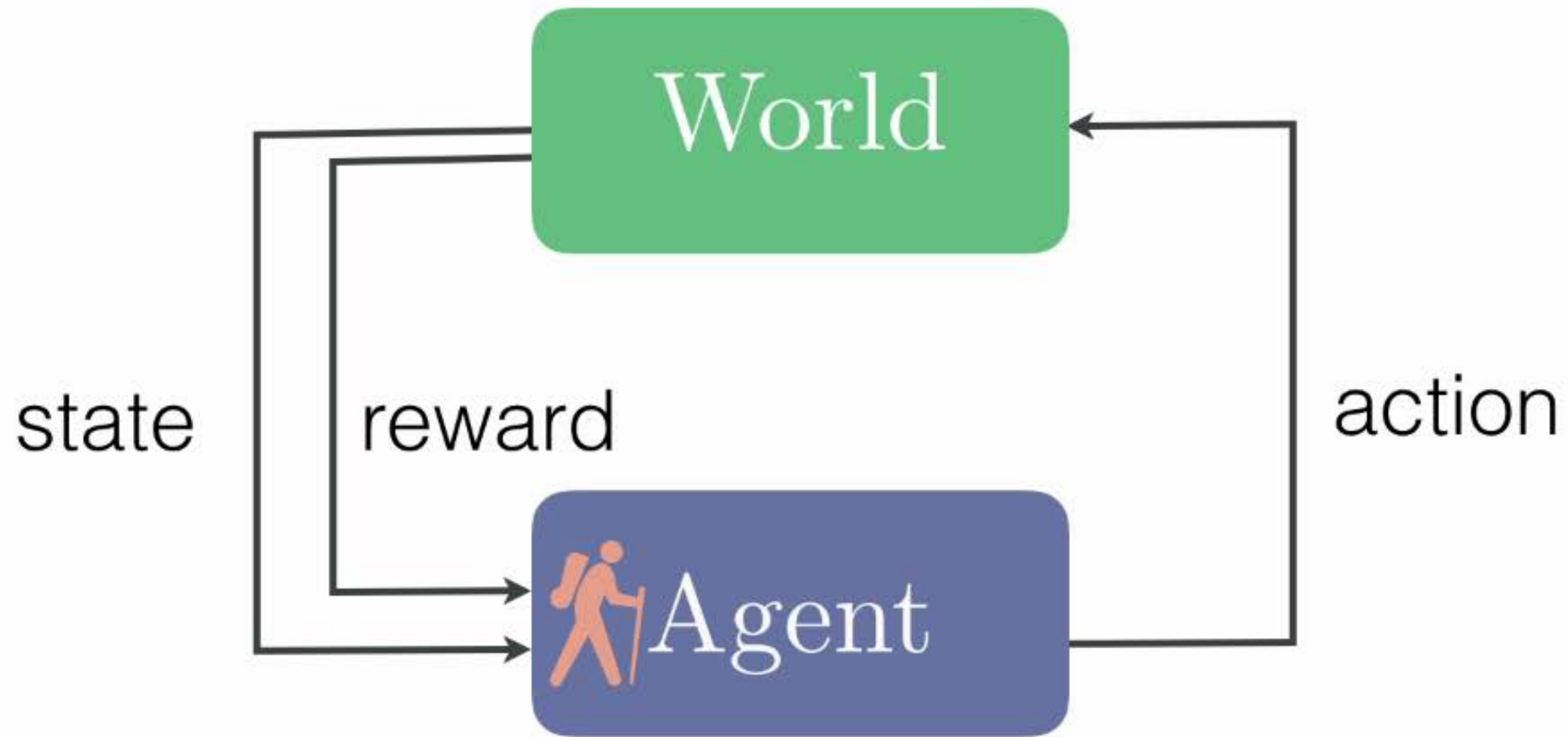
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$\gamma \in [0, 1)$	A discount factor.

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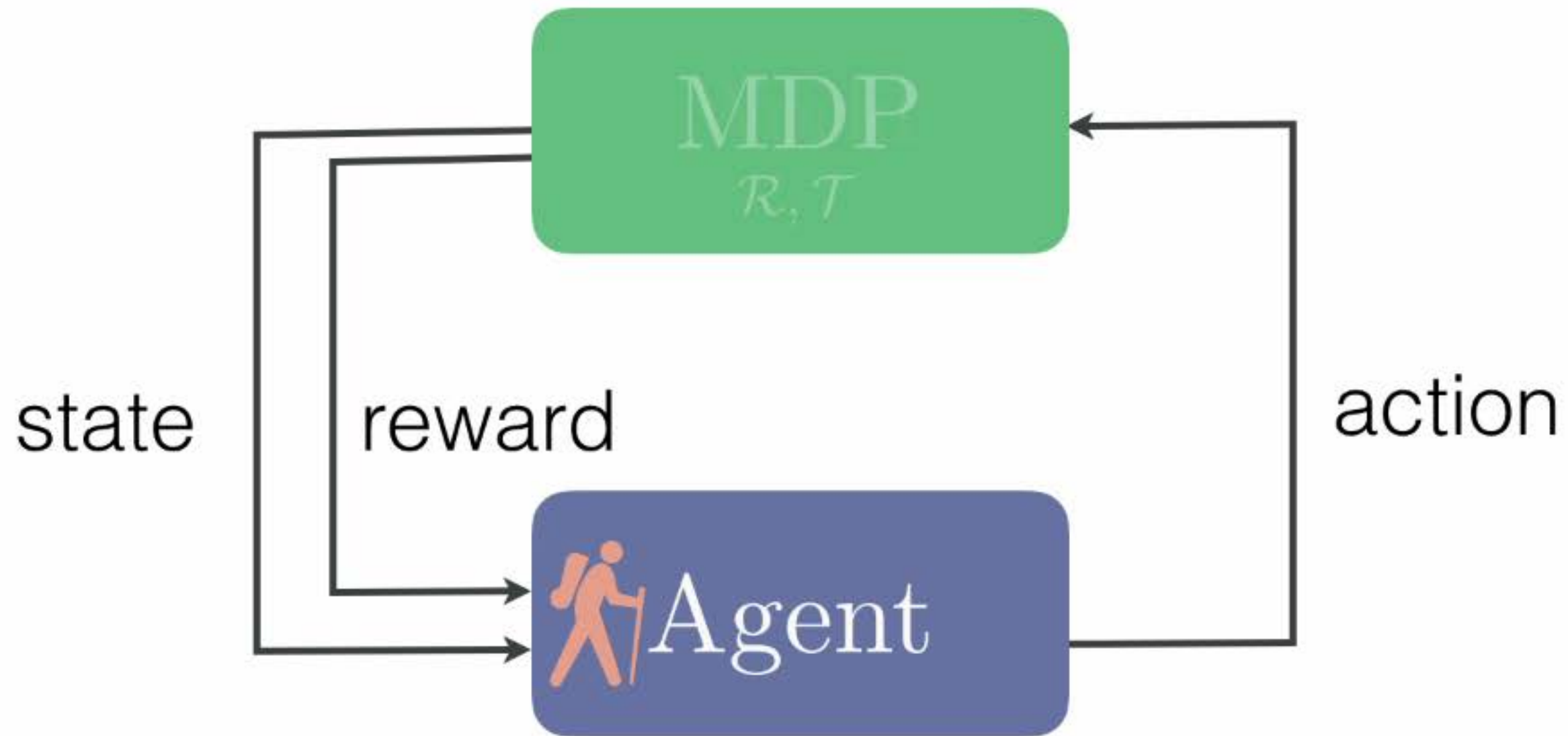




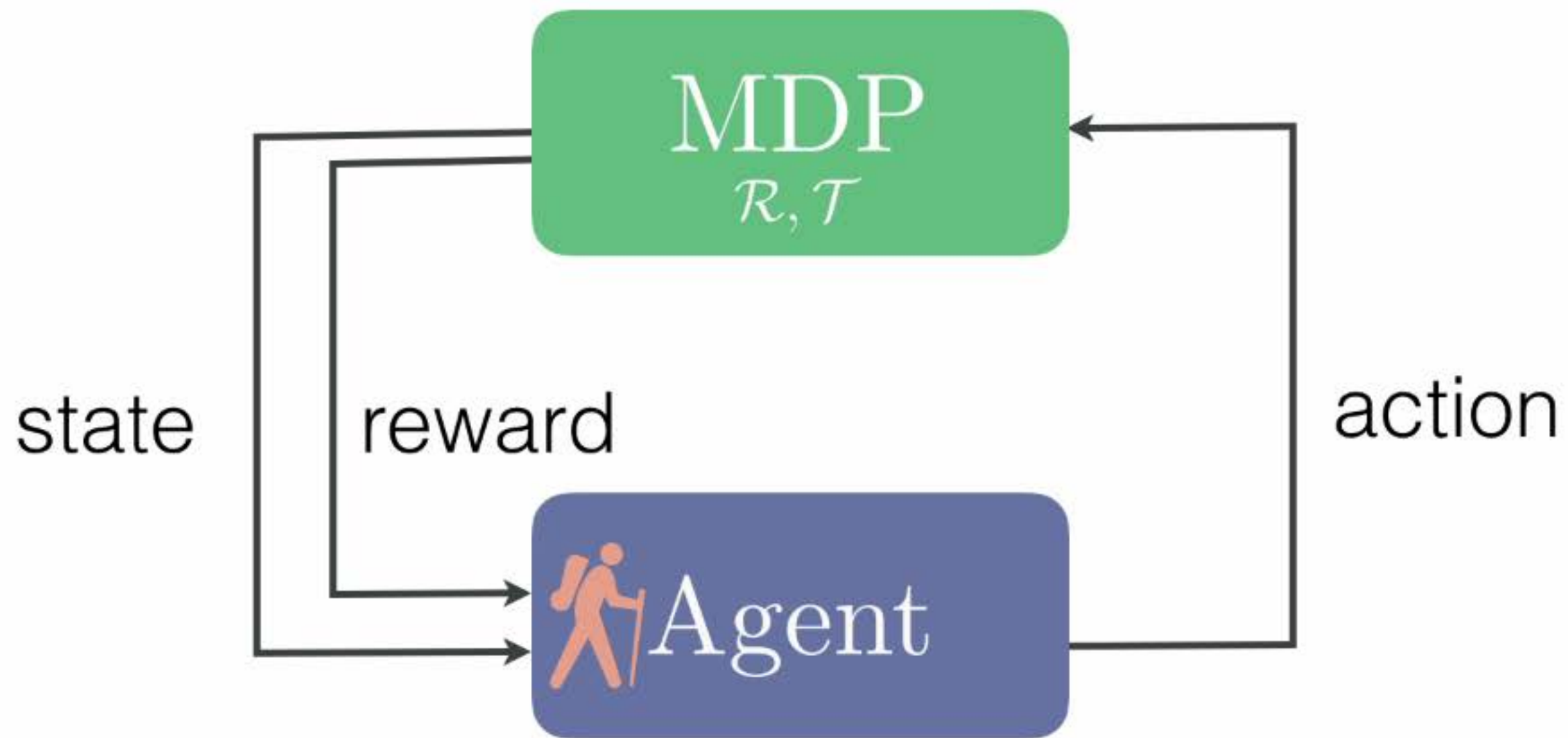
# Reinforcement Learning



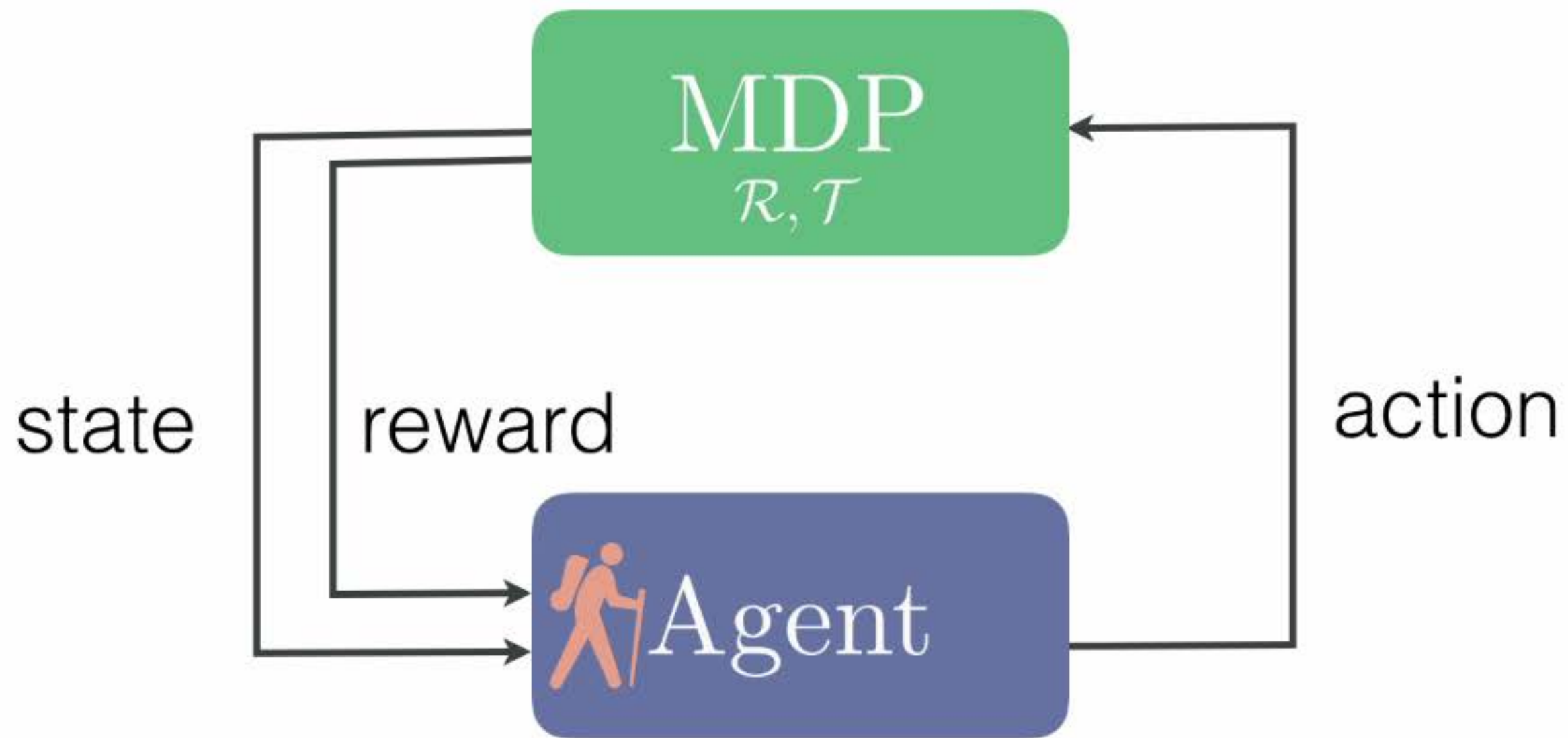
# Reinforcement Learning



# Reinforcement Learning



# Reinforcement Learning



**Goal:** Maximize long term expected reward.

# Reinforcement Learning

Policy:

$$\pi : \mathcal{S} \rightarrow \mathcal{A}$$

**Goal:** Maximize long term expected reward.

# Reinforcement Learning

Policy:  $\pi : \mathcal{S} \rightarrow \mathcal{A}$

Value:  $V^\pi(s) = \mathcal{R}(s, \pi(s)) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{T}(s, \pi(s), s') V^\pi(s')$

**Goal:** Maximize long term expected reward.

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Immediate  
Reward

**Goal:** Maximize long term expected reward.

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Immediate Reward

Discounted Expected Future Reward

**Goal:** Maximize long term expected reward.



# Reinforcement Learning

Policy:  $\pi : \mathcal{S} \rightarrow \mathcal{A}$

Value:  $V^\pi(s) = \mathcal{R}(s, \pi(s)) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{T}(s, \pi(s), s') V^\pi(s')$

Action Value:  $Q^\pi(s, a) = \mathcal{R}(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{T}(s, a, s') V^\pi(s')$

**Goal:** Maximize long term expected reward.

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Immediate Reward      Discounted Expected Future Reward

**Goal:** Maximize long term expected reward.

# Reinforcement Learning

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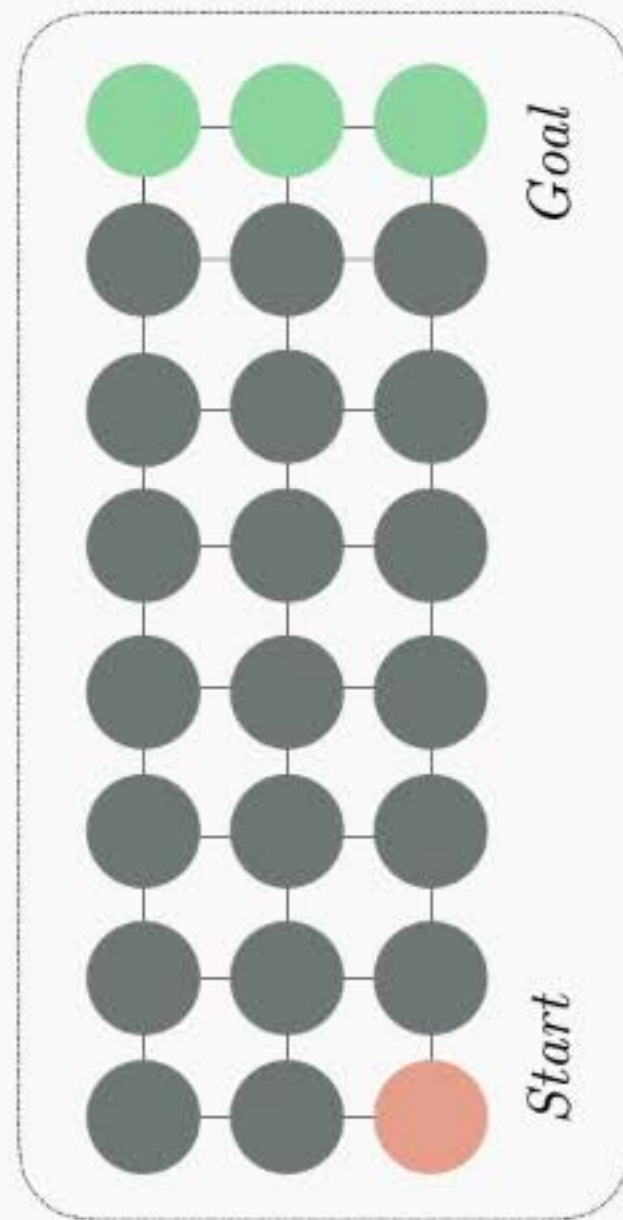
Value:  $V^\pi(s) = \mathcal{R}(s, \pi(s)) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{T}(s, \pi(s), s') V^\pi(s')$

Action Value:  $Q^\pi(s, a) = \mathcal{R}(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathcal{T}(s, a, s') V^\pi(s')$

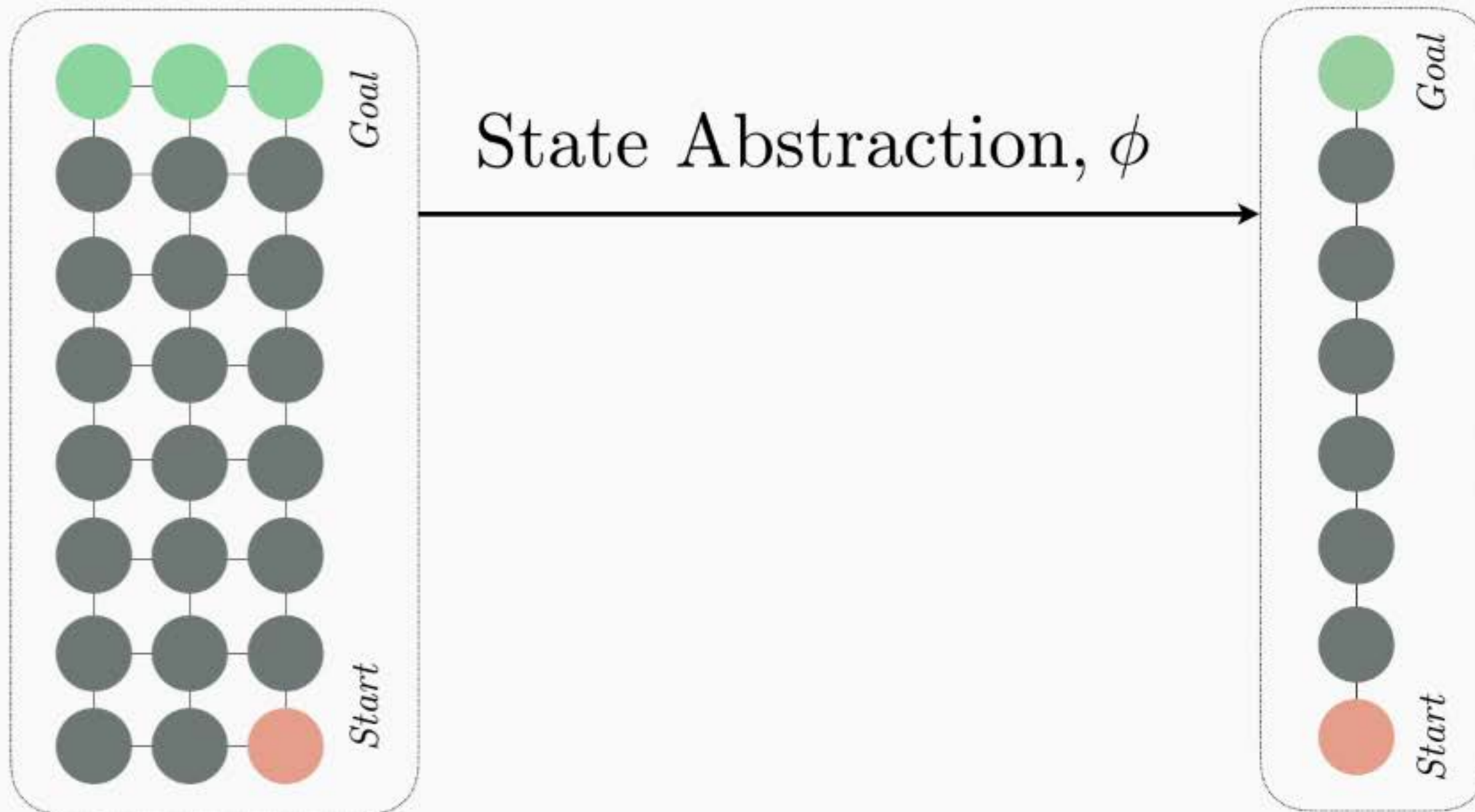
Optimal:  $V^*(s) = \max_{\pi} V^\pi(s)$        $Q^*(s, a) = \max_{\pi} Q^\pi(s, a)$

**Goal:** Maximize long term expected reward.

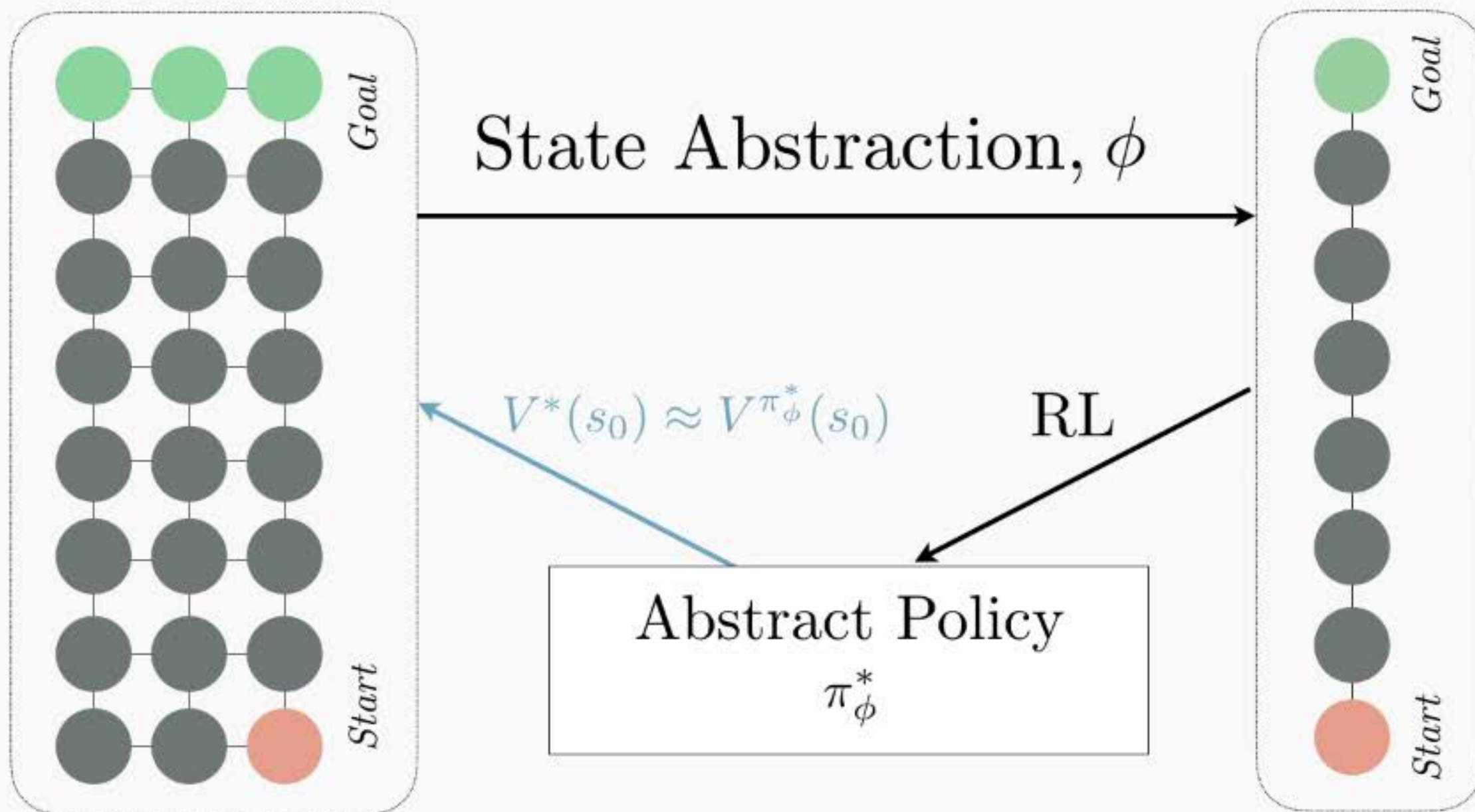
# State Abstraction



# State Abstraction



# State Abstraction



# State Abstraction

**Definition** (State Abstraction): A state abstraction,  $\phi : \mathcal{S} \rightarrow \mathcal{S}_\phi$ , is a function that maps every ground state to an abstract state.

[Fox '73]

[Whitt '78]

[Singh et al. '95]

[Dean, Givan '97]

[Dieterich '00]

[Andre, Russell '02]

[Ravindran, Barto '03, '04]

[Jong, Stone '05] [Ortner et al. '07, '14, '19]

[Ferns et al., '04, '06] [Hutter '14, '16, '19]

[Li et al. '06] [Jiang et al., '14, '15]

[Whiteson et al. '07] [Akroun et al., '18]

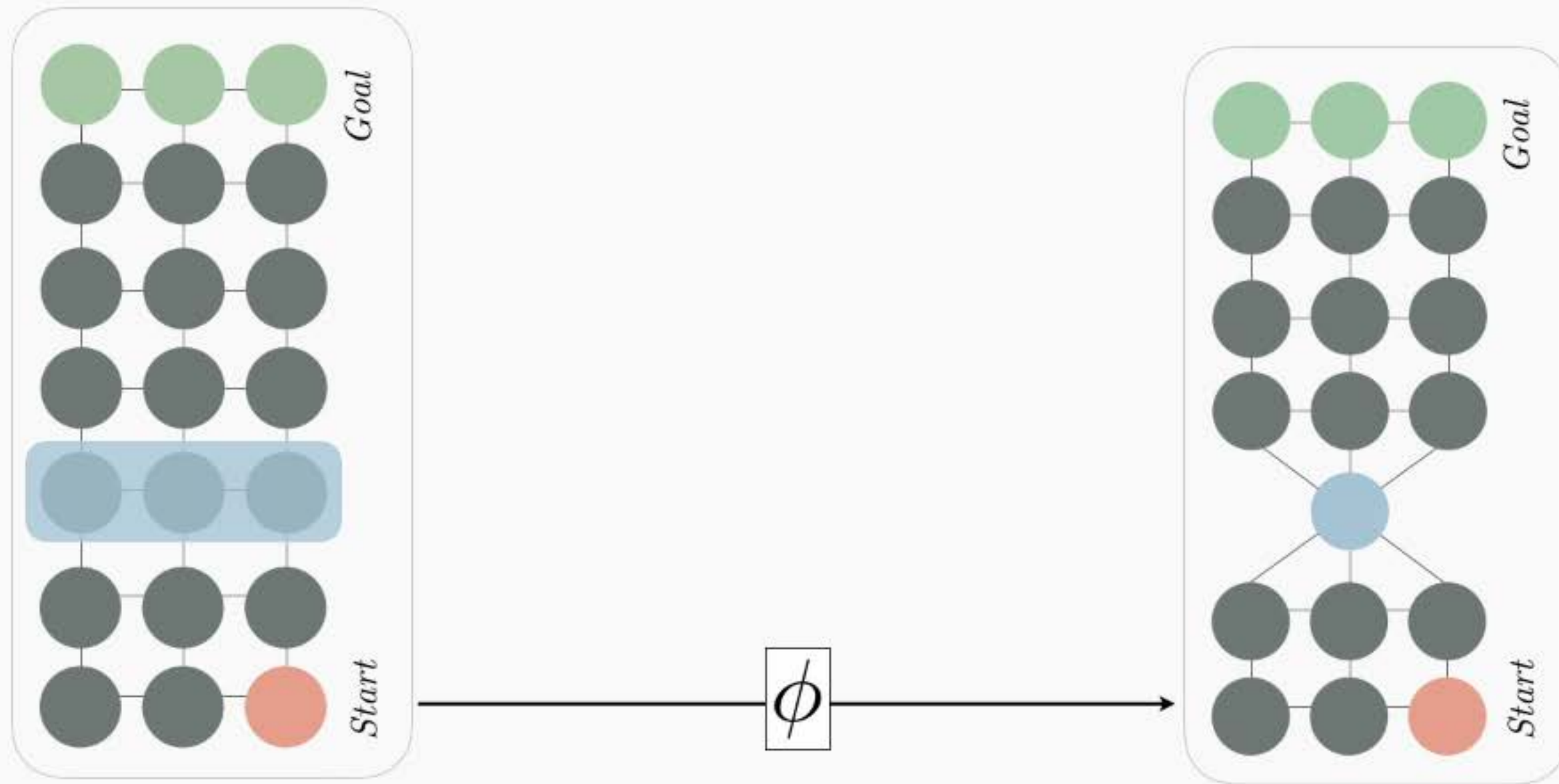
[Castro, Precup '09] [Menashe, Stone '18]

[Van Seijen et al. '14] [Misra et al. '19]

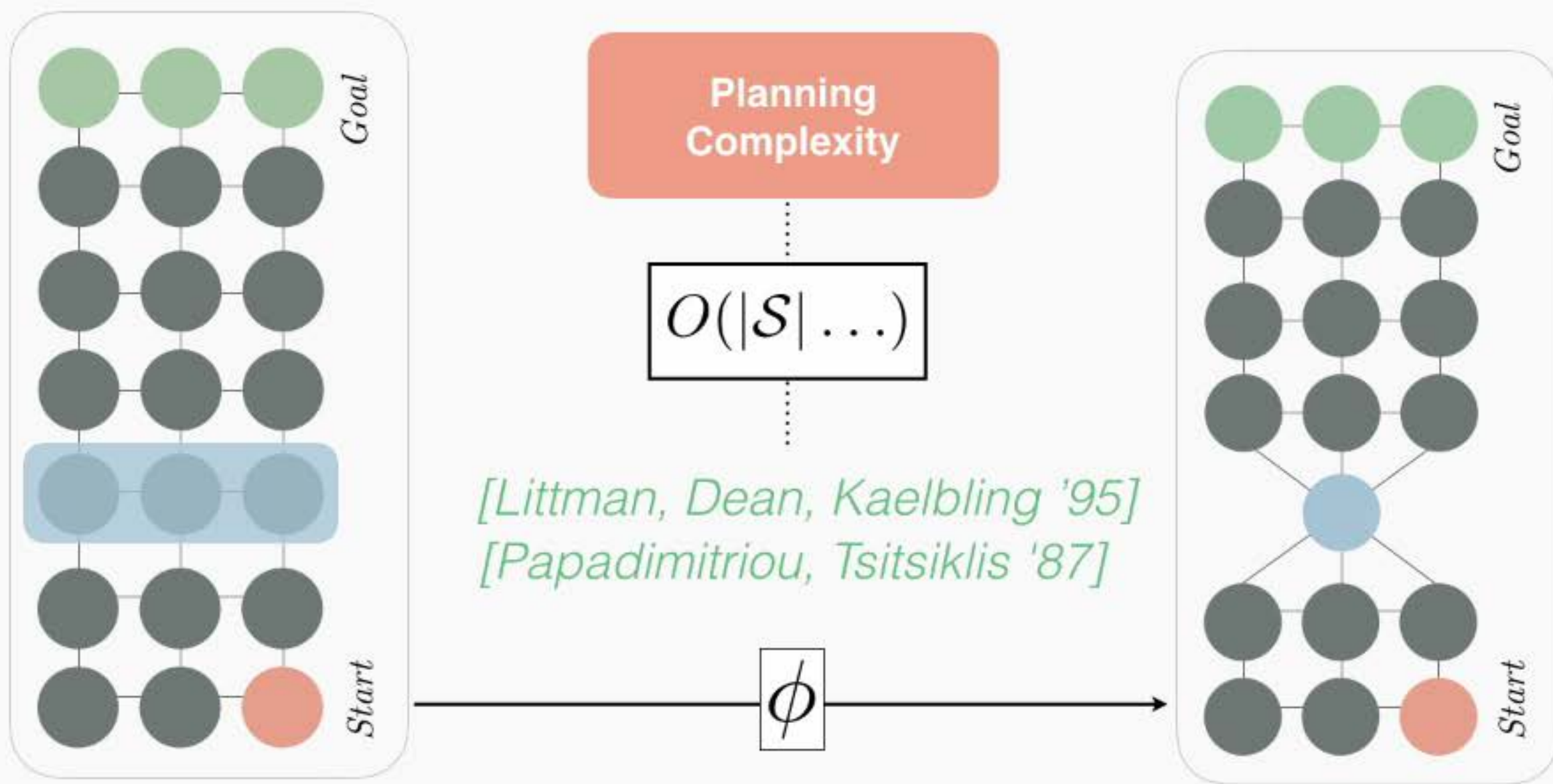
[Hostetler et al. '14, '15, '17]



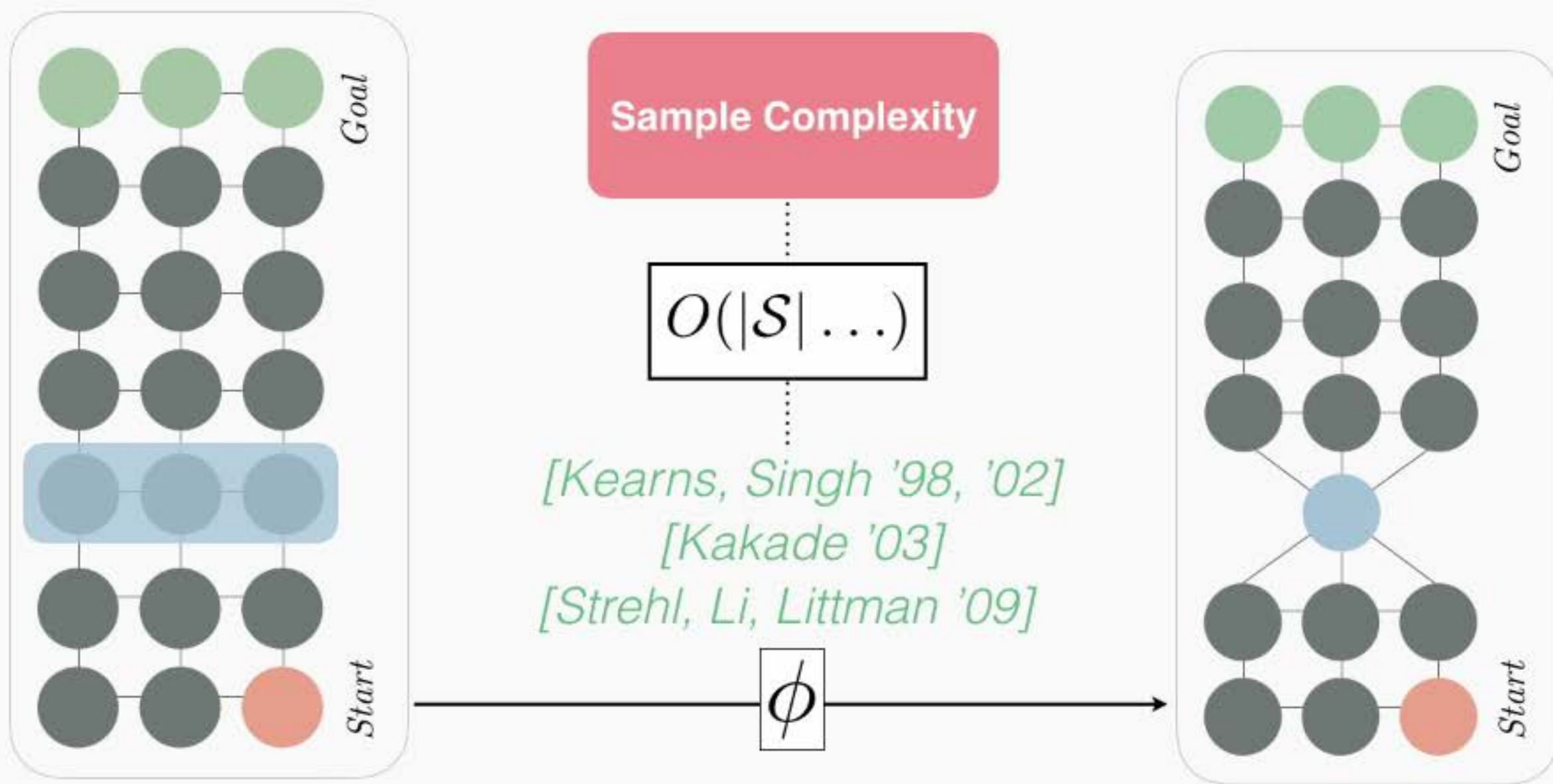
# State Abstraction



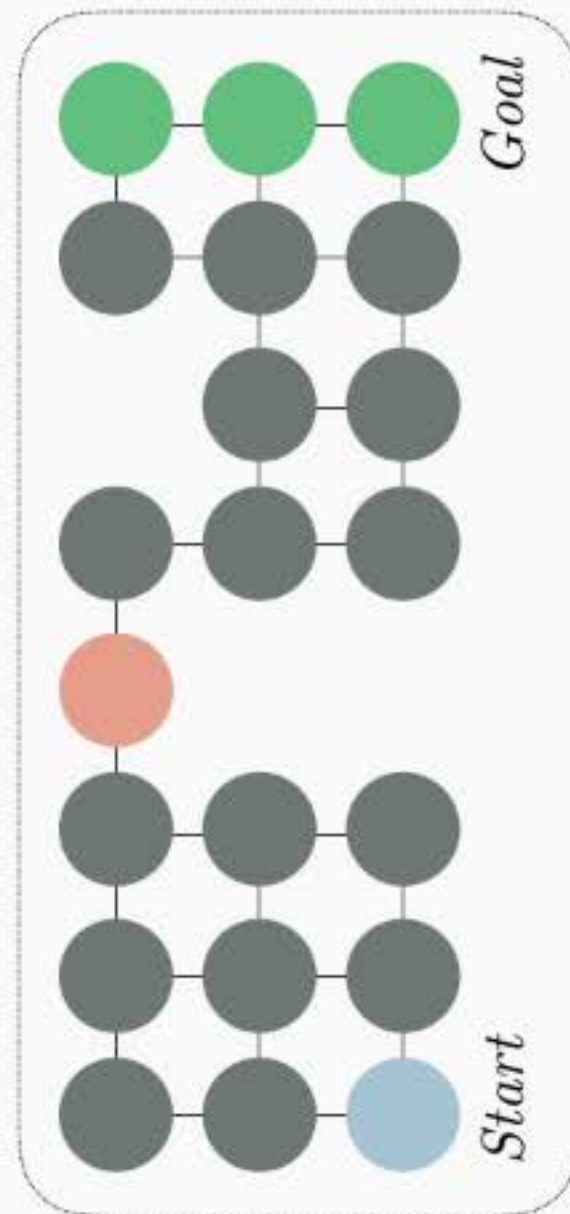
# State Abstraction



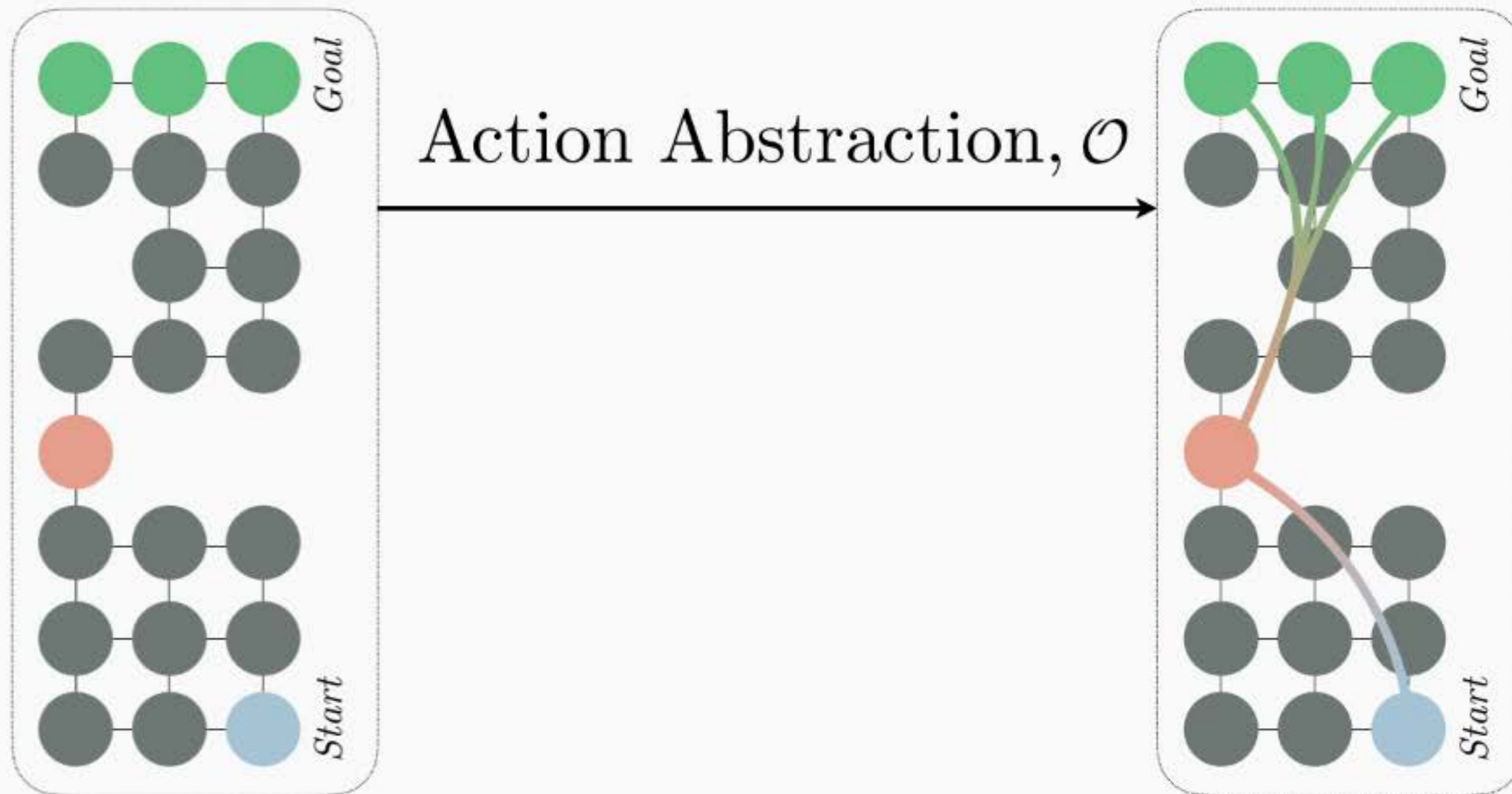
# State Abstraction



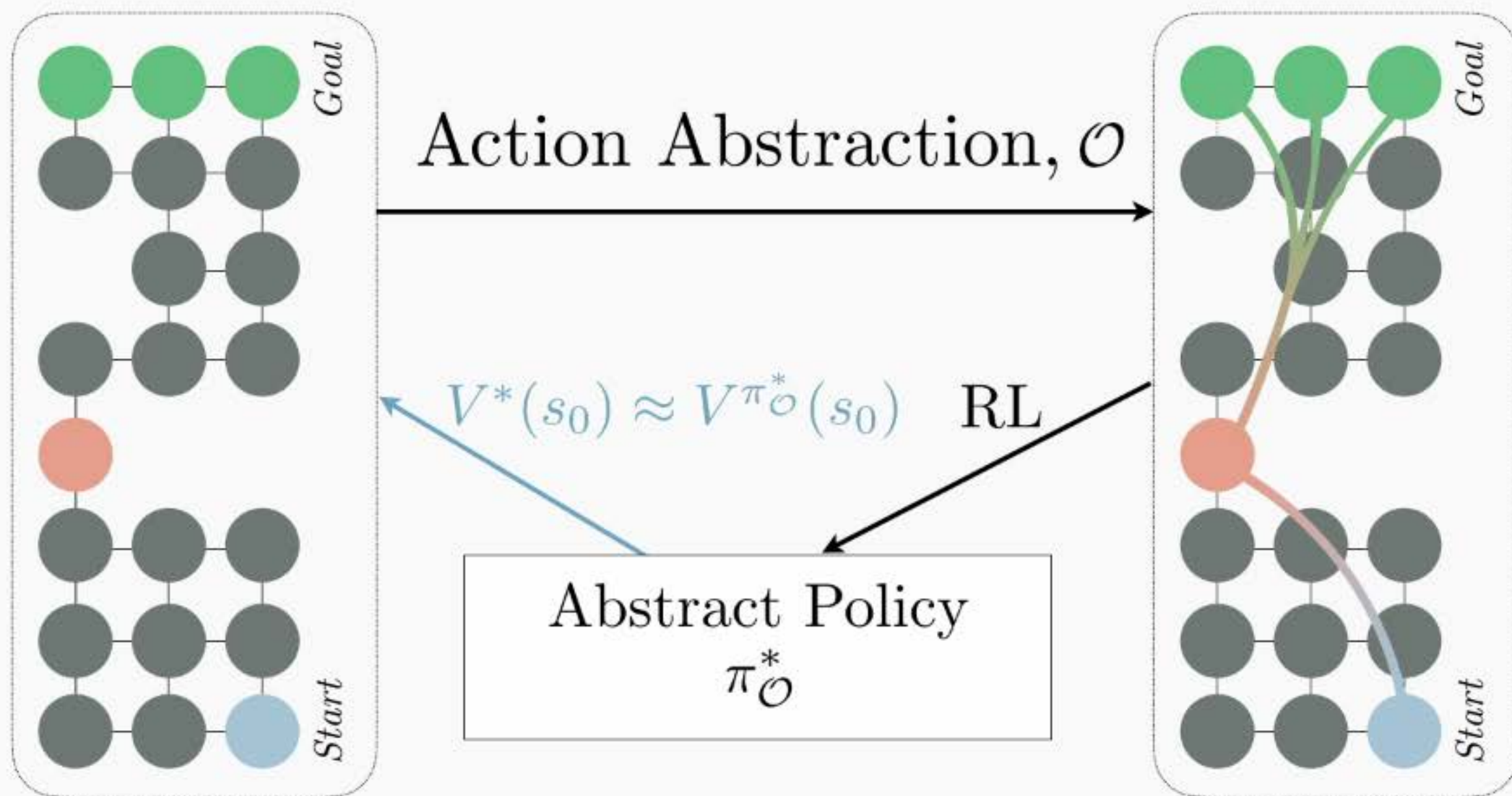
# Action Abstraction



# Action Abstraction



# Action Abstraction



# Action Abstraction

*[Sutton, Precup, Singh 1999]*

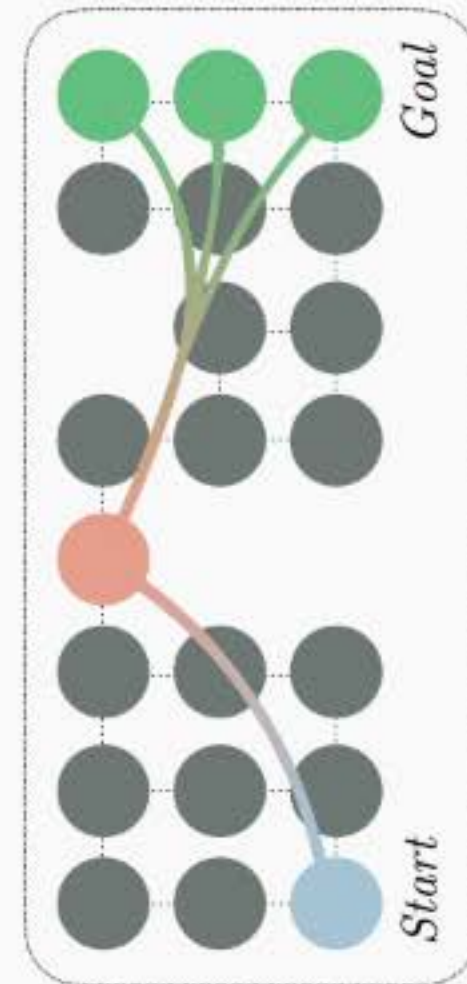
**Definition** (Option): A start condition, end condition, and a policy.

# Action Abstraction

*Example:*

$$o_1 = (\text{blue}, \text{red}, \pi)$$

$$o_2 = (\text{red}, \text{green}, \pi)$$



*[Sutton, Precup, Singh 1999]*

**Definition (Option):** A start condition, end condition, and a policy.



# Action Abstraction

**Definition** (Action Abstraction): An action abstraction extends the primitive actions with the option set  $\mathcal{O}$ .

*[McGovern et al. '97]*

*[Durugkar et al. '16]*

*[Sutton, Precup, Singh '99]*

*[Bacon et al. '17]*

*[Konidaris, Barto '05, '06]*

*[Fruit et al. '17, '17]*

*[Jong, Hester, Stone '08]*

*[Machado et al. '17]*

*[Mugan, Kuipers '09, '12]*

*[Harutyunyan et al. '18]*

*[Brunskill, Li '14]*

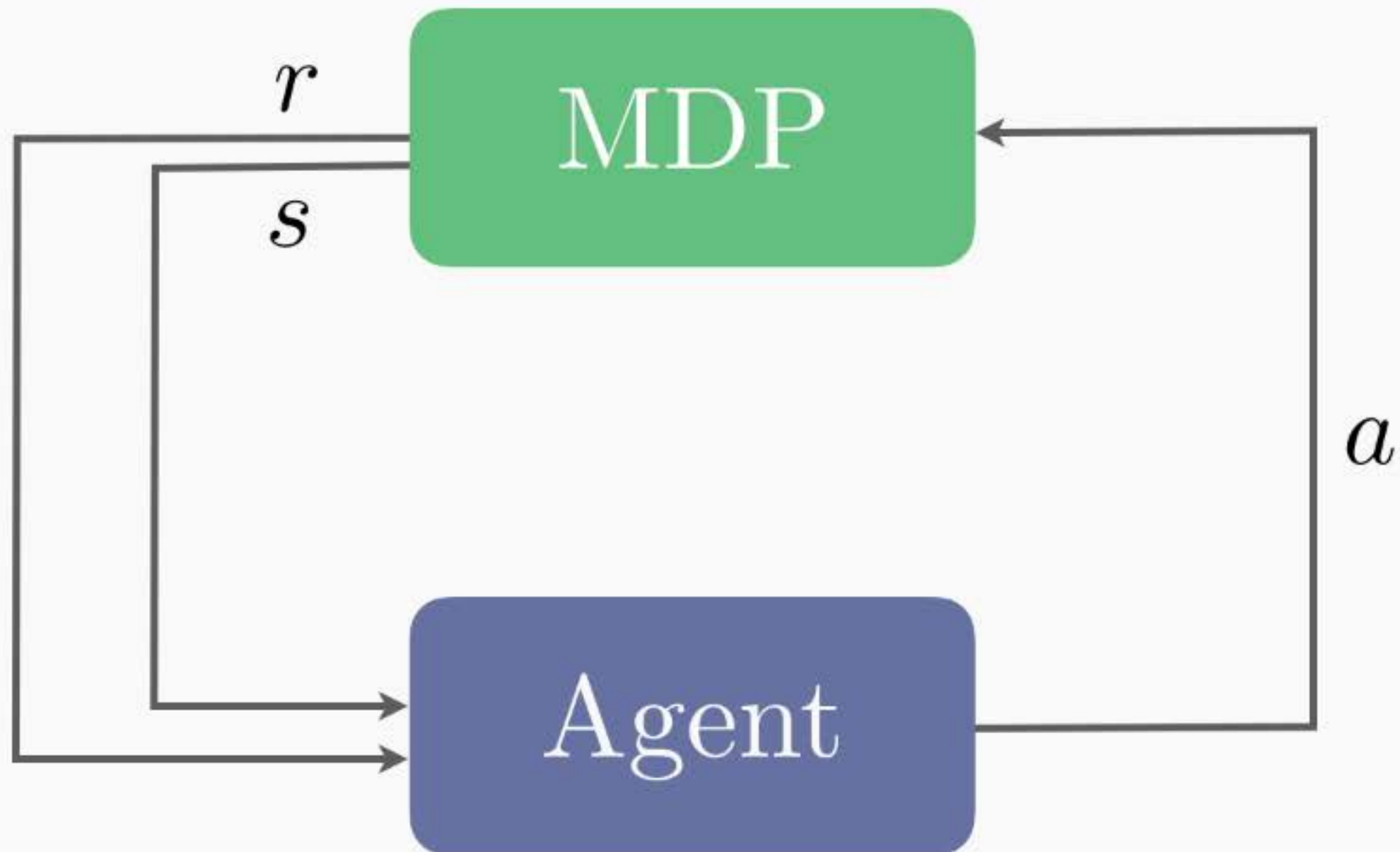
*[Eysenbach et al. '18]*

*[Ciosek, Silver '15]*

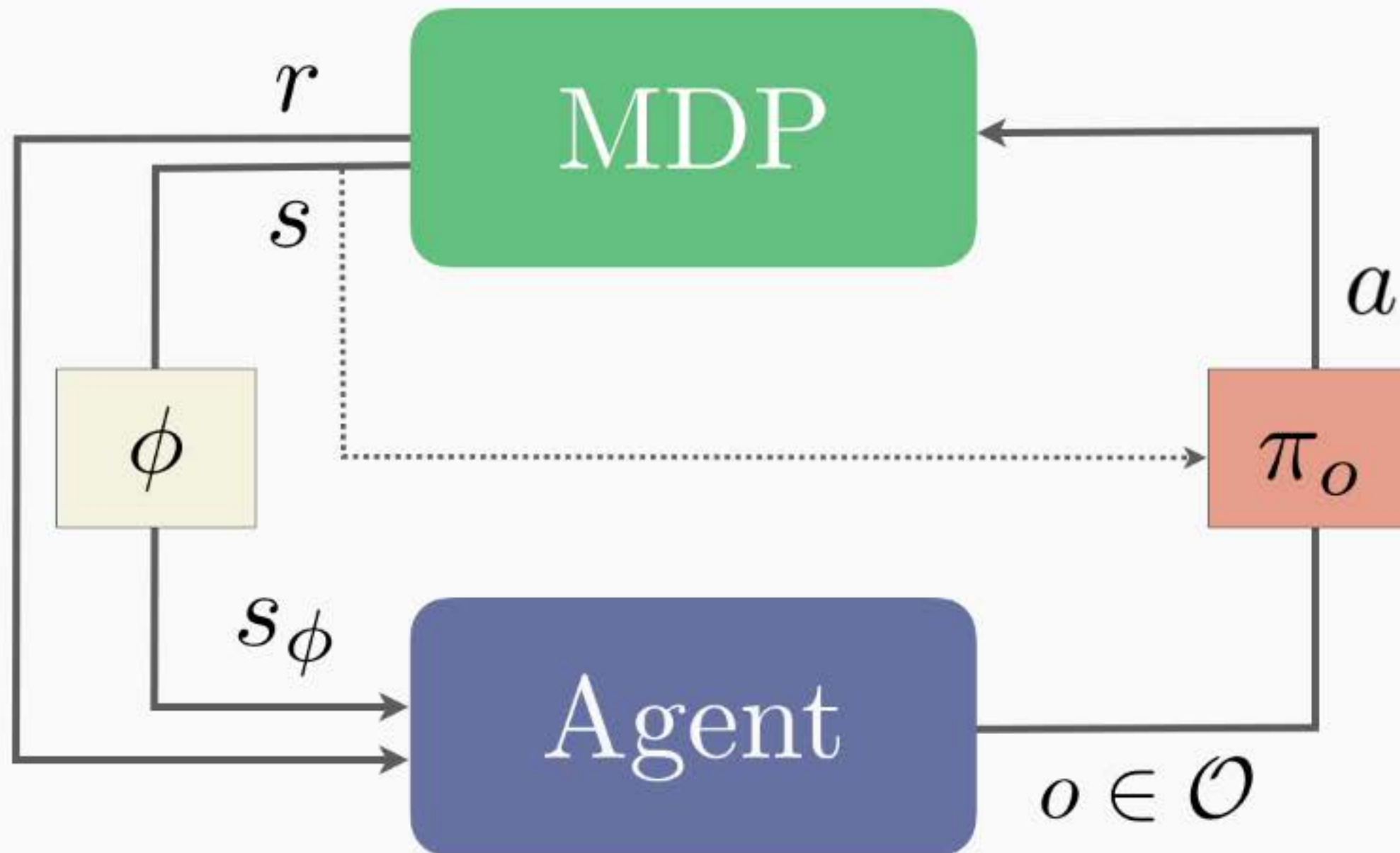
*[Barreto et al. '19]*

*...and more!*

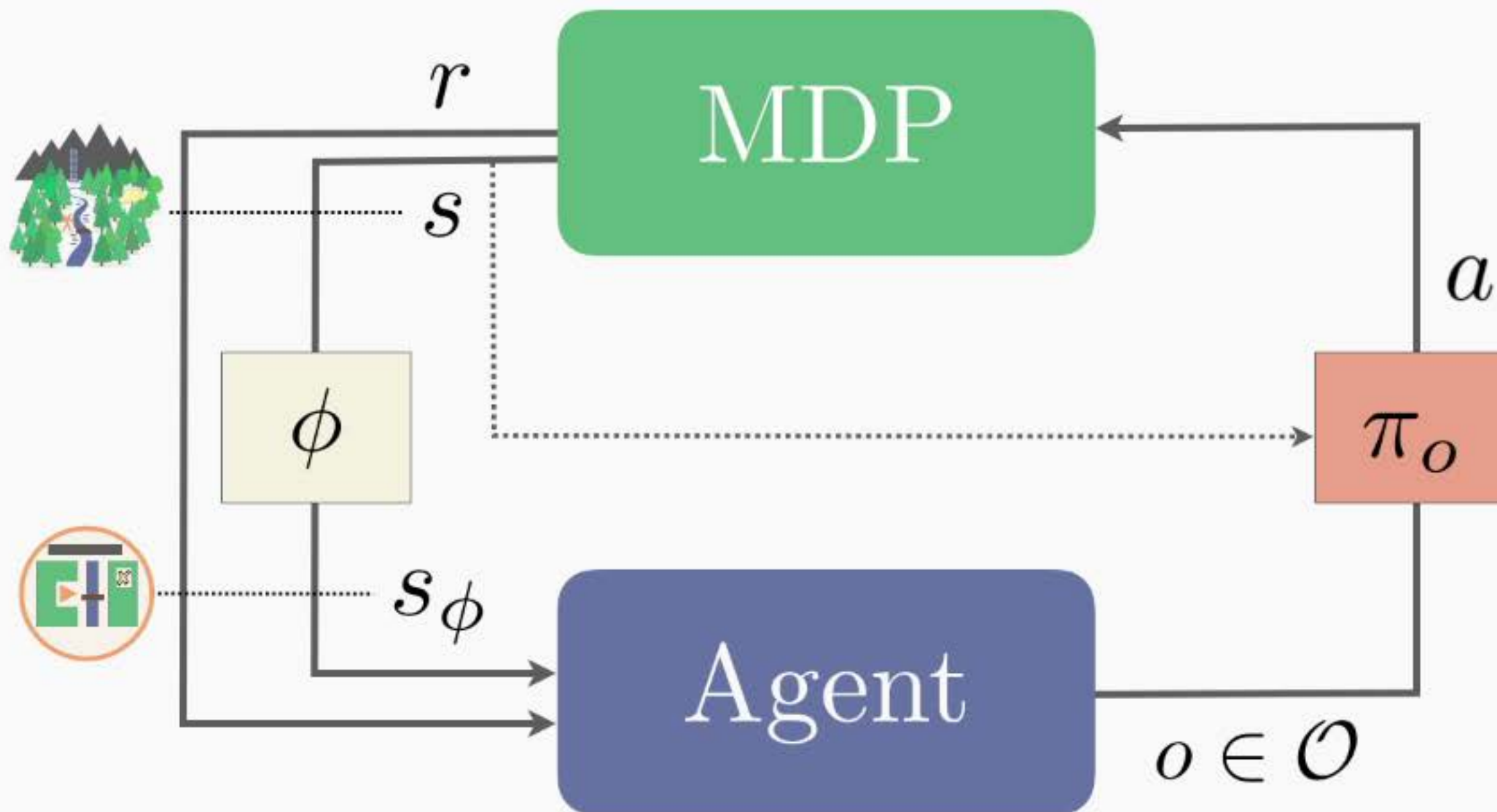
# Abstraction in RL



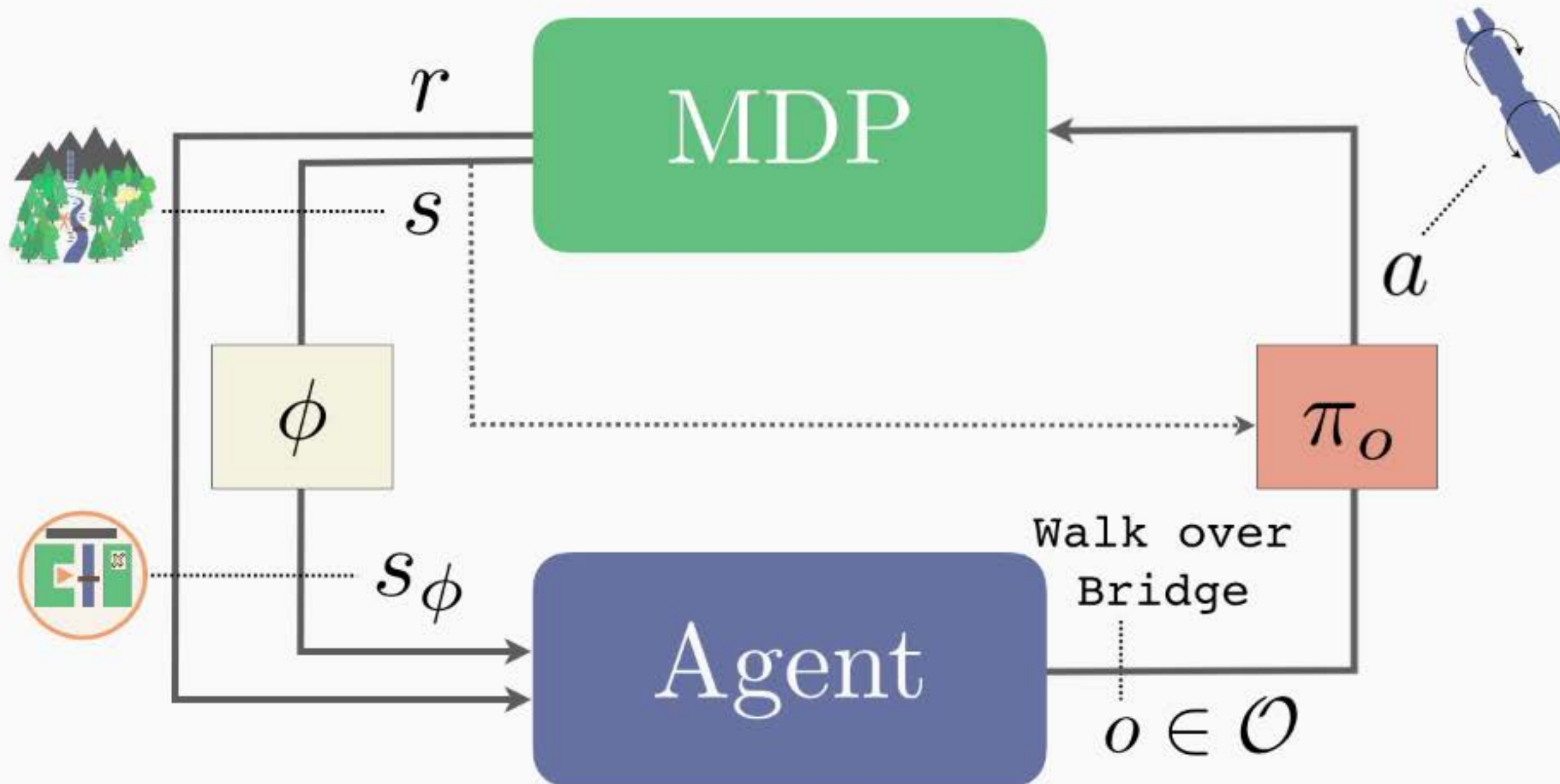
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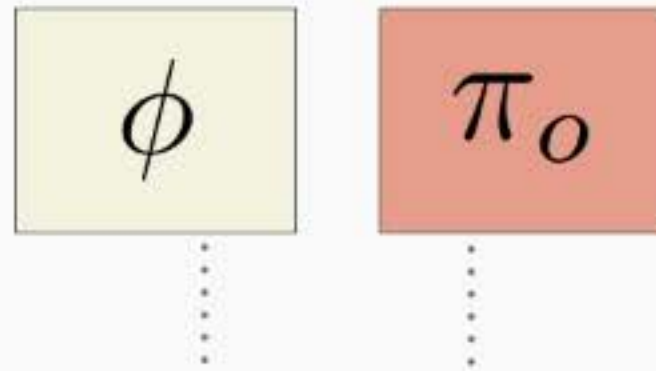
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# Abstraction in RL

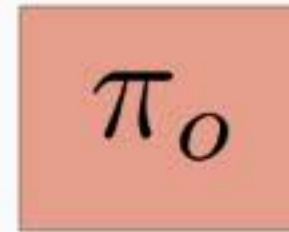


# Desirable Abstractions



**Q: Which kinds of abstractions are desirable?**

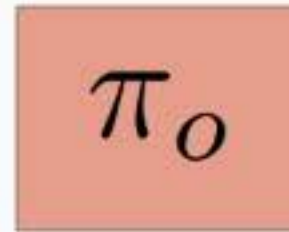
# Desirable Abstractions



Q: Which kinds of abstractions are desirable?

Easy To Construct

# Desirable Abstractions



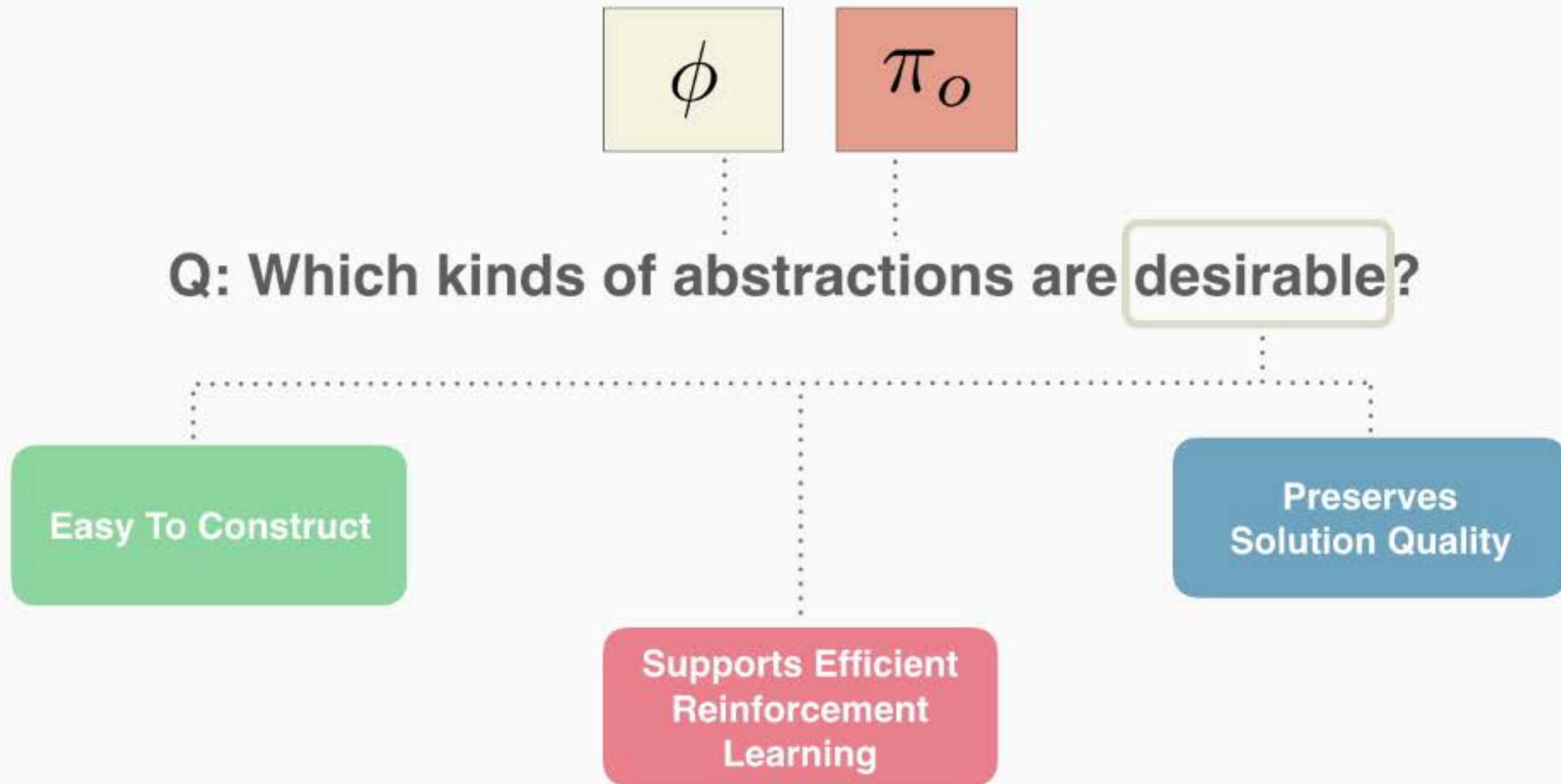
Q: Which kinds of abstractions are **desirable**?

Easy To Construct

Supports Efficient  
Reinforcement  
Learning



# Desirable Abstractions



# Abstraction Desiderata

Easy To Construct

Supports Efficient  
Reinforcement  
Learning

Preserves  
Solution Quality

# Abstraction Desiderata

Easy To Construct

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# Abstraction Desiderata

Easy To Construct



Supports Efficient  
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# Abstraction Desiderata

*Easy To Construct*

State (& Action)  
Abstractions

Effective RL

MDPs

?

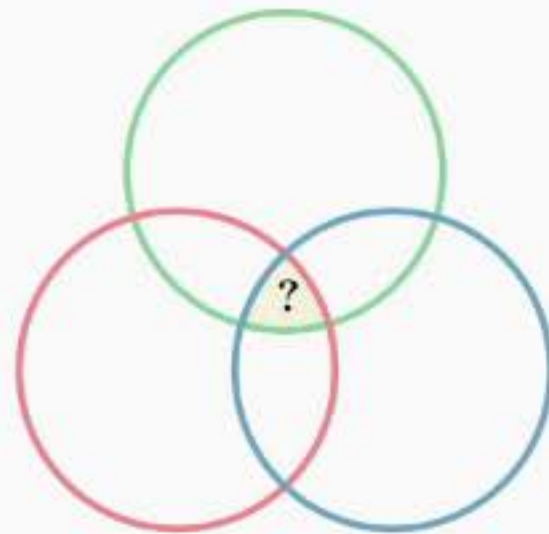
**Question:** How do *intelligent* agents come up with the *right* *abstract understanding* of the *worlds* they inhabit?

*Supports Efficient  
Reinforcement  
Learning*

*Preserves  
Solution Quality*

# Talk Overview

*Easy To Construct*





*Supports Efficient  
Reinforcement  
Learning*

*Preserves  
Solution Quality*

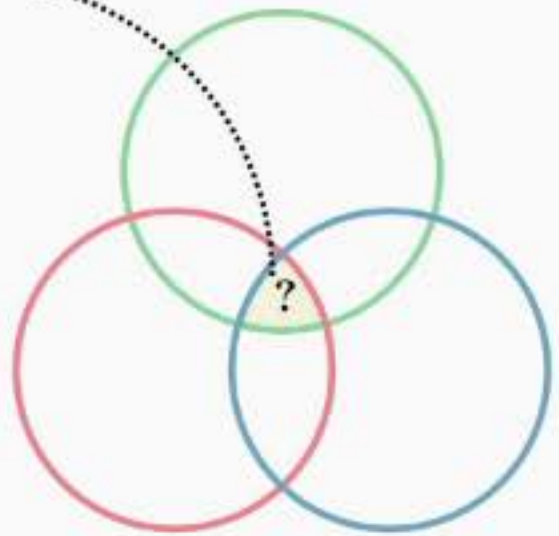
# Talk Overview

**1 State Abstraction**



[AAAJLW AAAI '19]

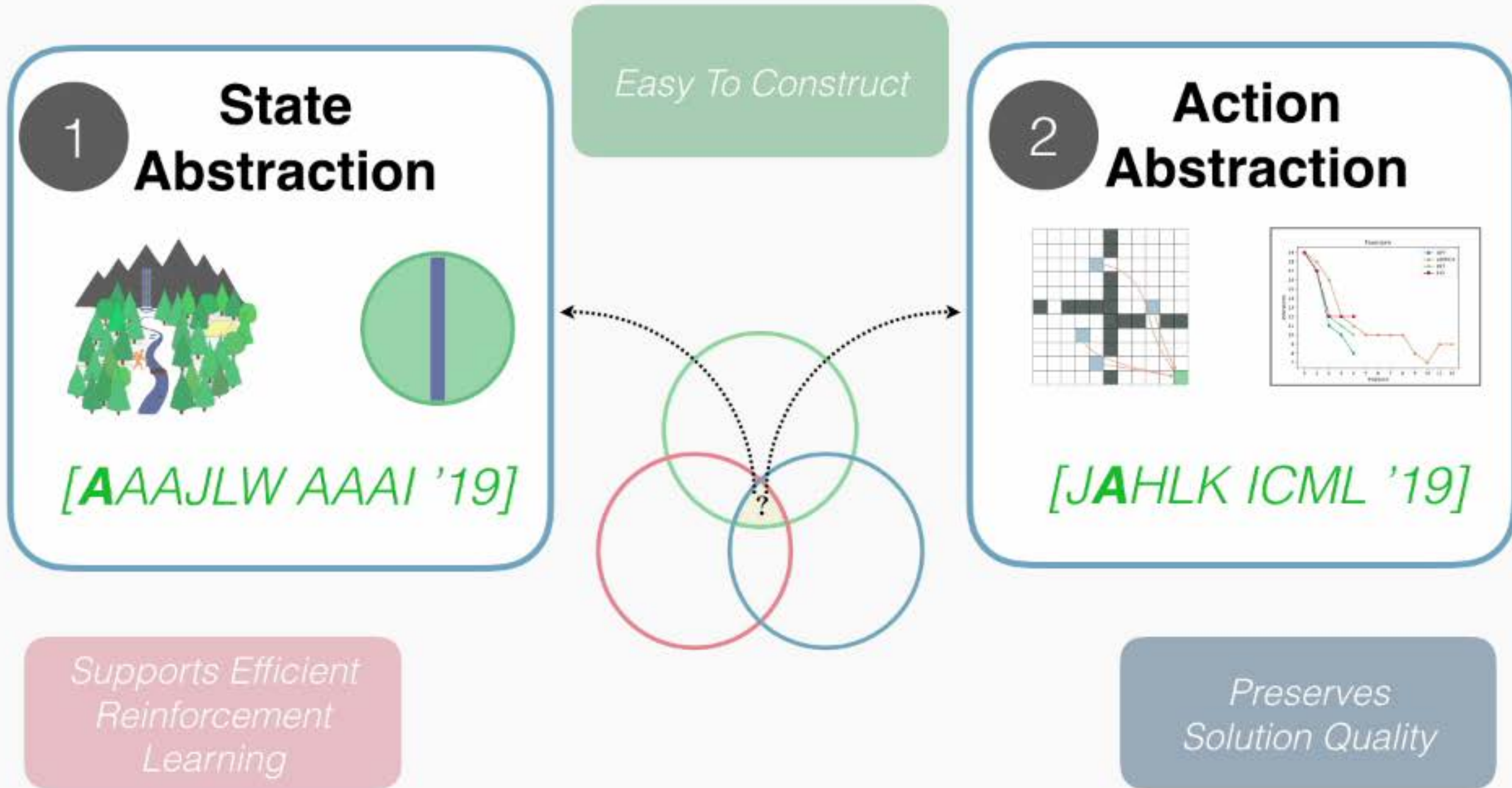
*Easy To Construct*



*Supports Efficient Reinforcement Learning*

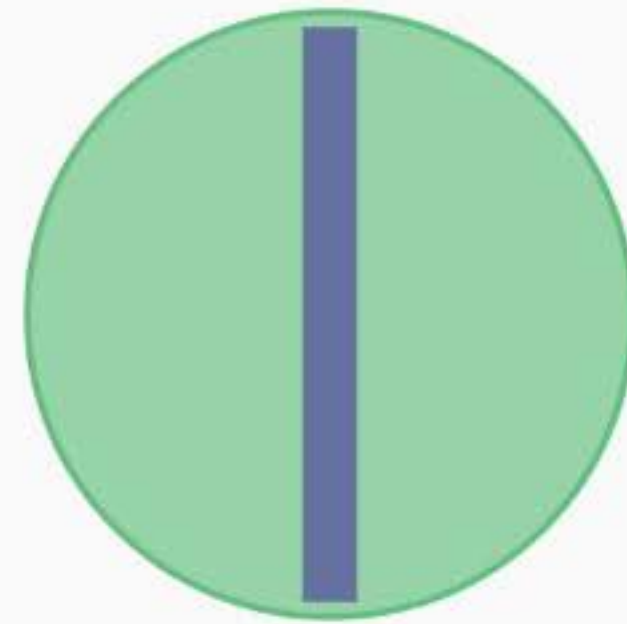
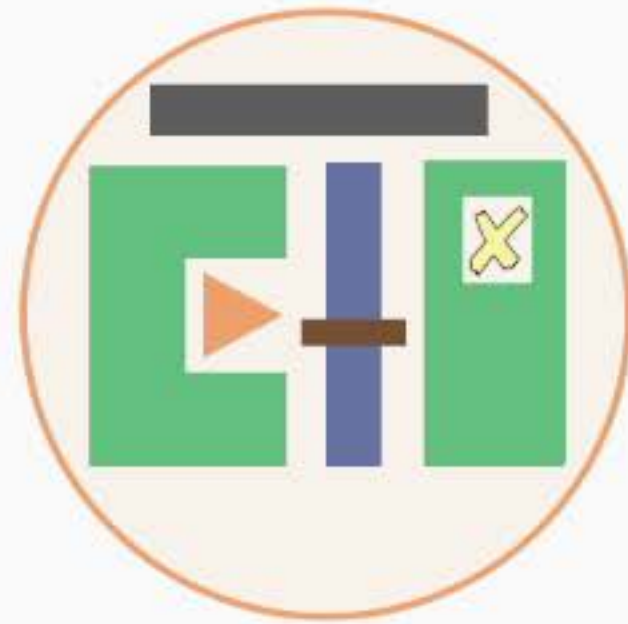
*Preserves Solution Quality*

# Talk Overview





# Compression vs. Value



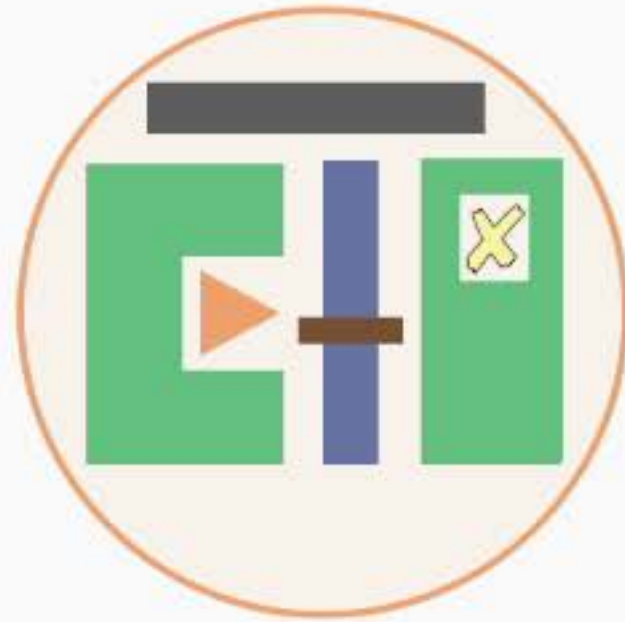
*[Abel, Arumugam, Asadi, Jinnai, Littman, Wong; AAAI 2019]*

# Compression vs. Value

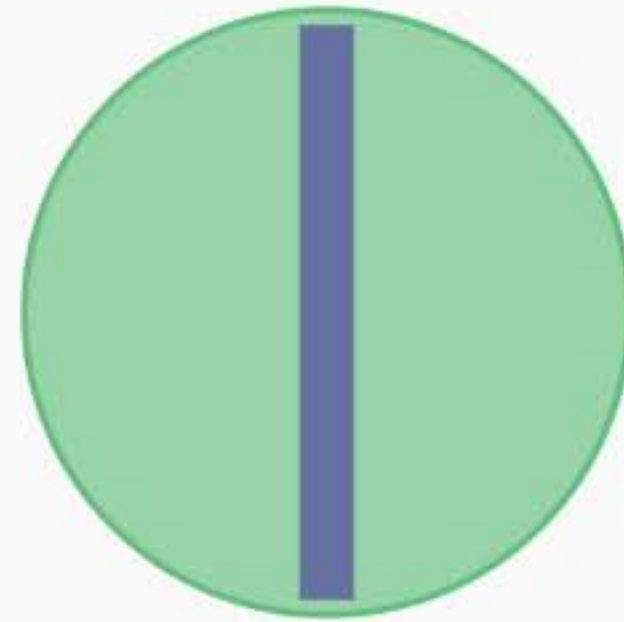
*High Value  
No Compression*



*Some Value  
Some Compression*



*No Value  
High Compression*



**Question:** *Can we find state abstractions that minimize  $|\mathcal{S}_\phi|$  while still representing good policies?*

# Compression vs. Value

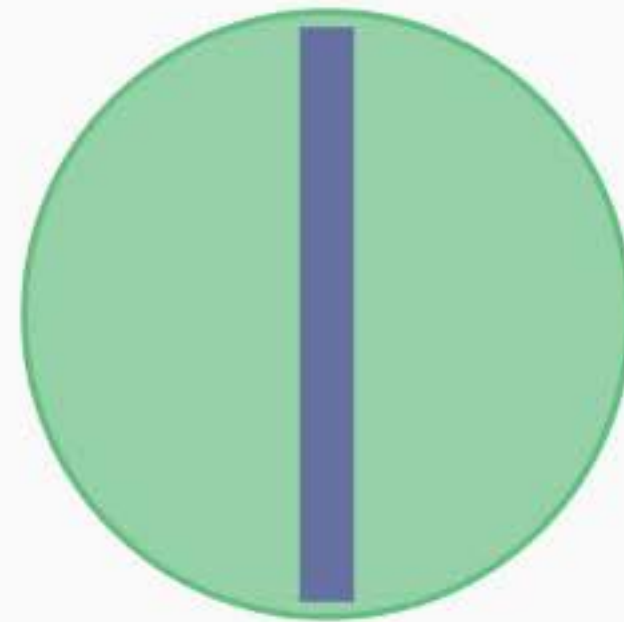
High Value  
No Compression



Some Value  
Some Compression



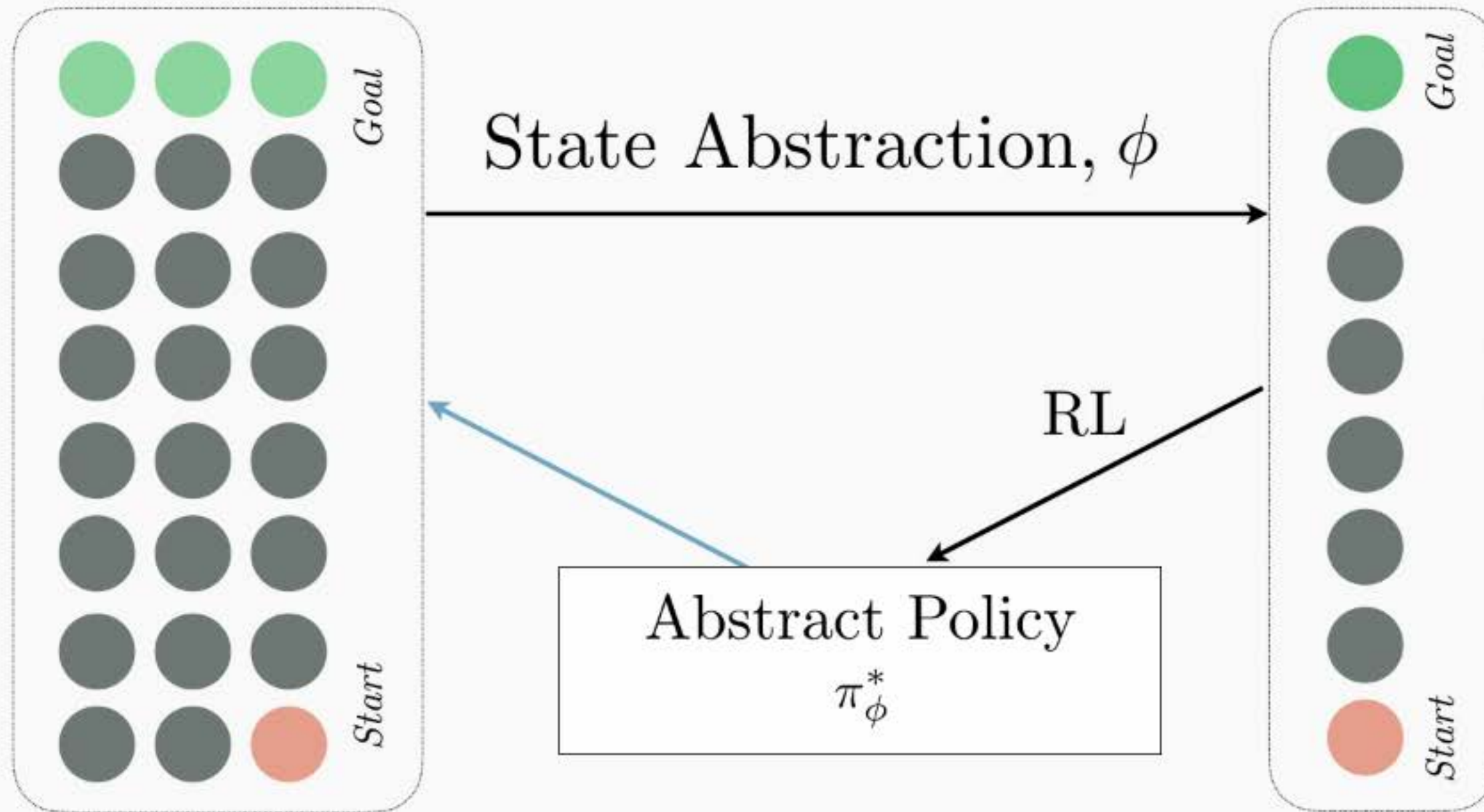
No Value  
High Compression



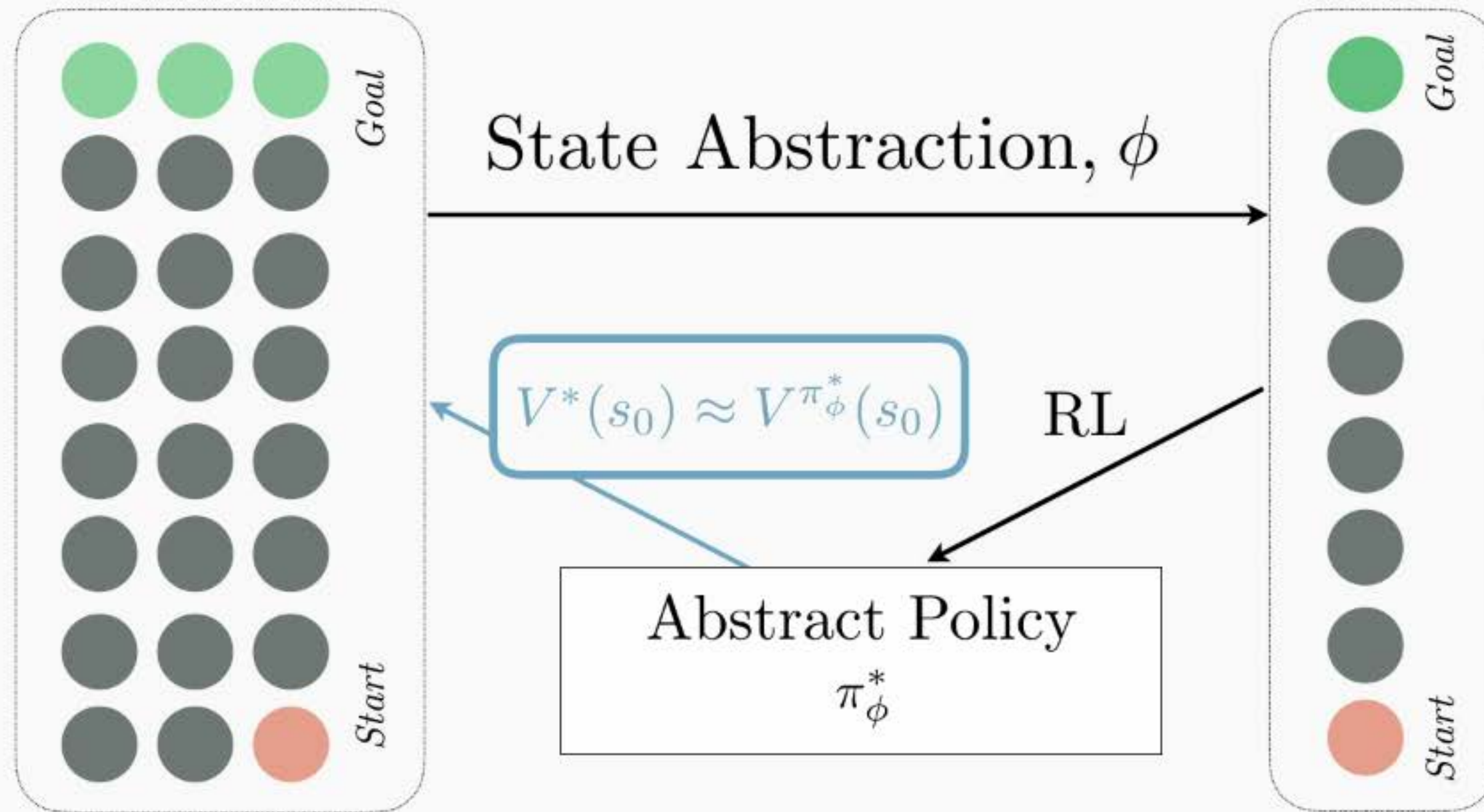
**Question:** Can we find state abstractions that minimize  $|\mathcal{S}_\phi|$  while still representing good policies?

Preserves  
Solution Quality

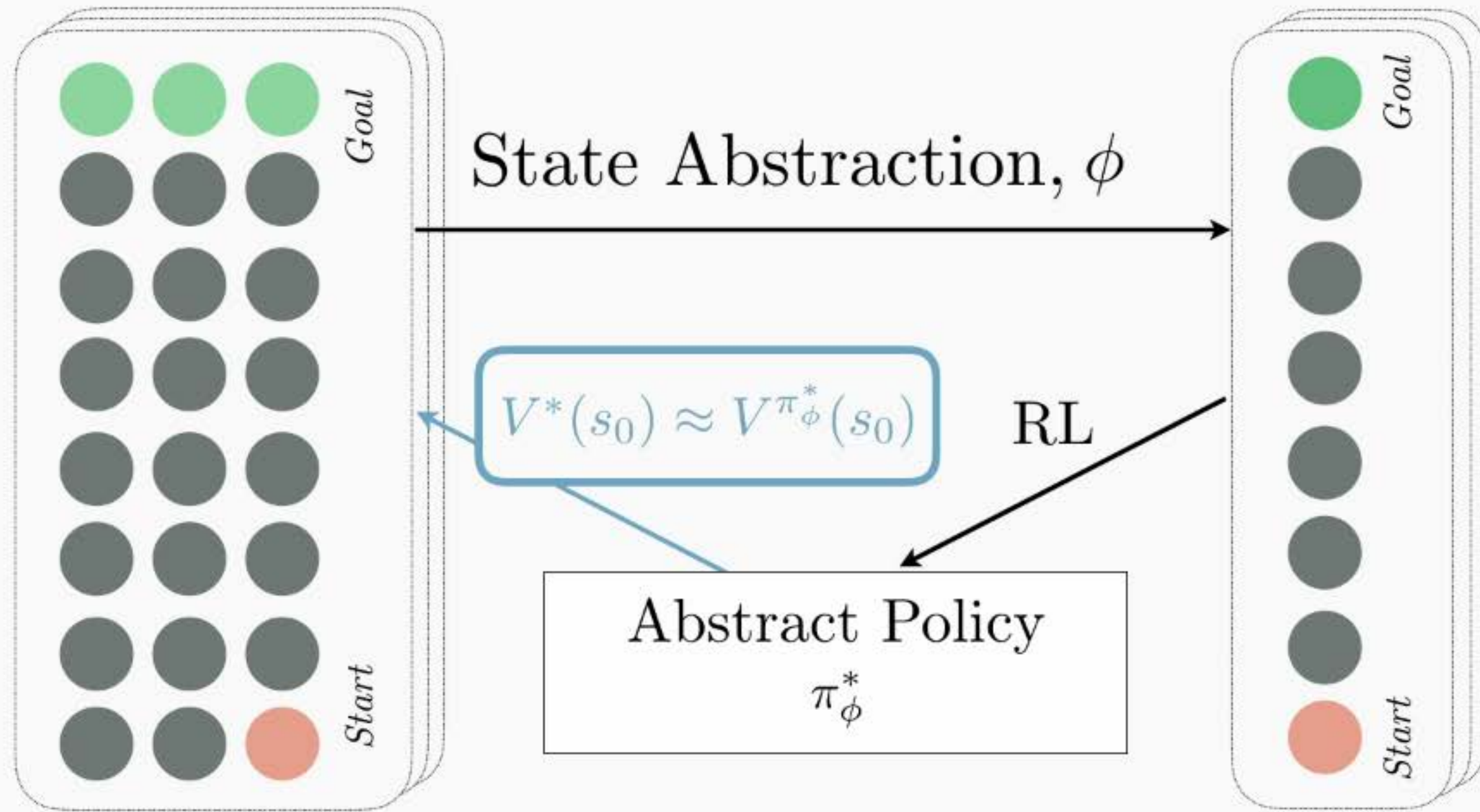
# Preserves Solution Quality



# Preserves Solution Quality



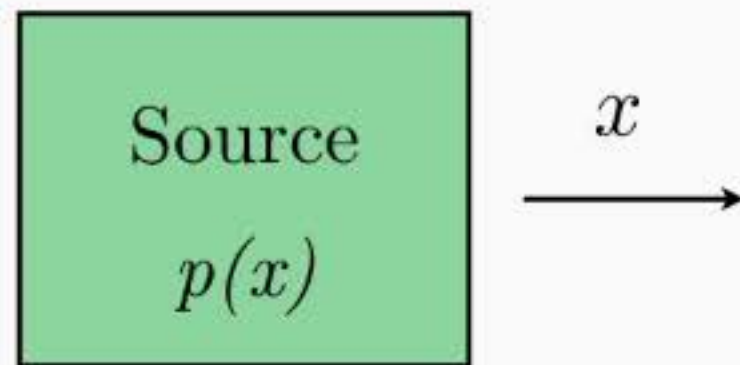
# Preserves Solution Quality



# Rate-Distortion Theory

*[Shannon '48, Berger '03]*

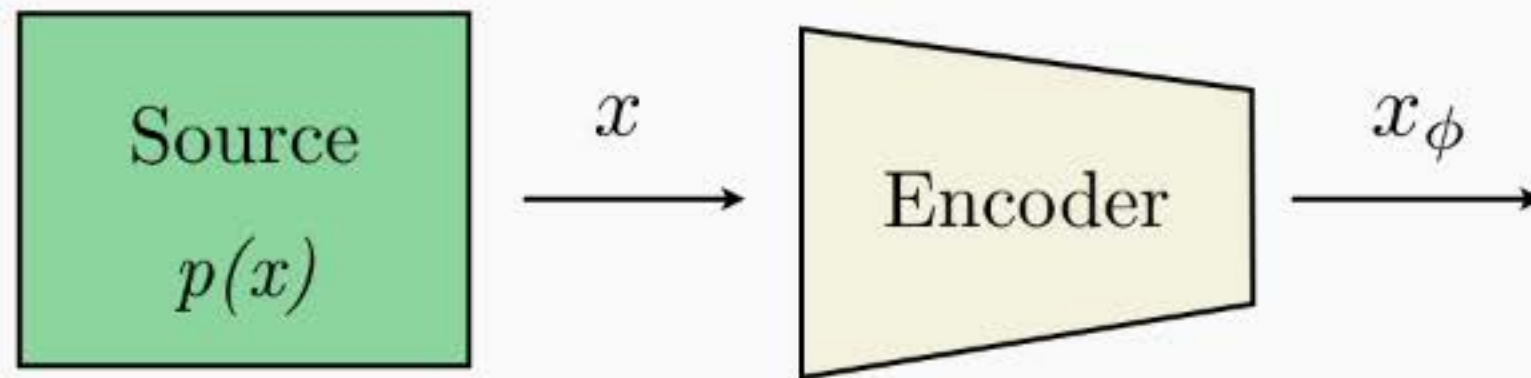
# Rate-Distortion Theory



*[Shannon '48, Berger '03]*

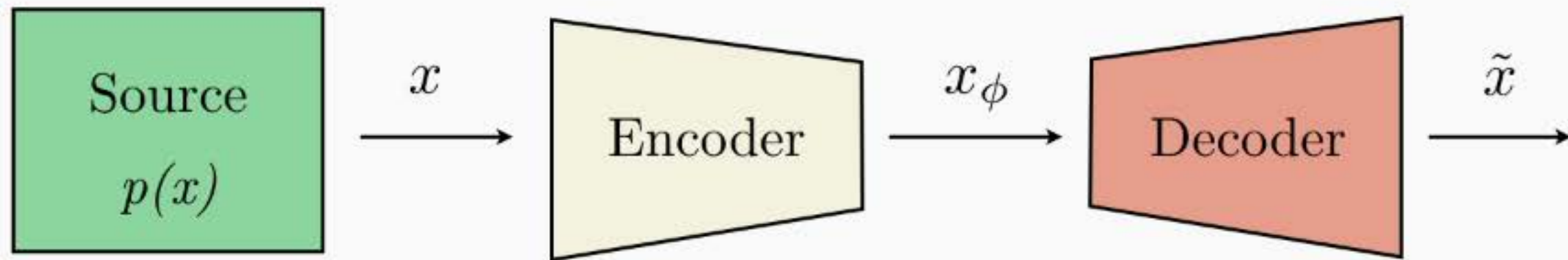


# Rate-Distortion Theory



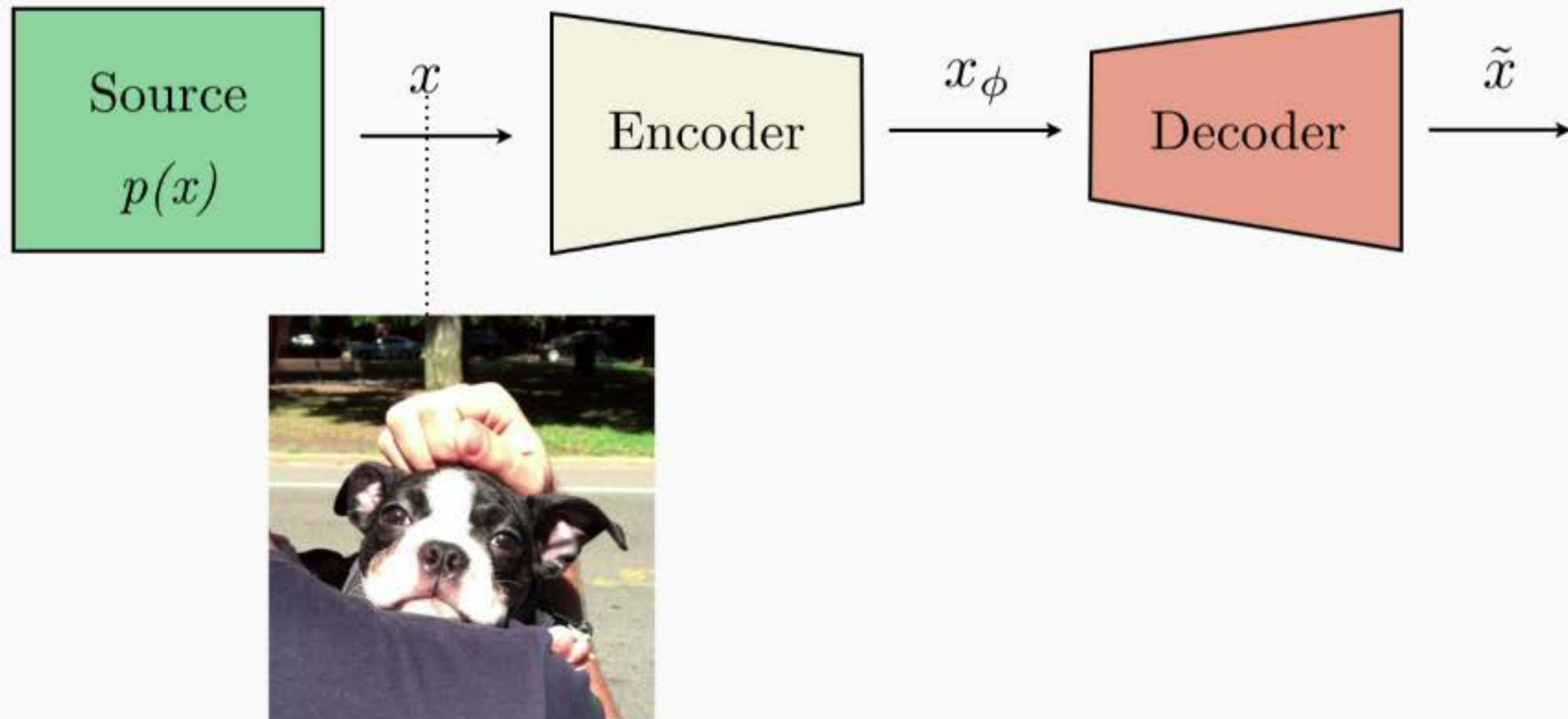
*[Shannon '48, Berger '03]*

# Rate-Distortion Theory



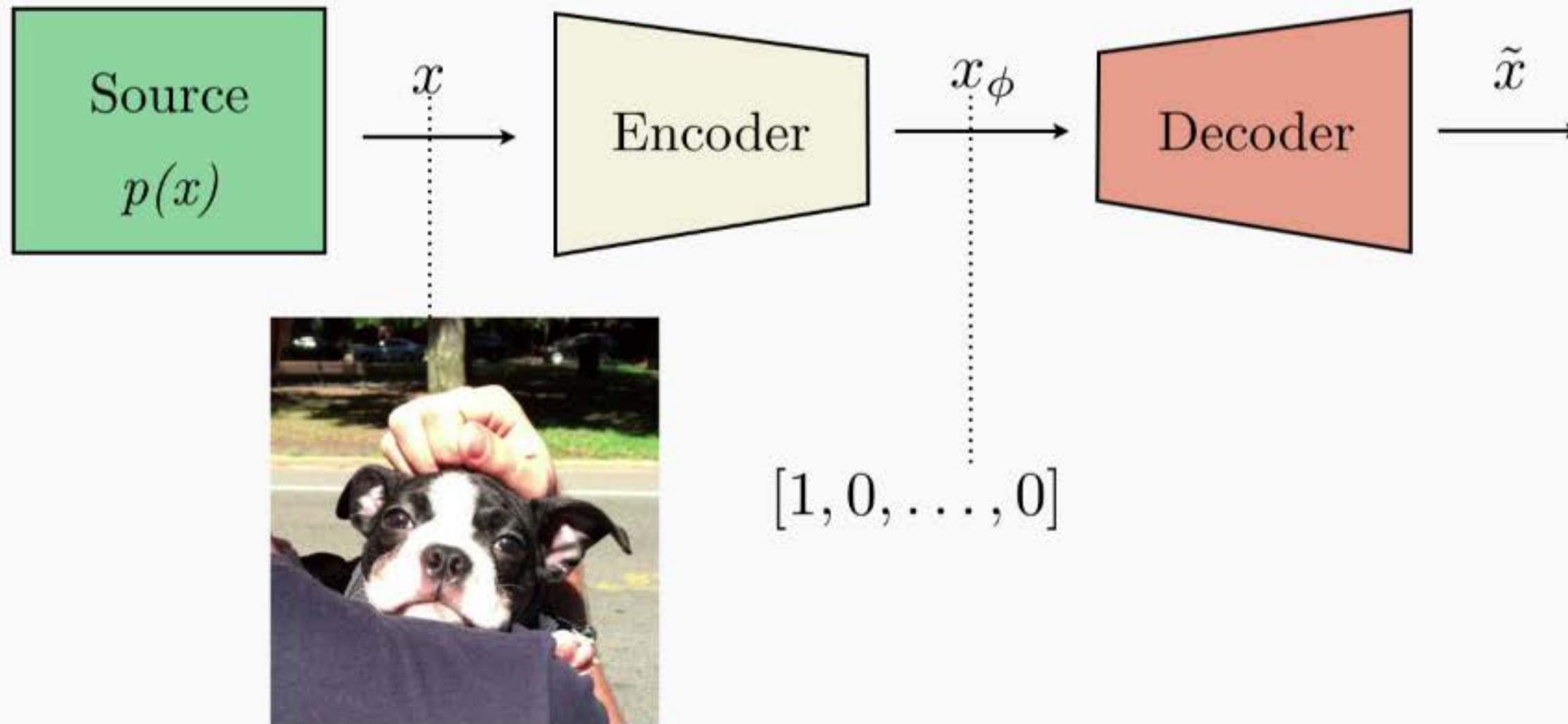
*[Shannon '48, Berger '03]*

# Rate-Distortion Theory



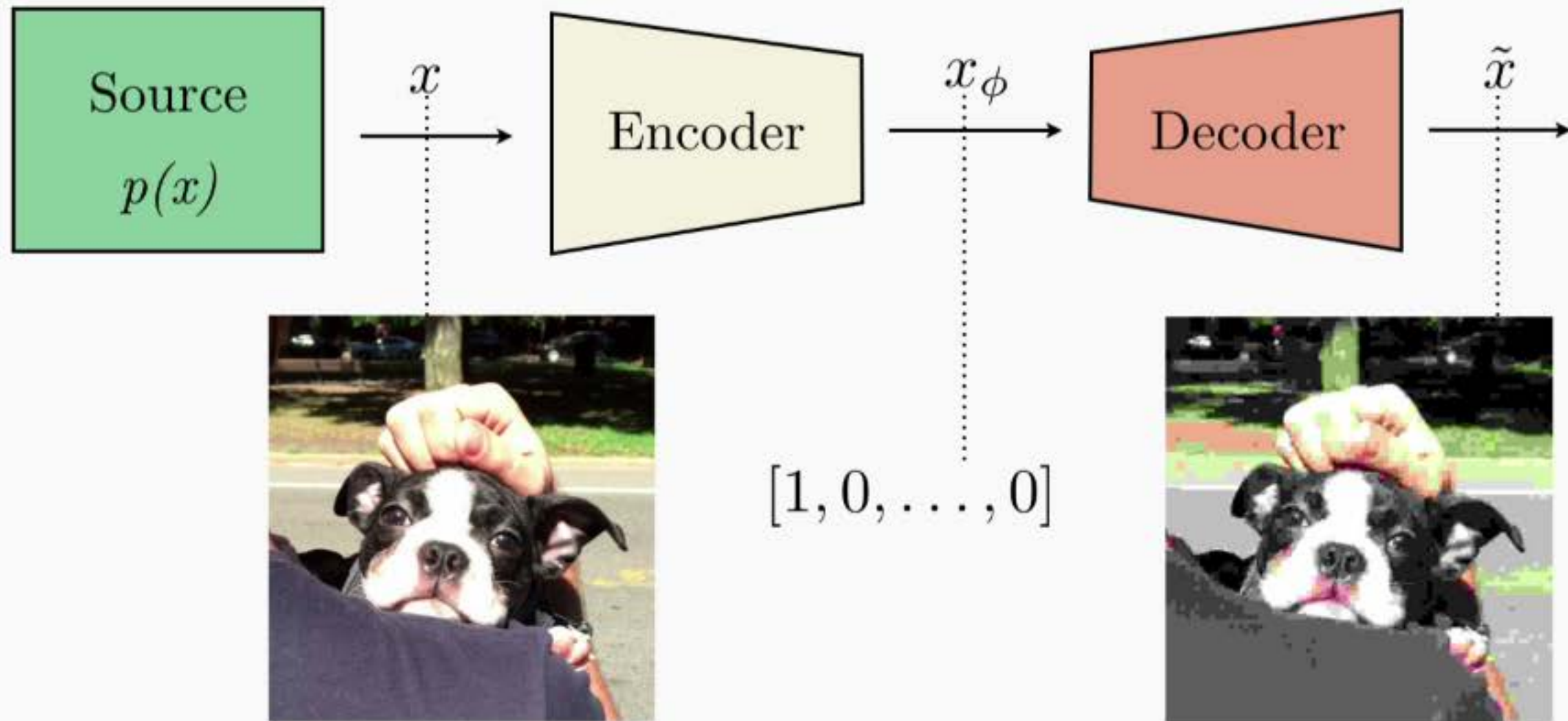
*[Shannon '48, Berger '03]*

# Rate-Distortion Theory



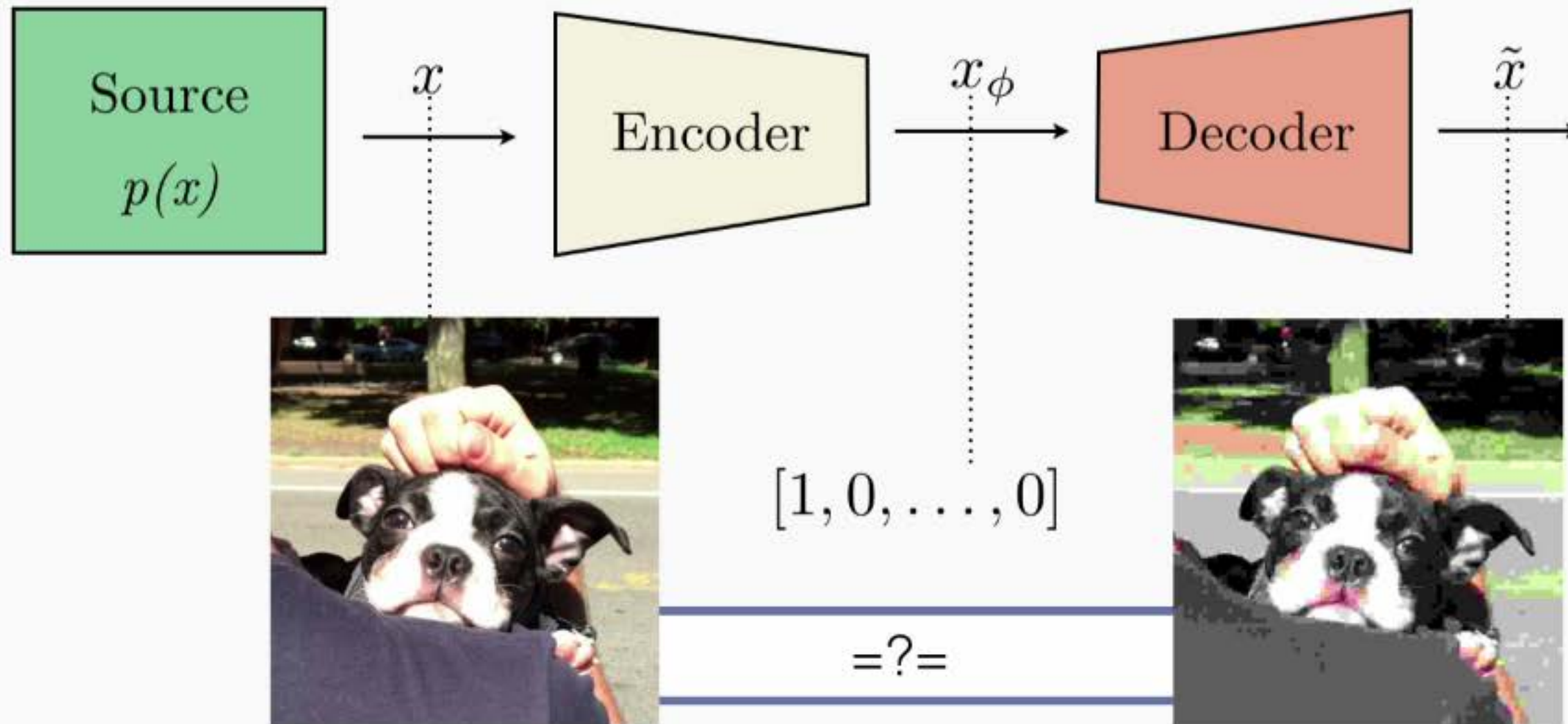
*[Shannon '48, Berger '03]*

# Rate-Distortion Theory



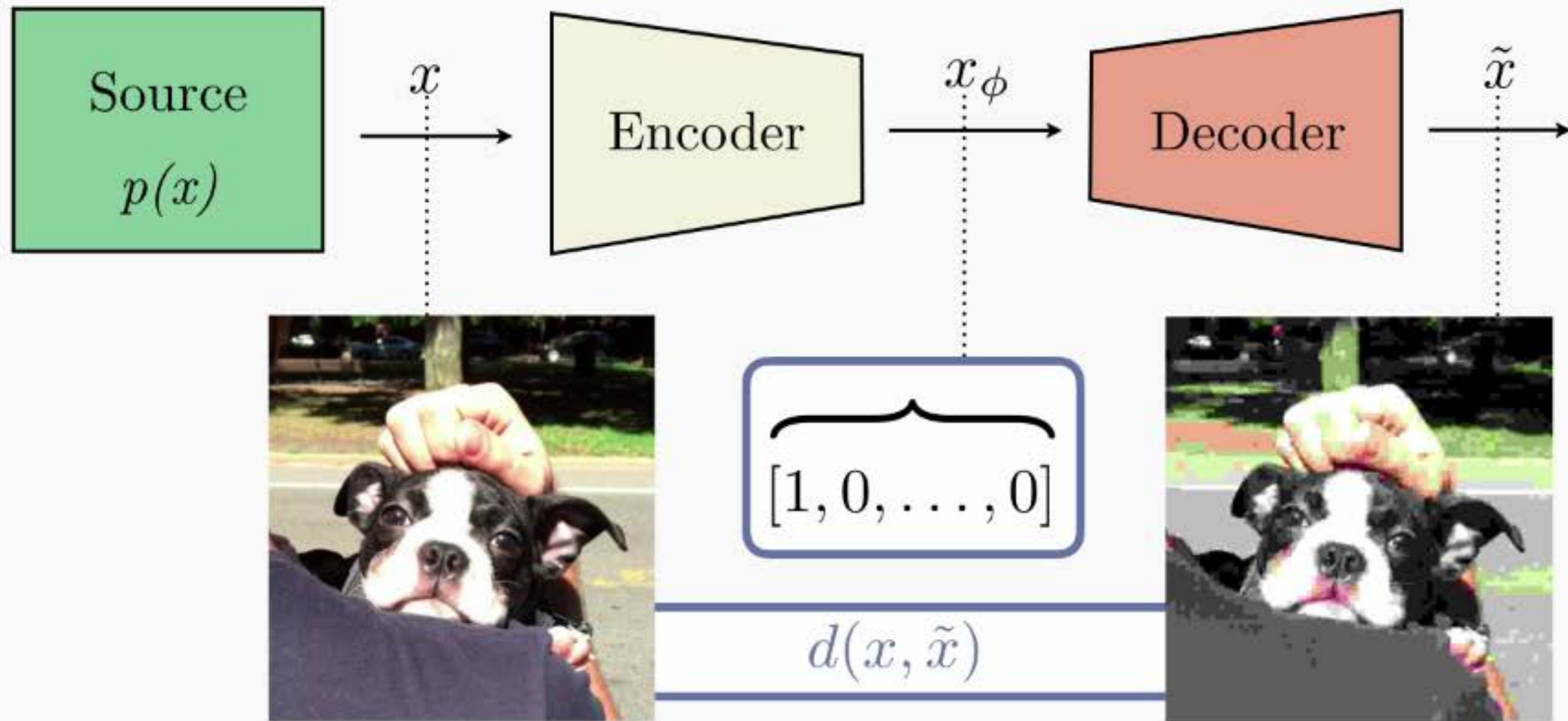
*[Shannon '48, Berger '03]*

# Rate-Distortion Theory



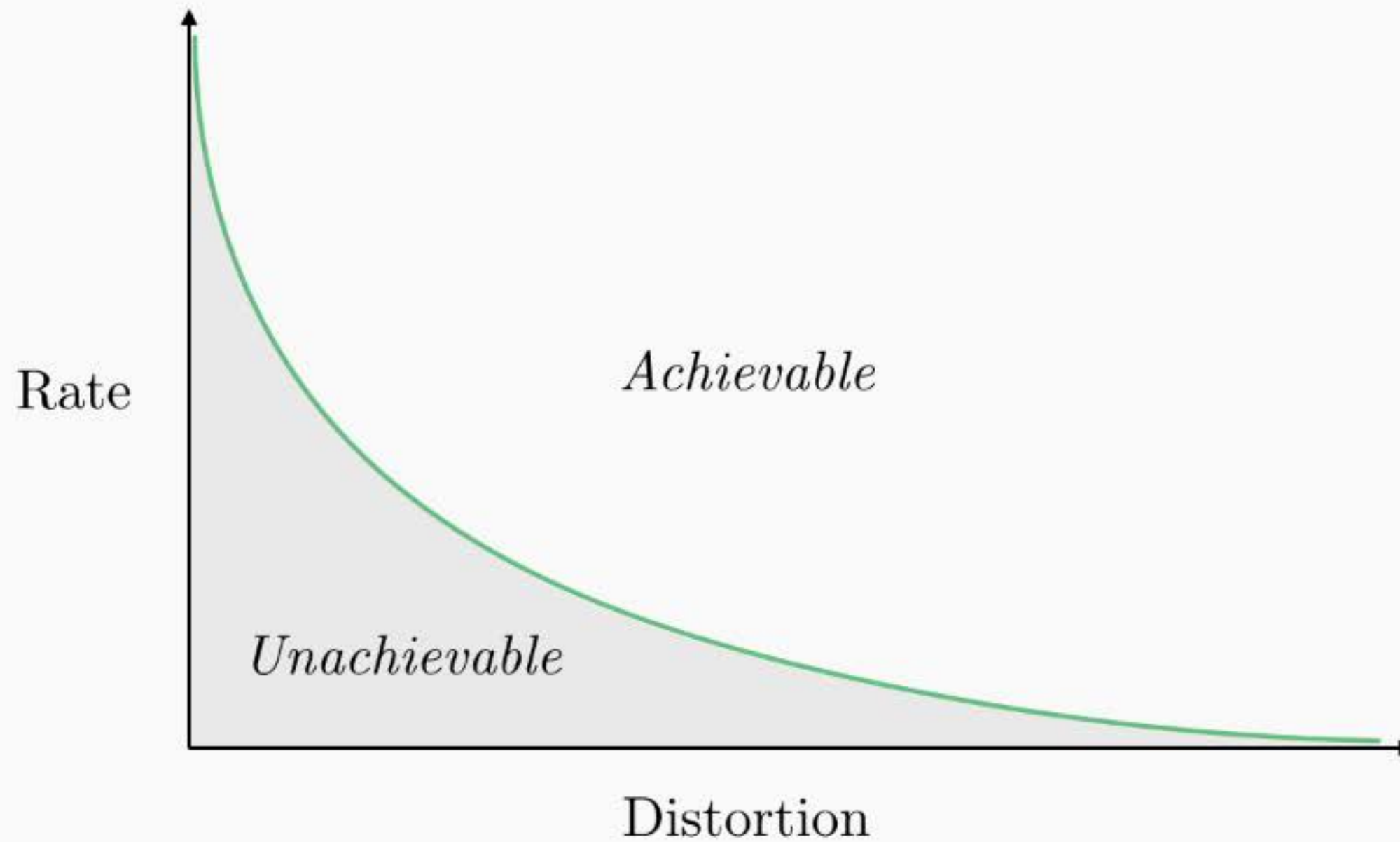
[Shannon '48, Berger '03]

# Rate-Distortion Theory



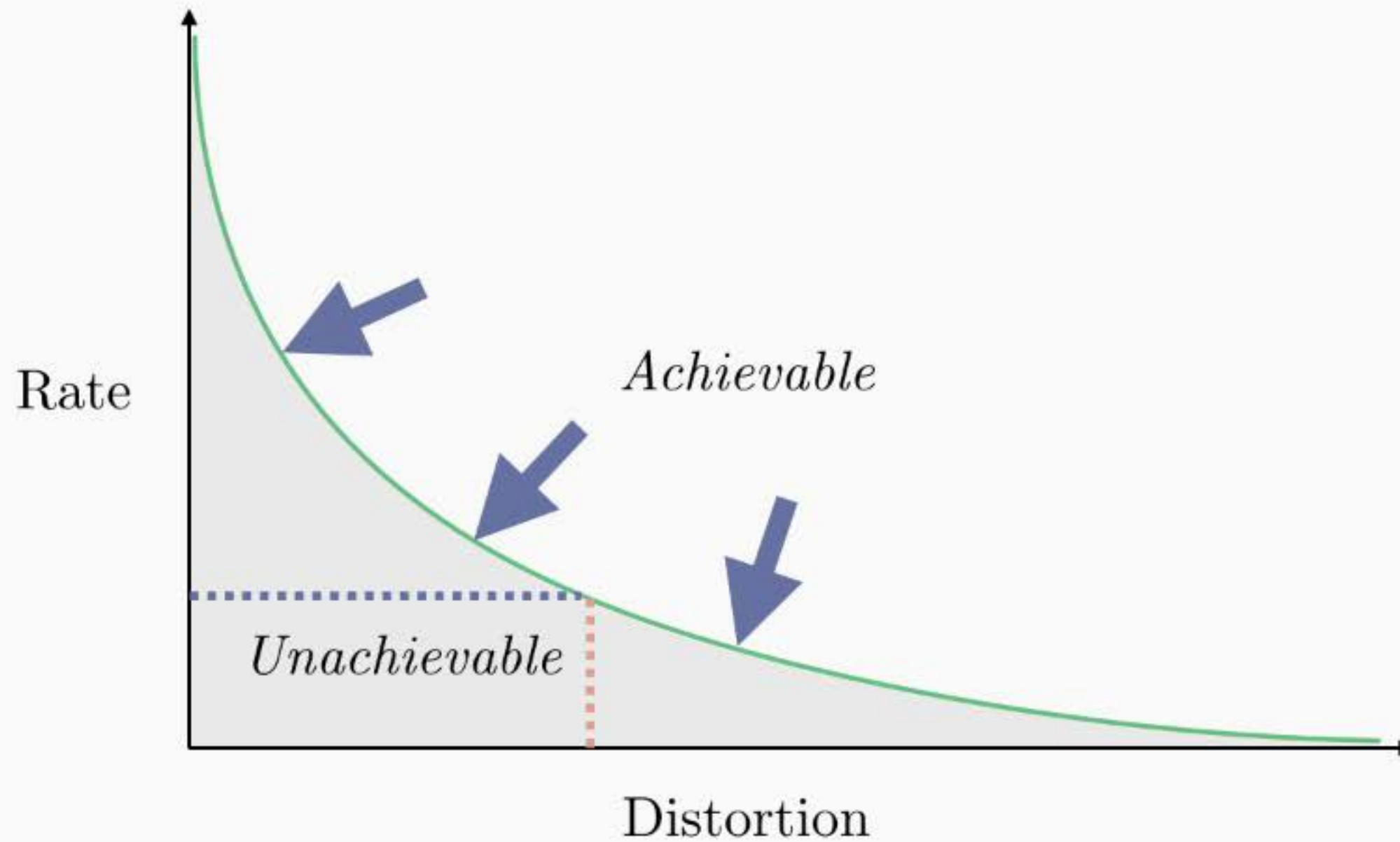
[Shannon '48, Berger '03]

# Rate-Distortion Theory

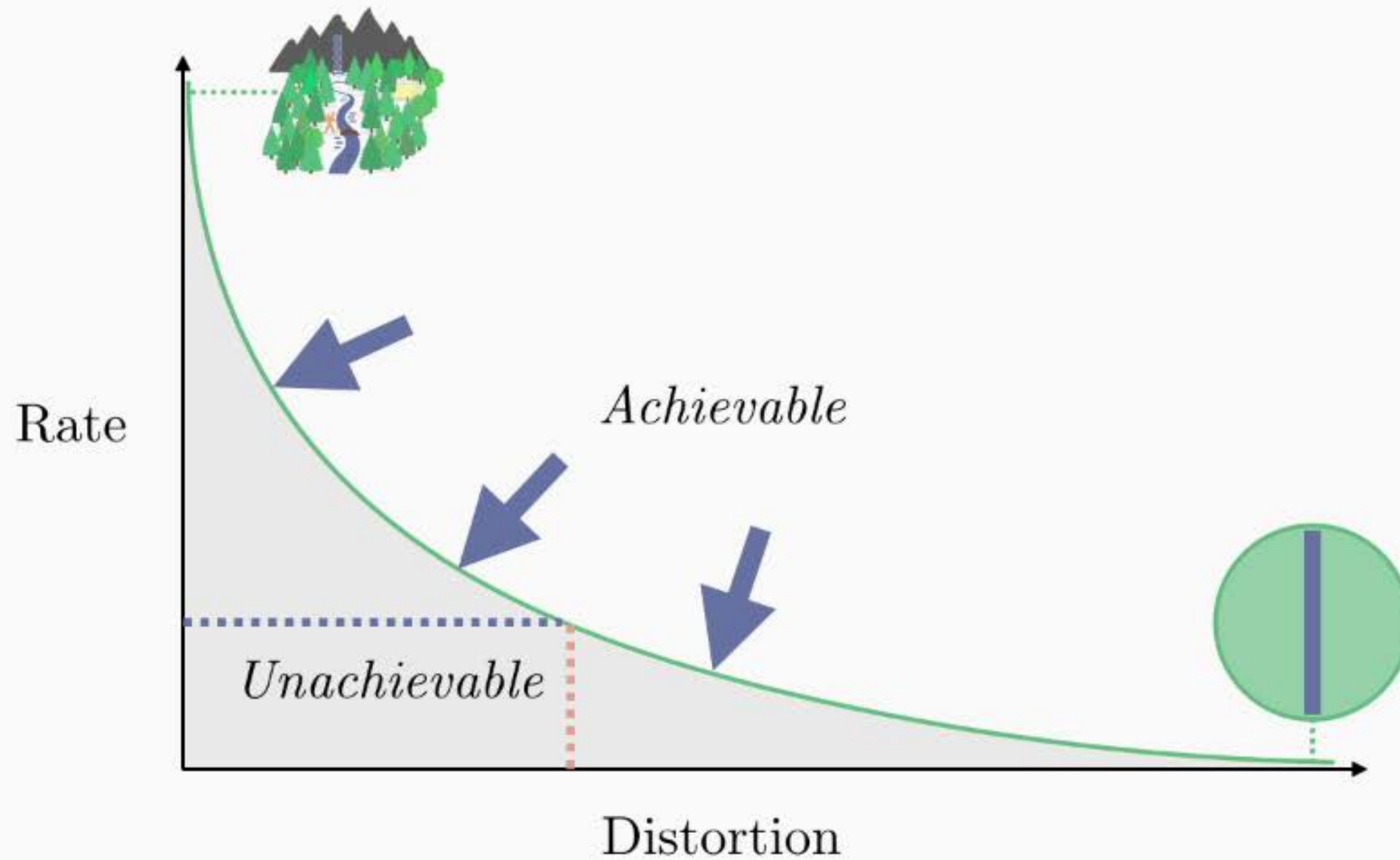




# Rate-Distortion Theory



# Rate-Distortion Theory



# Blahut-Arimoto

**Problem:** 
$$\min_{p(\tilde{x}|x)} \underbrace{I(X; \tilde{X})}_{\text{Rate}} + \beta \underbrace{\mathbb{E}[d(x, \tilde{x})]}_{\text{Distortion}}.$$

[Blahut 1972, Arimoto, 1972]

# Blahut-Arimoto

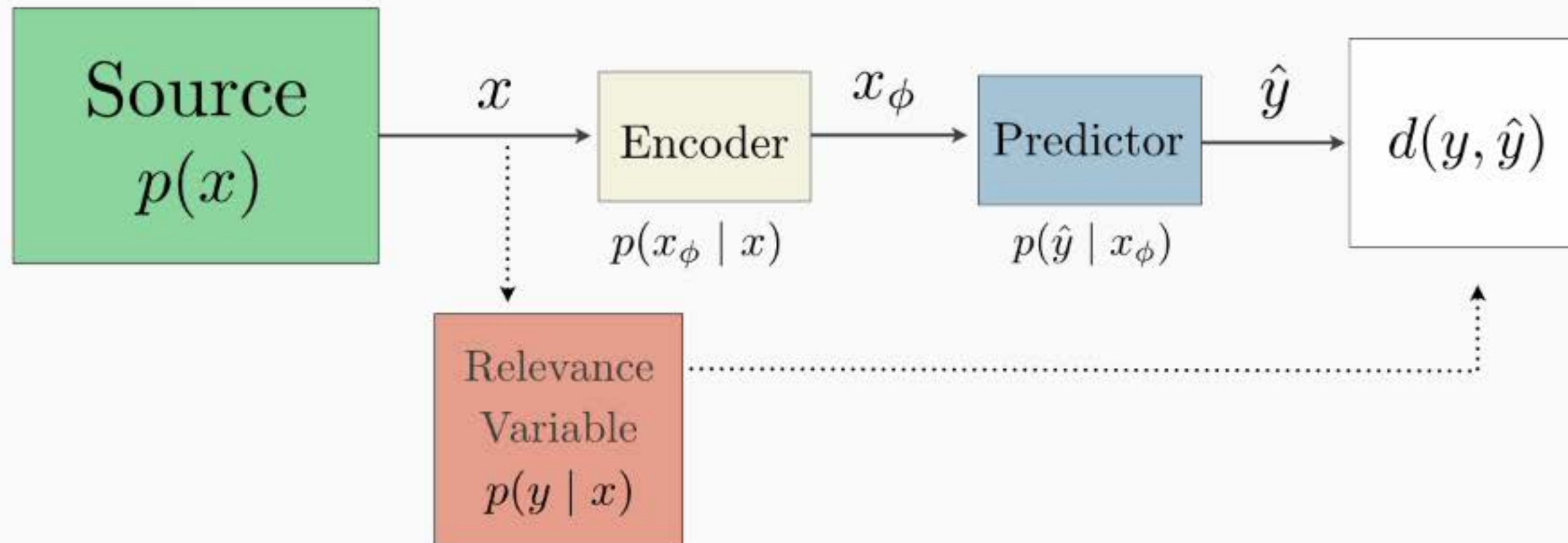
**Problem:** 
$$\min_{p(\tilde{x}|x)} \underbrace{I(X; \tilde{X})}_{\text{Rate}} + \beta \underbrace{\mathbb{E}[d(x, \tilde{x})]}_{\text{Distortion}}.$$

**Solution:**

$$p_{t+1}(\tilde{x}) = \sum_x p_t(x) p_t(\tilde{x} | x)$$
$$p_{t+1}(\tilde{x} | x) = \frac{p_t(\tilde{x}) \exp\{-\beta d(x, \tilde{x})\}}{\sum_x p_t(x) \exp\{-\beta d(x, \tilde{x})\}}$$

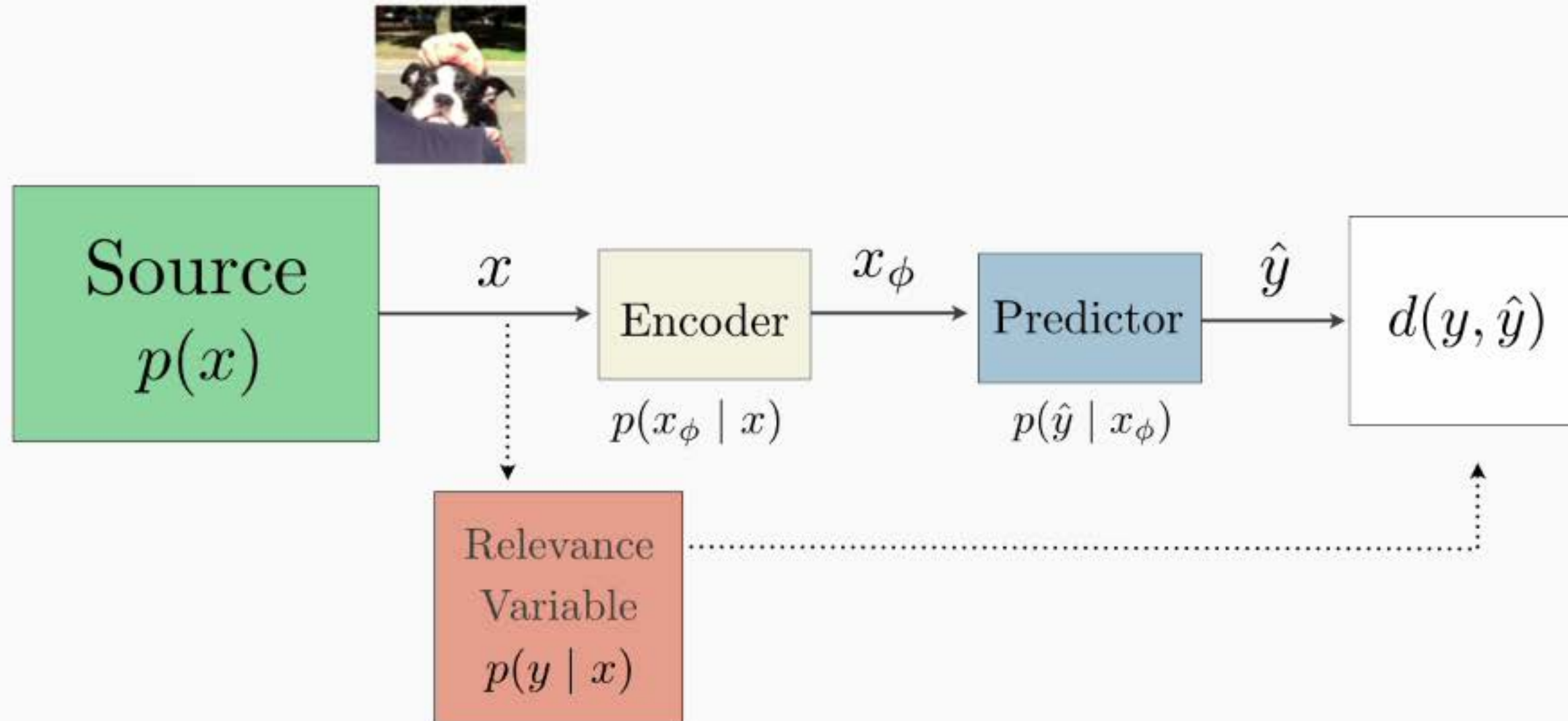
[Blahut 1972, Arimoto, 1972]

# Information Bottleneck



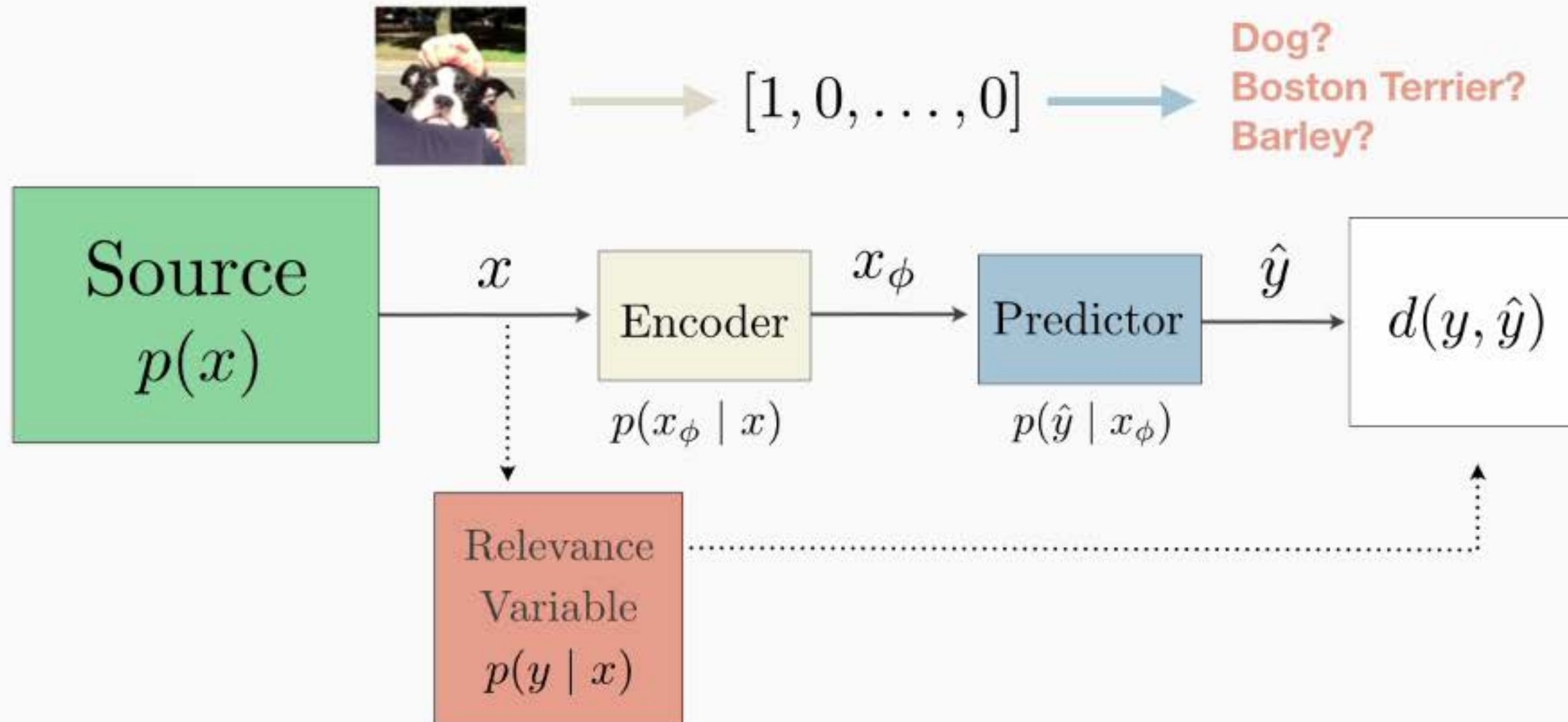
[Tishby, Pereira, Bialek '99]

# Information Bottleneck



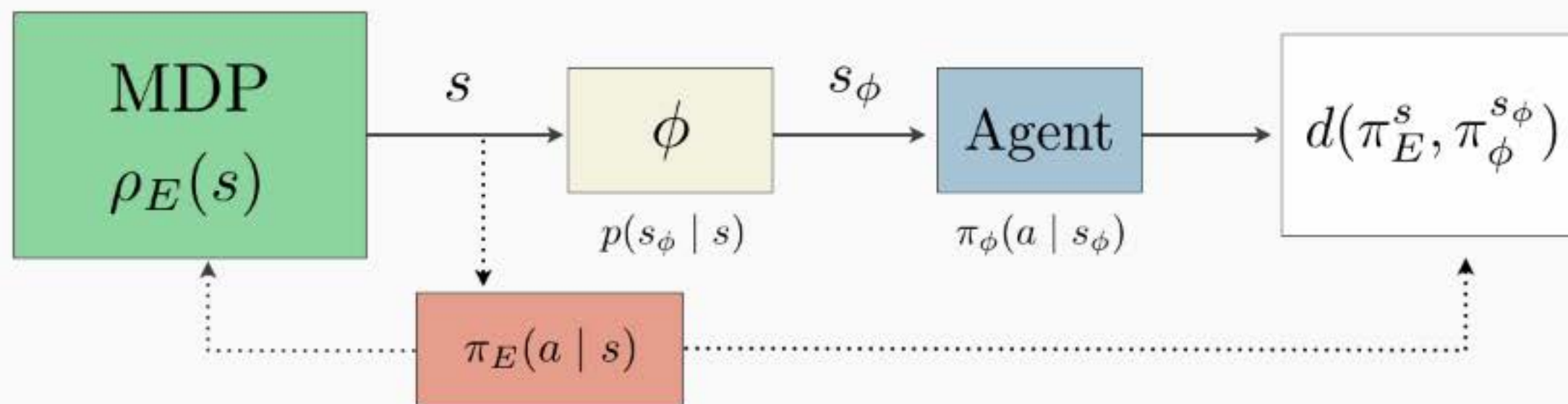
[Tishby, Pereira, Bialek '99]

# Information Bottleneck



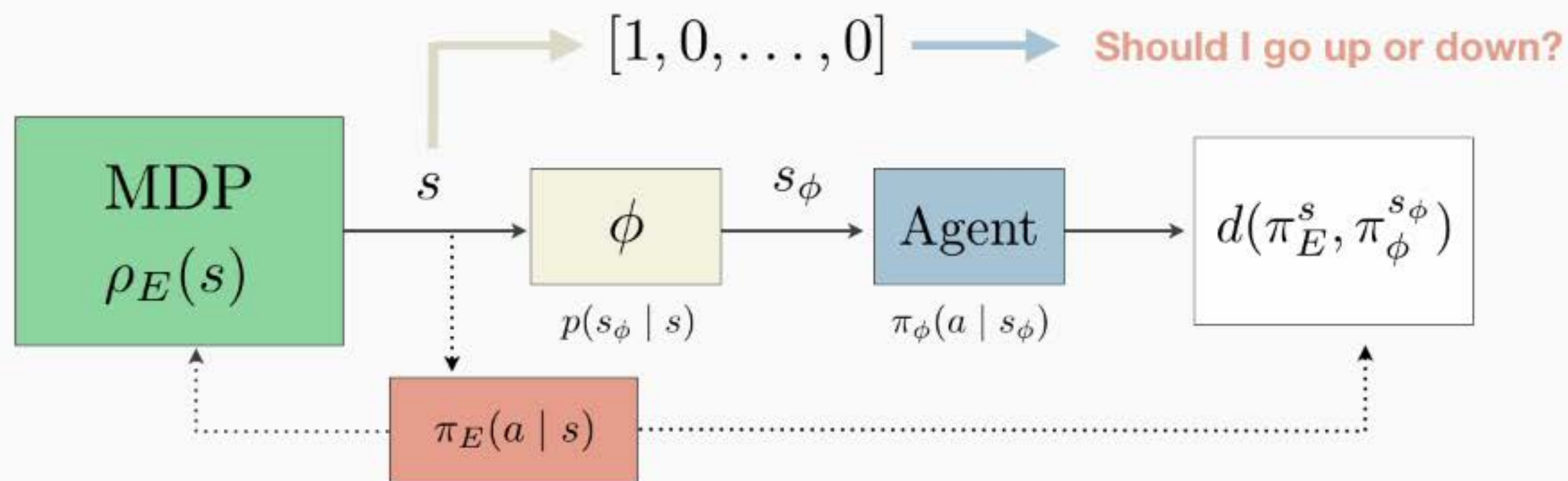
[Tishby, Pereira, Bialek '99]

# State Abstraction as Compression

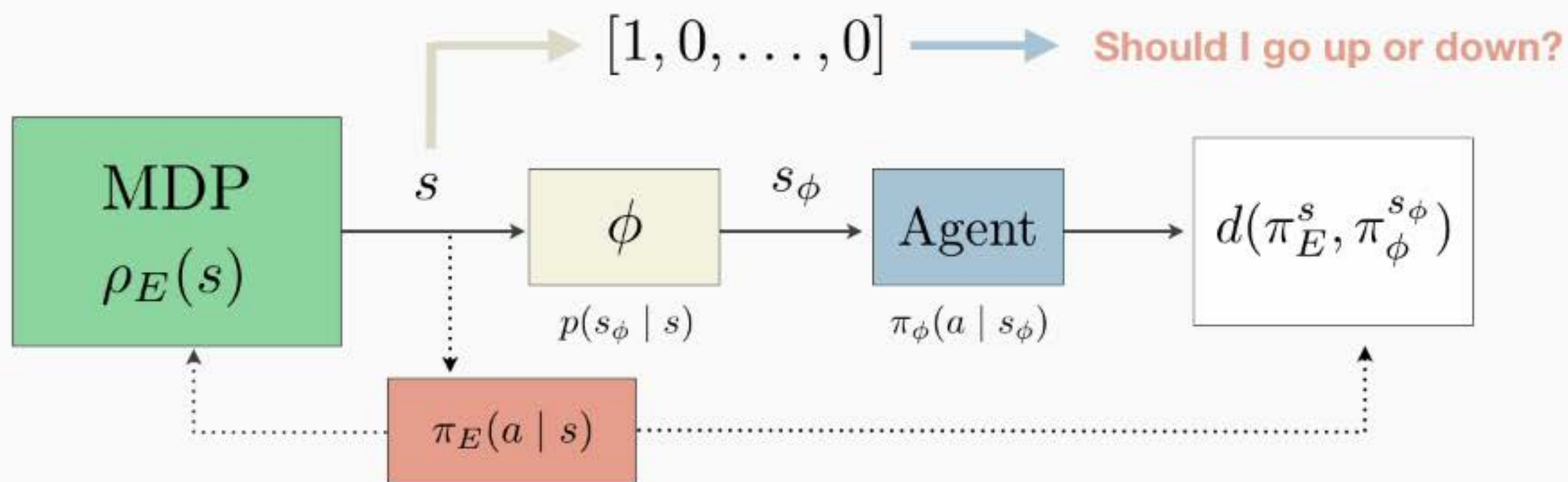




# State Abstraction as Compression



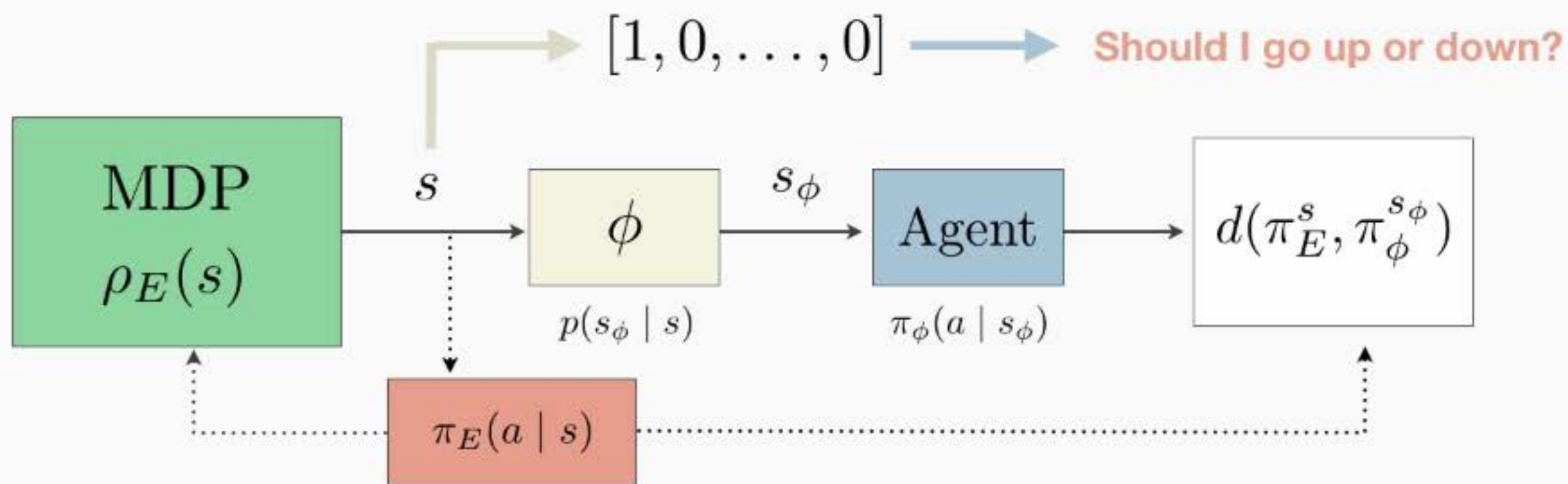
# State Abstraction as Compression



$$\min_{\phi} \left( |\mathcal{S}_\phi| + \beta \mathbb{E}_{\rho_E(s)} \left[ V^{\pi_E}(s) - V^{\pi_\phi^*}(s) \right] \right)$$

*Our Objective*

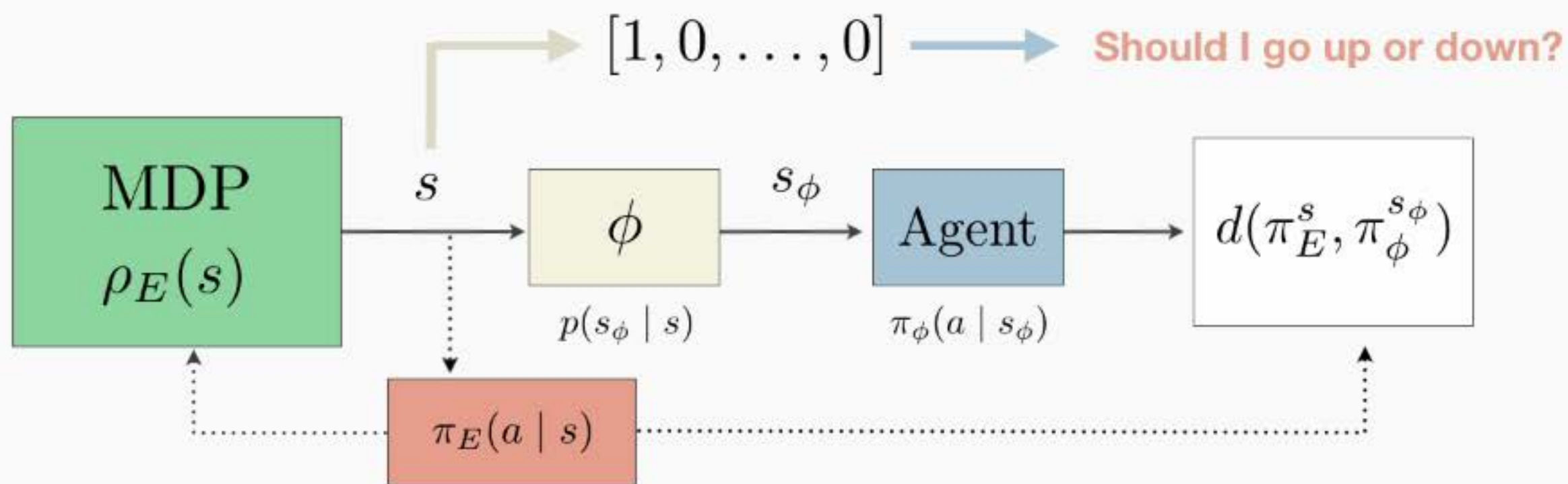
# State Abstraction as Compression



*Our Objective*

Preserves  
Solution Quality

# State Abstraction as Compression



$$\min_{\phi} \left( |\mathcal{S}_\phi| + \beta \mathbb{E}_{\rho_E(s)} \left[ V^{\pi_E}(s) - V^{\pi_\phi^*}(s) \right] \right)$$

*Our Objective*

Preserves Solution Quality

# State Abstraction as Compression

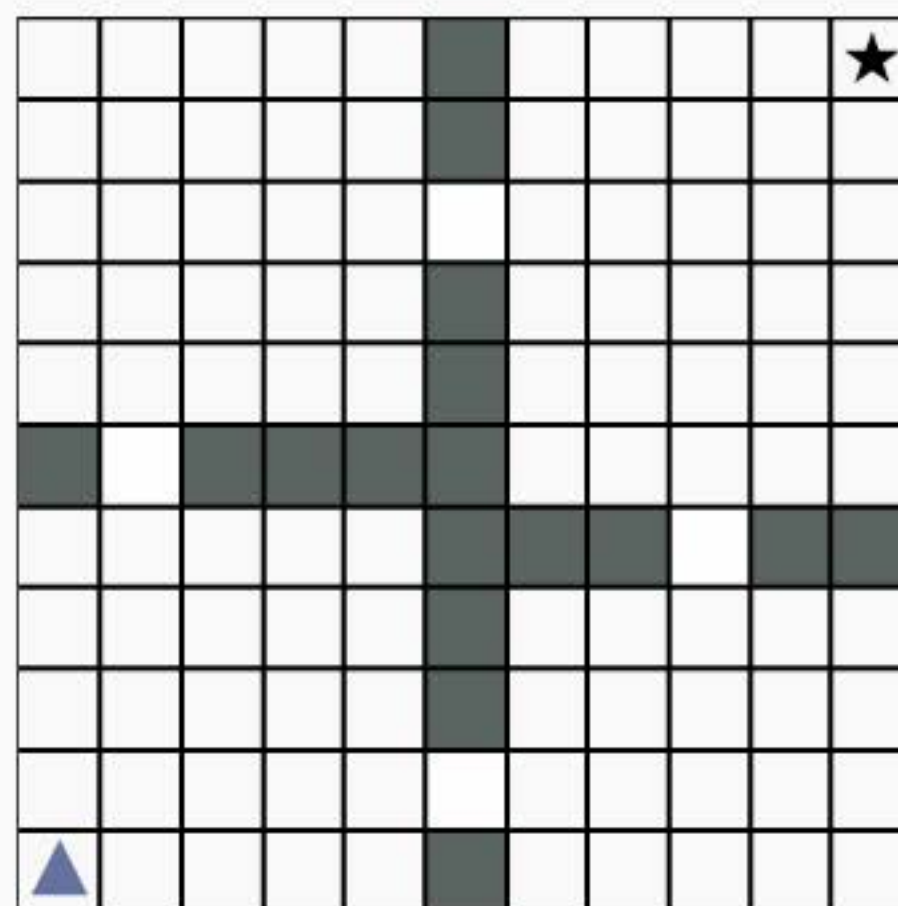
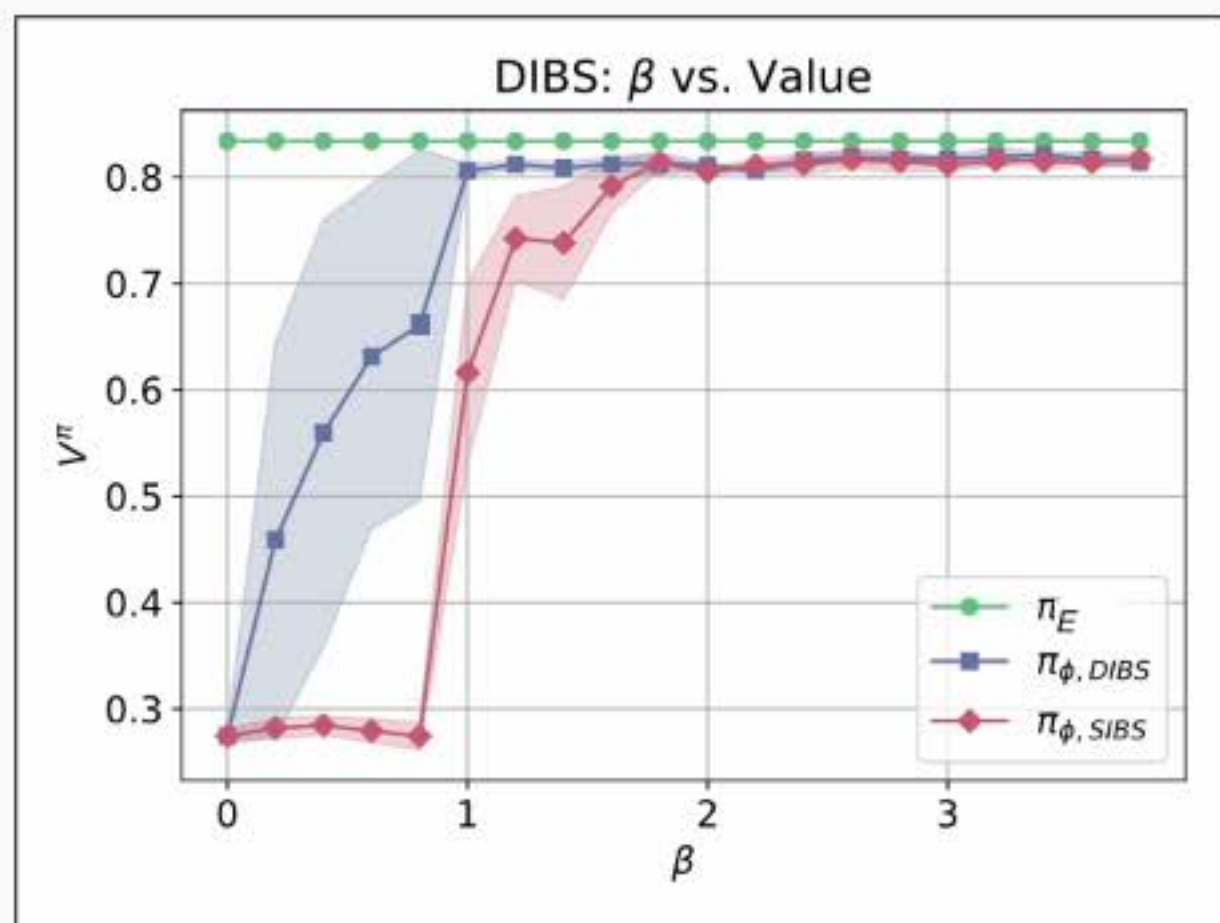
**Theorem.**

$$\min_{\phi} \left( |\mathcal{S}_{\phi}| + \beta \mathbb{E}_{\rho_E(s)} \left[ V^{\pi_E}(s) - V^{\pi_{\phi}^*}(s) \right] \right)$$

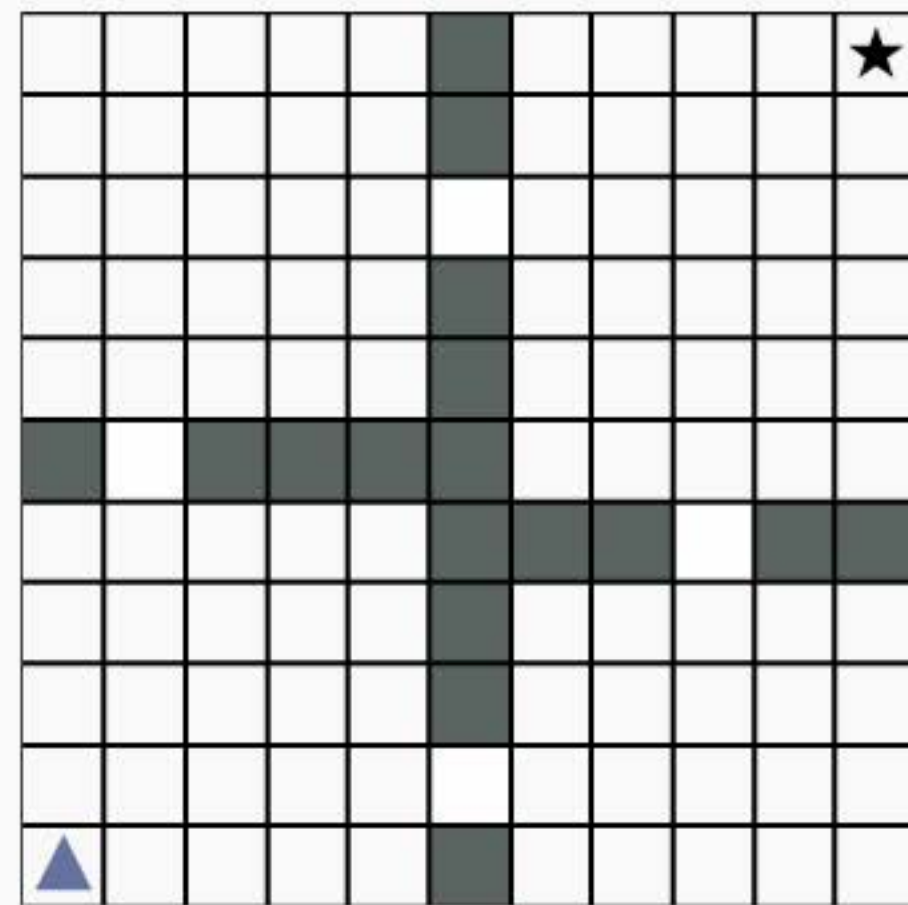
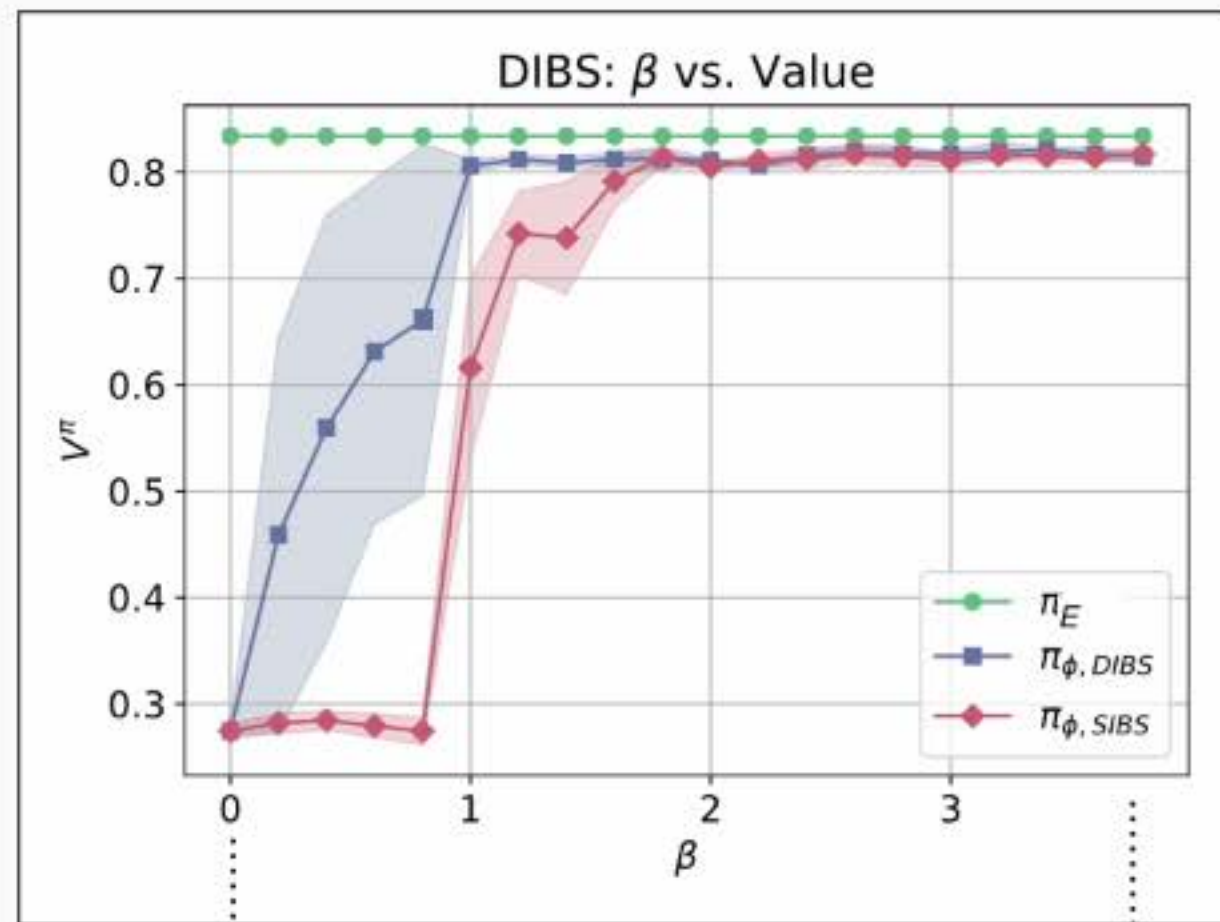
*Our Objective*  $\leq$  *DIB Objective*

$$\min_{\phi} \left( C_1 H(\rho_{\phi}) + C_2 \beta \mathbb{E}_{\rho_E(s)} \left[ D_{\text{KL}}(\pi_E(s) \parallel \pi_{\phi}^*(s)) \right] \right)$$

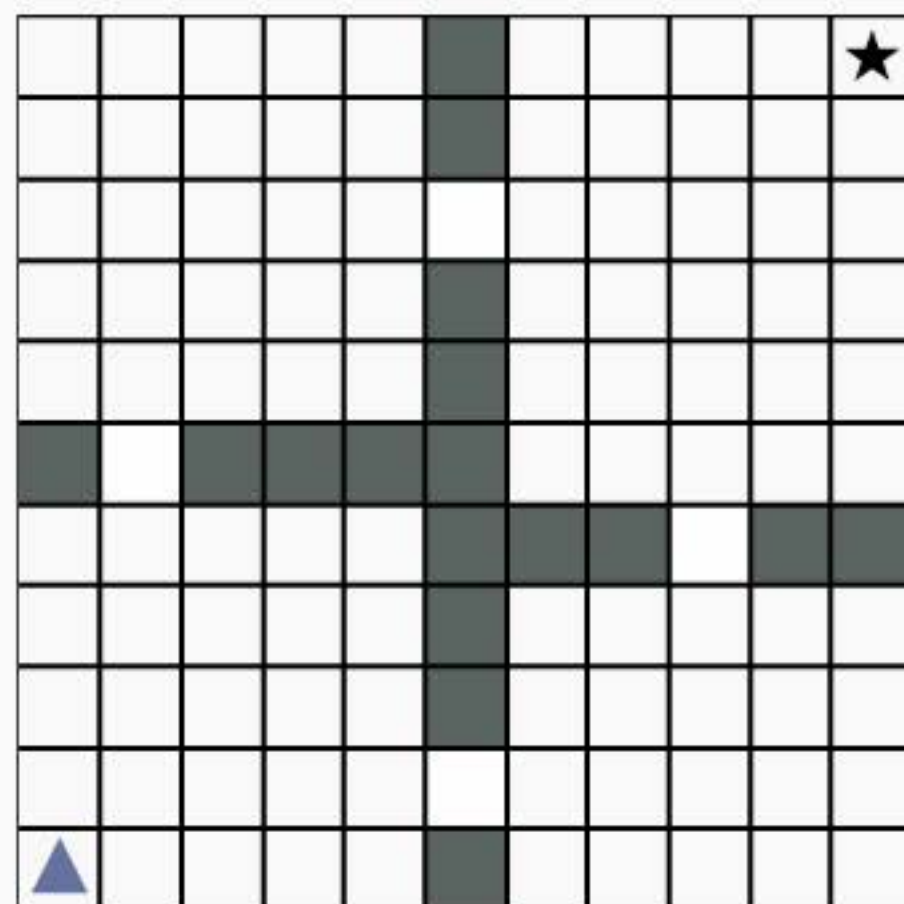
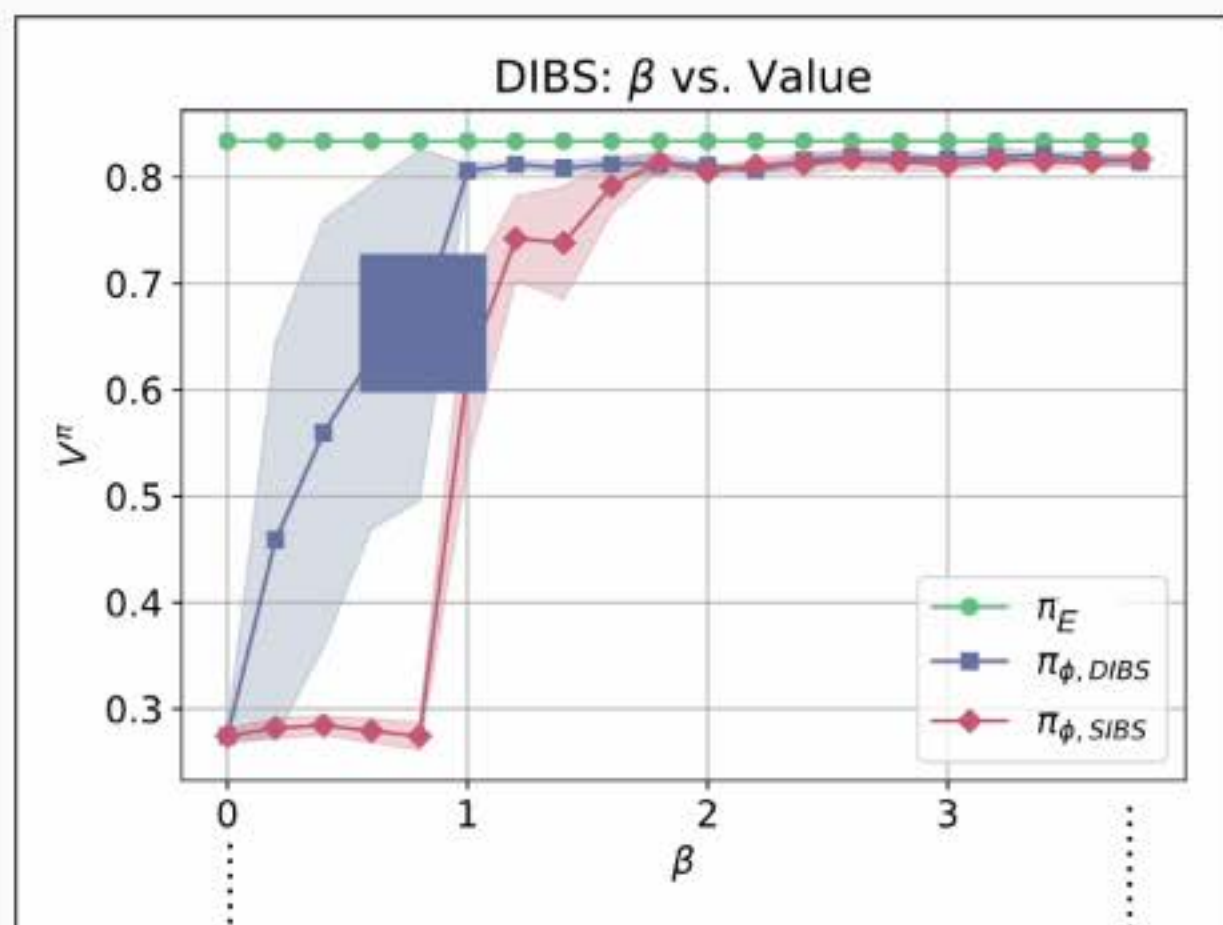
# State Abstraction as Compression



# State Abstraction as Compression



# State Abstraction as Compression





# State Abstraction as Compression

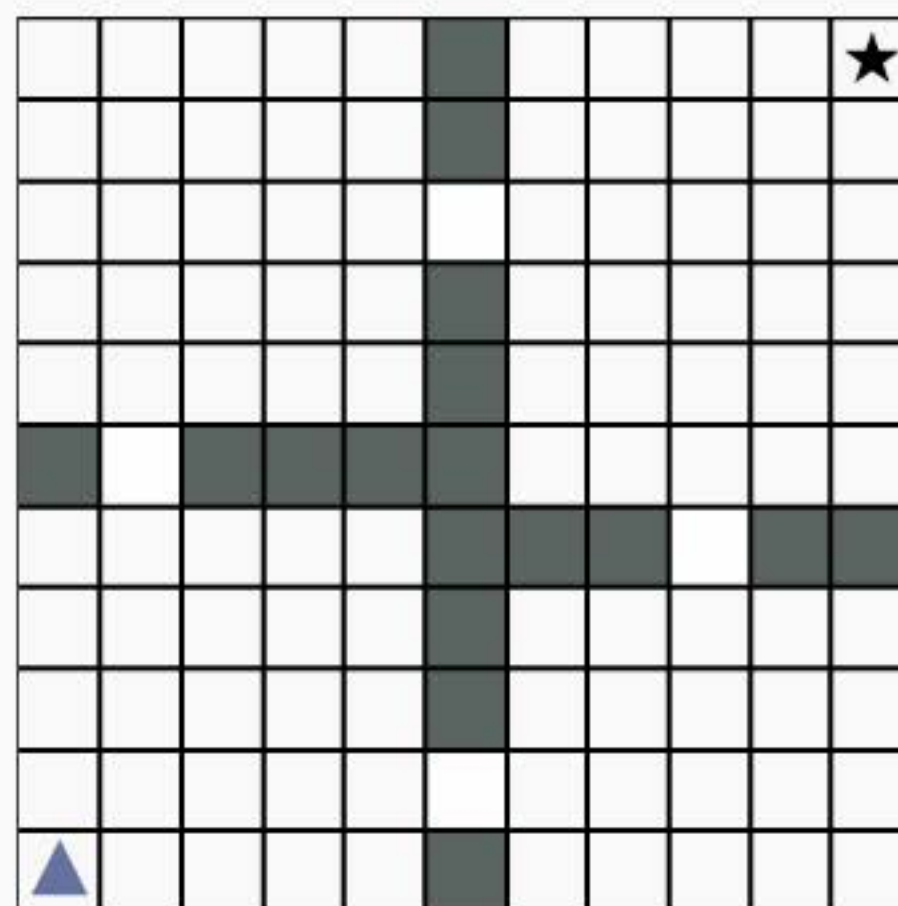
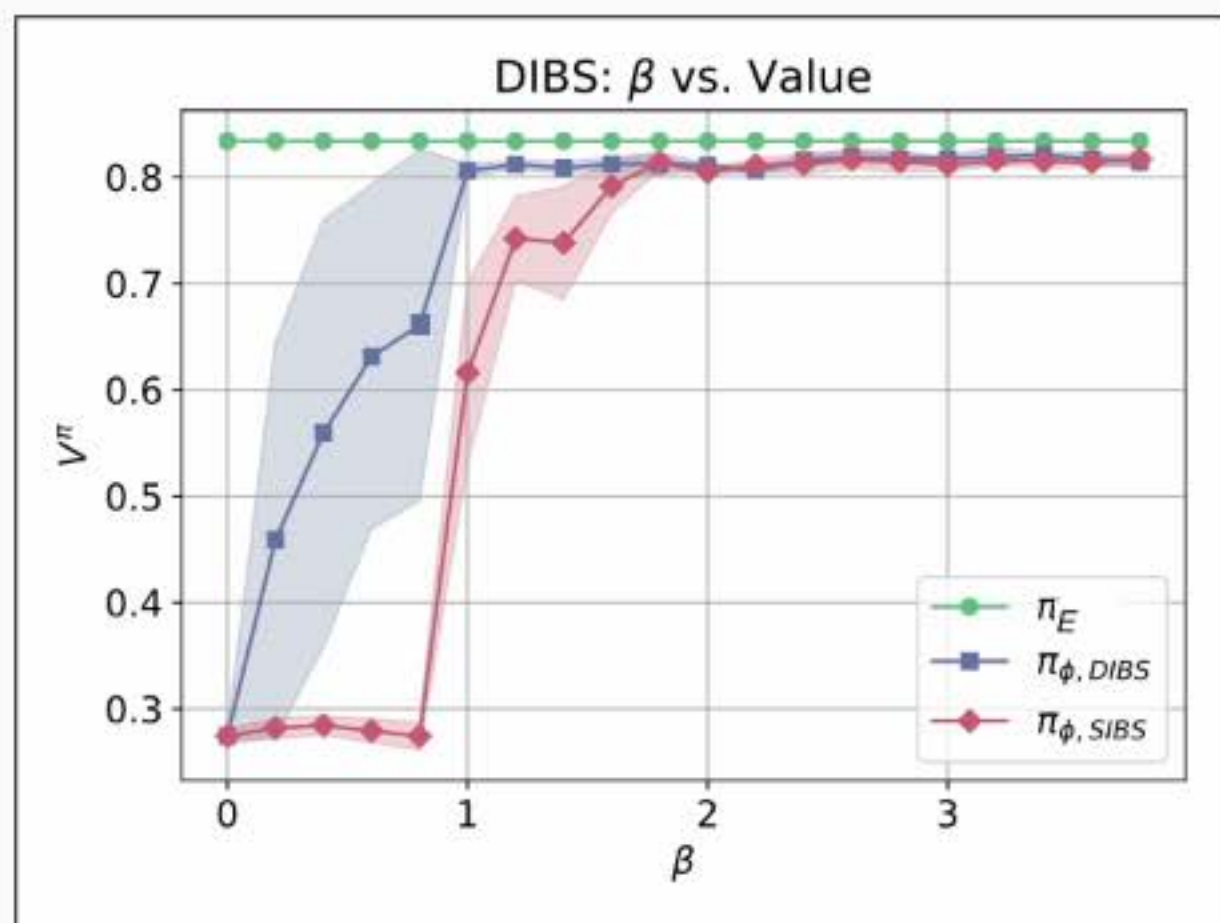
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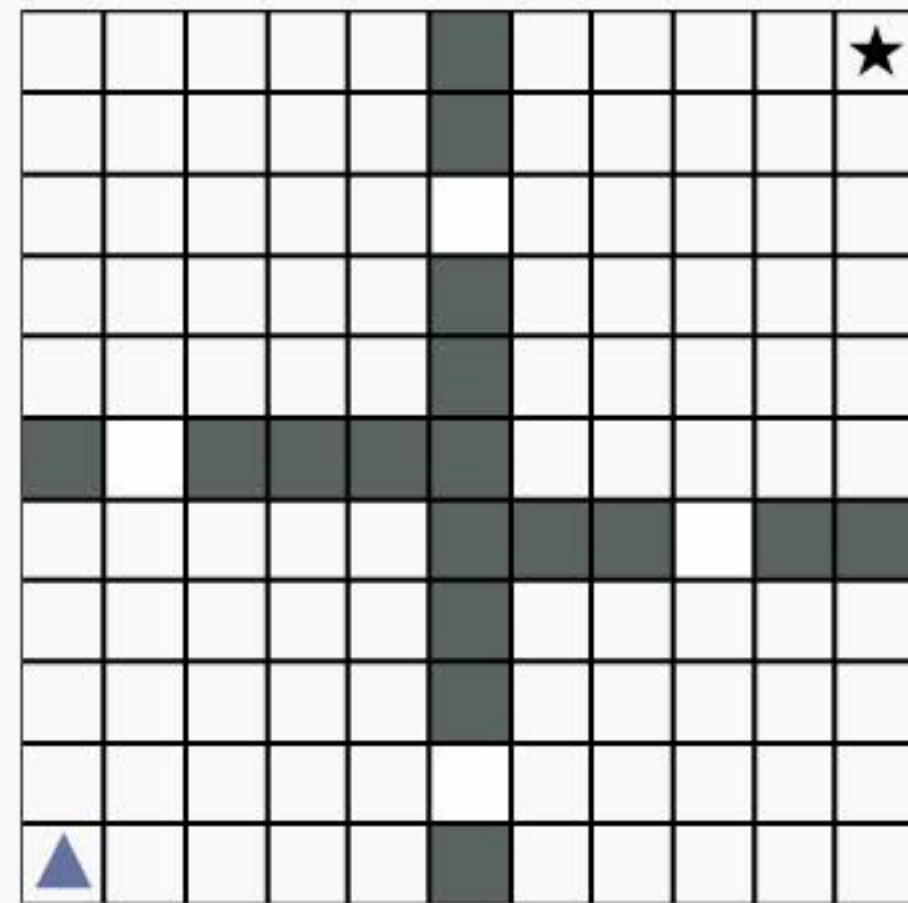
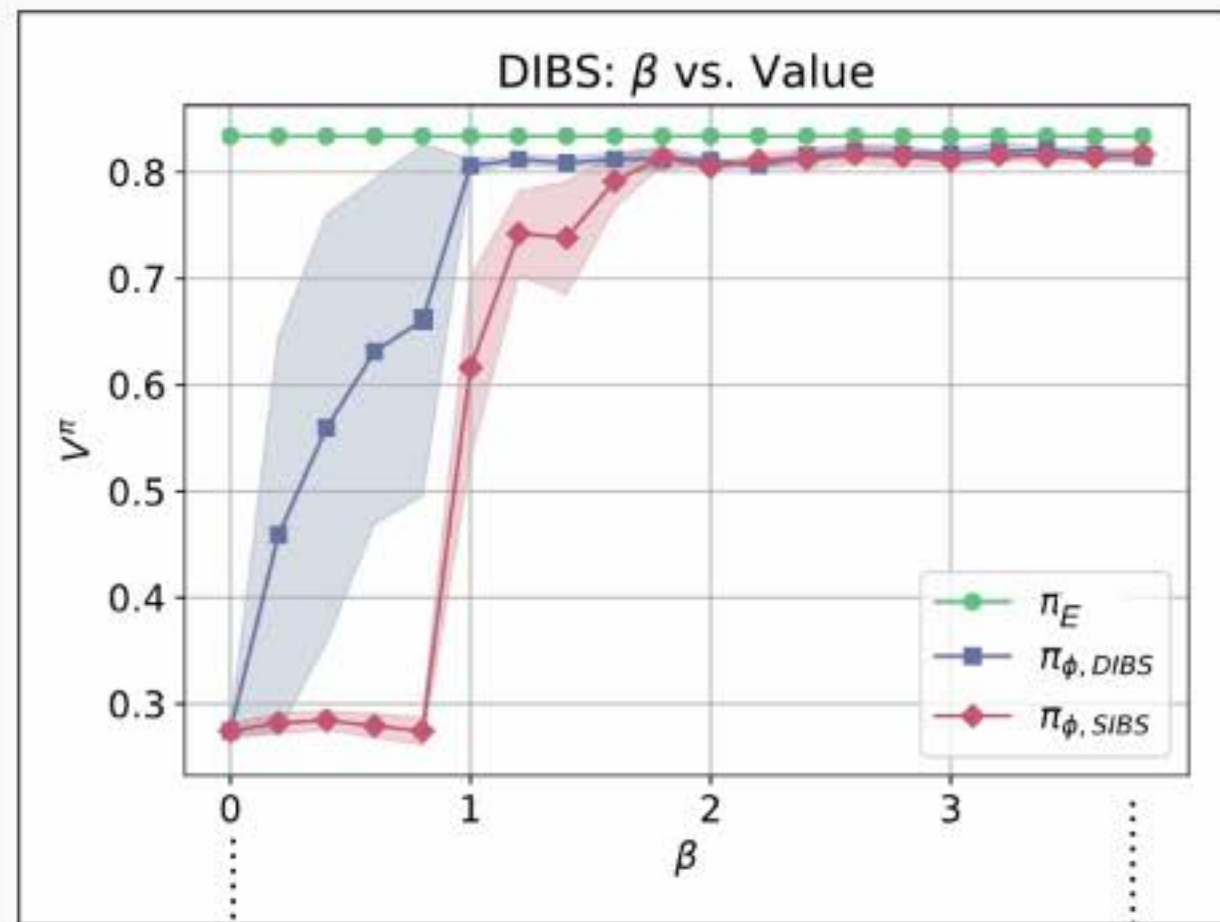
*Our Objective*  $\leq$  *DIB Objective*

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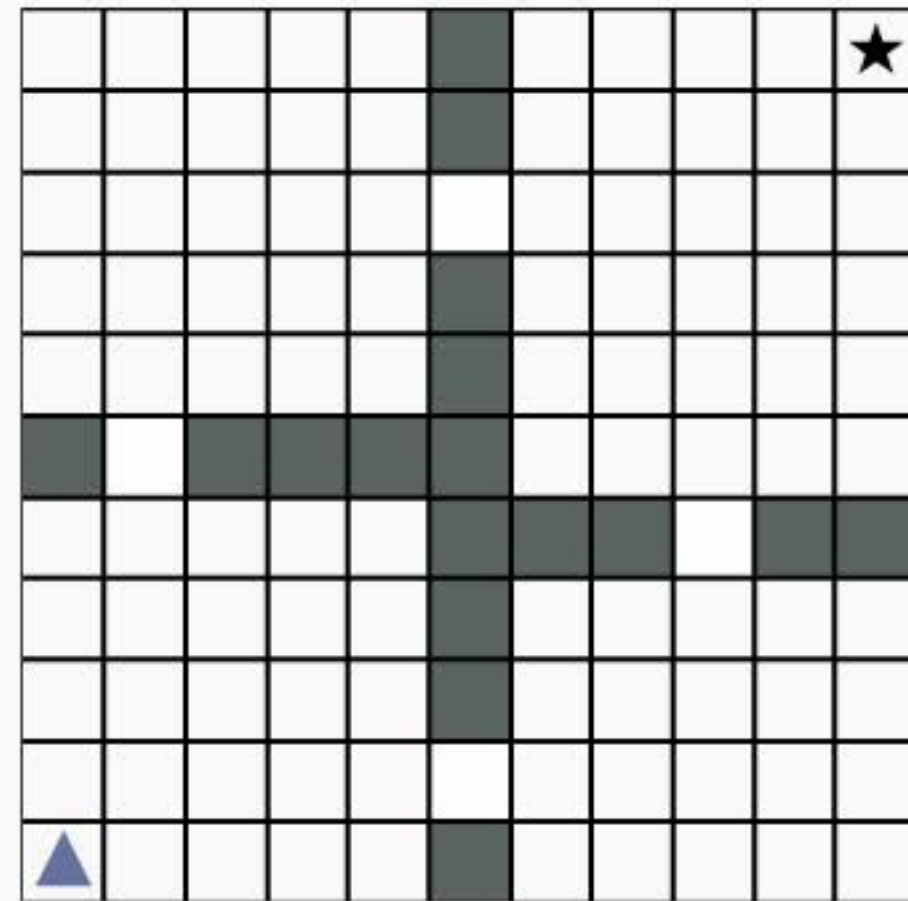
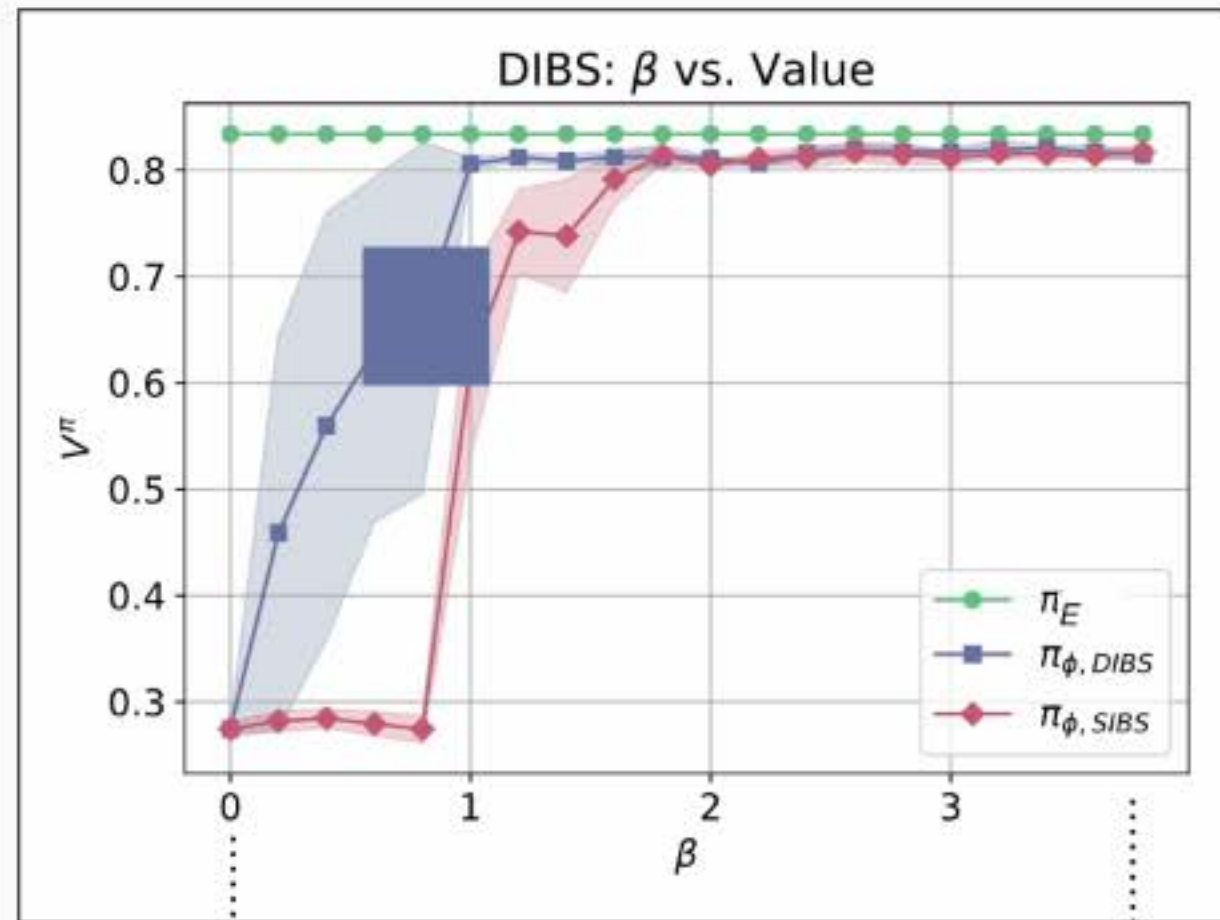
# State Abstraction as Compression



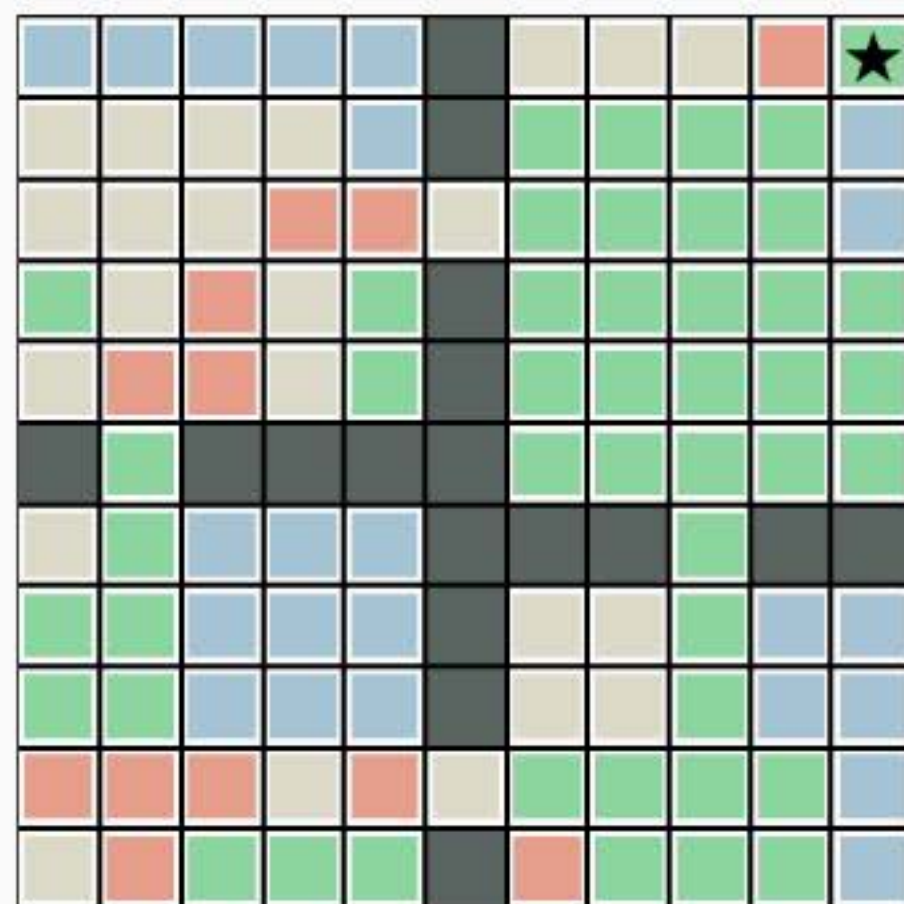
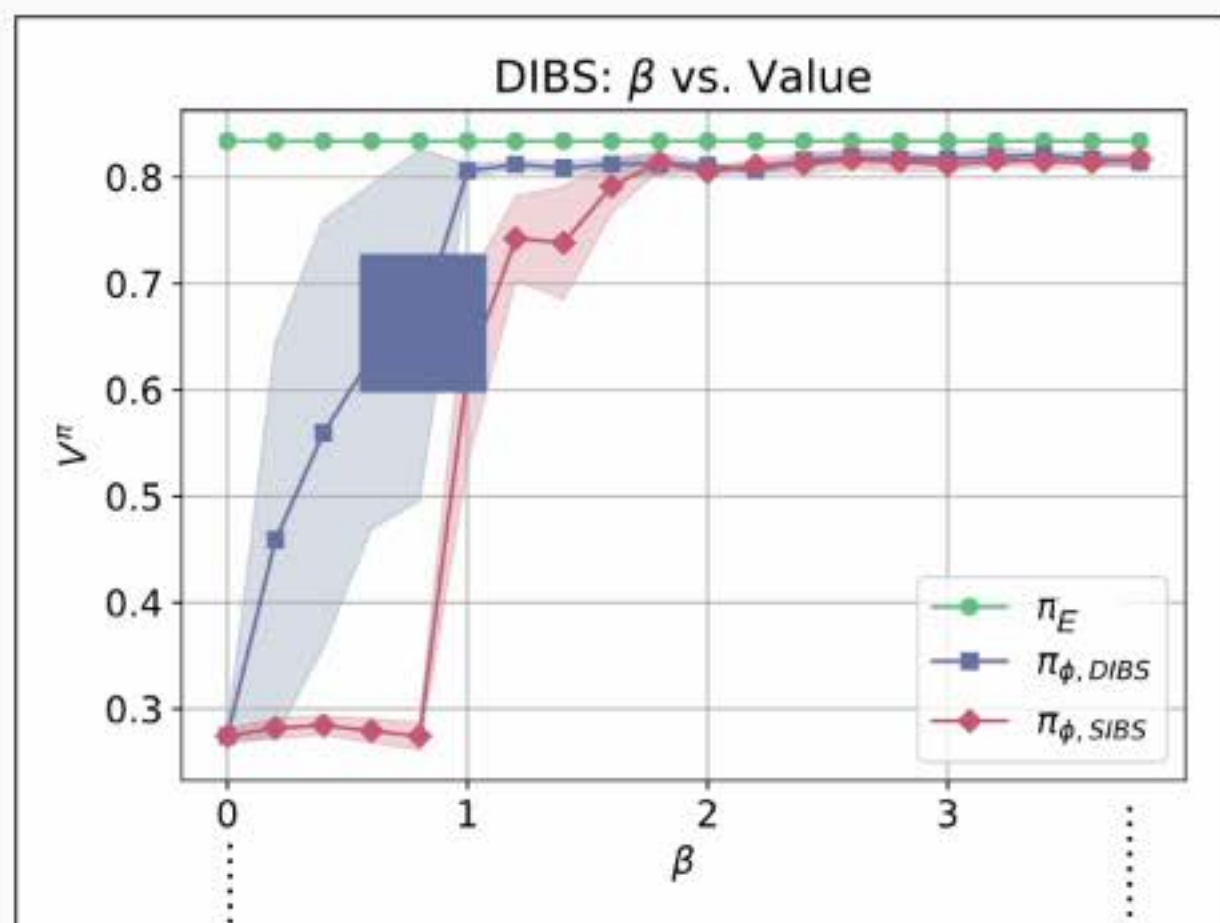
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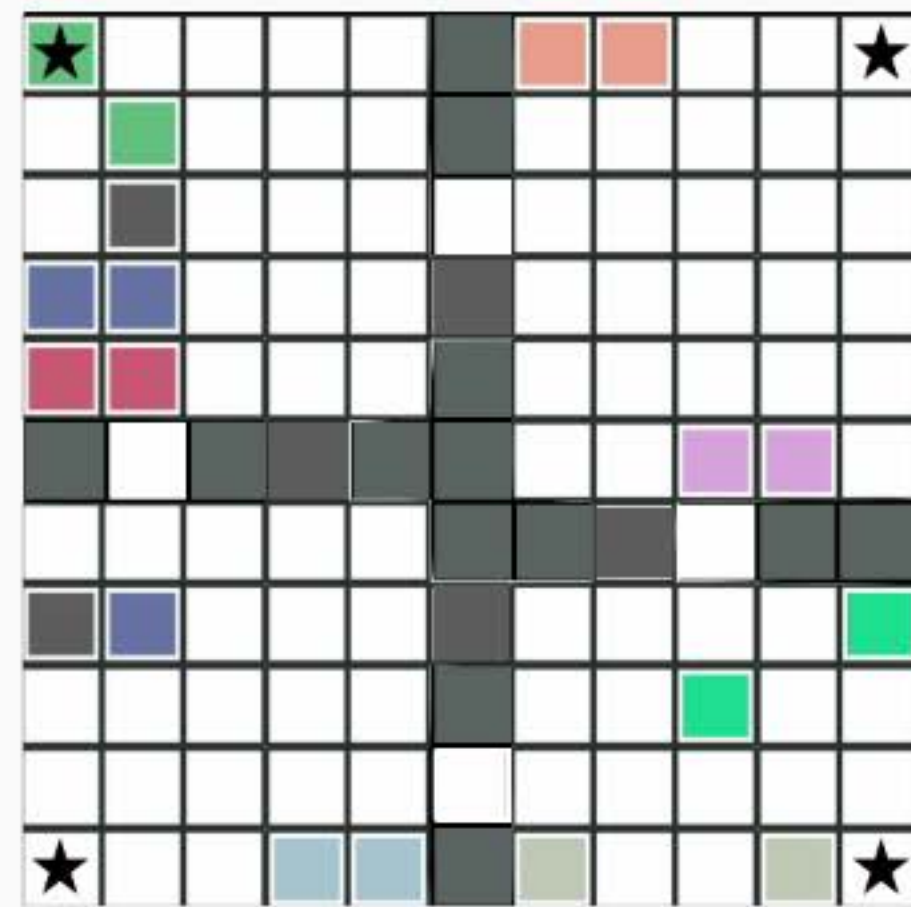
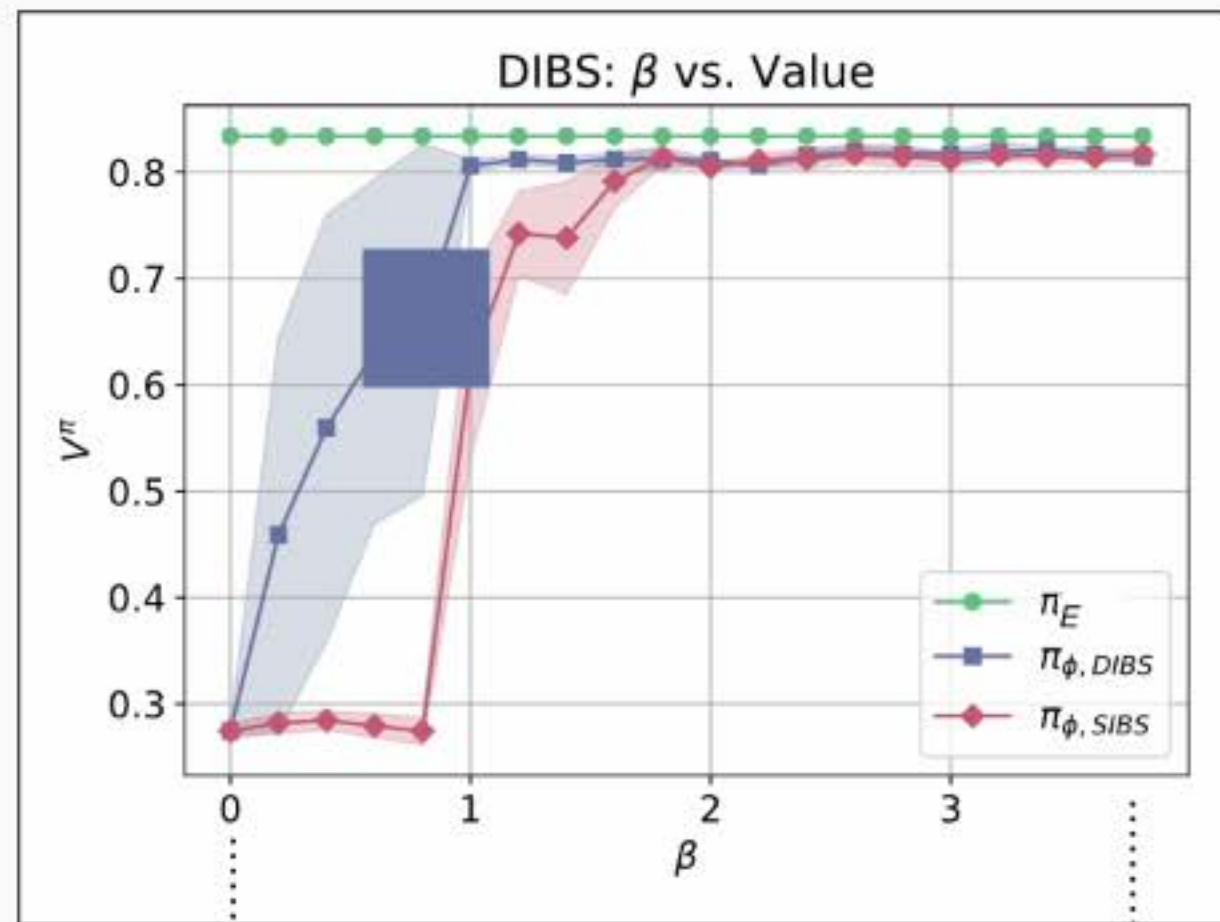
# State Abstraction as Compression



# State Abstraction as Compression

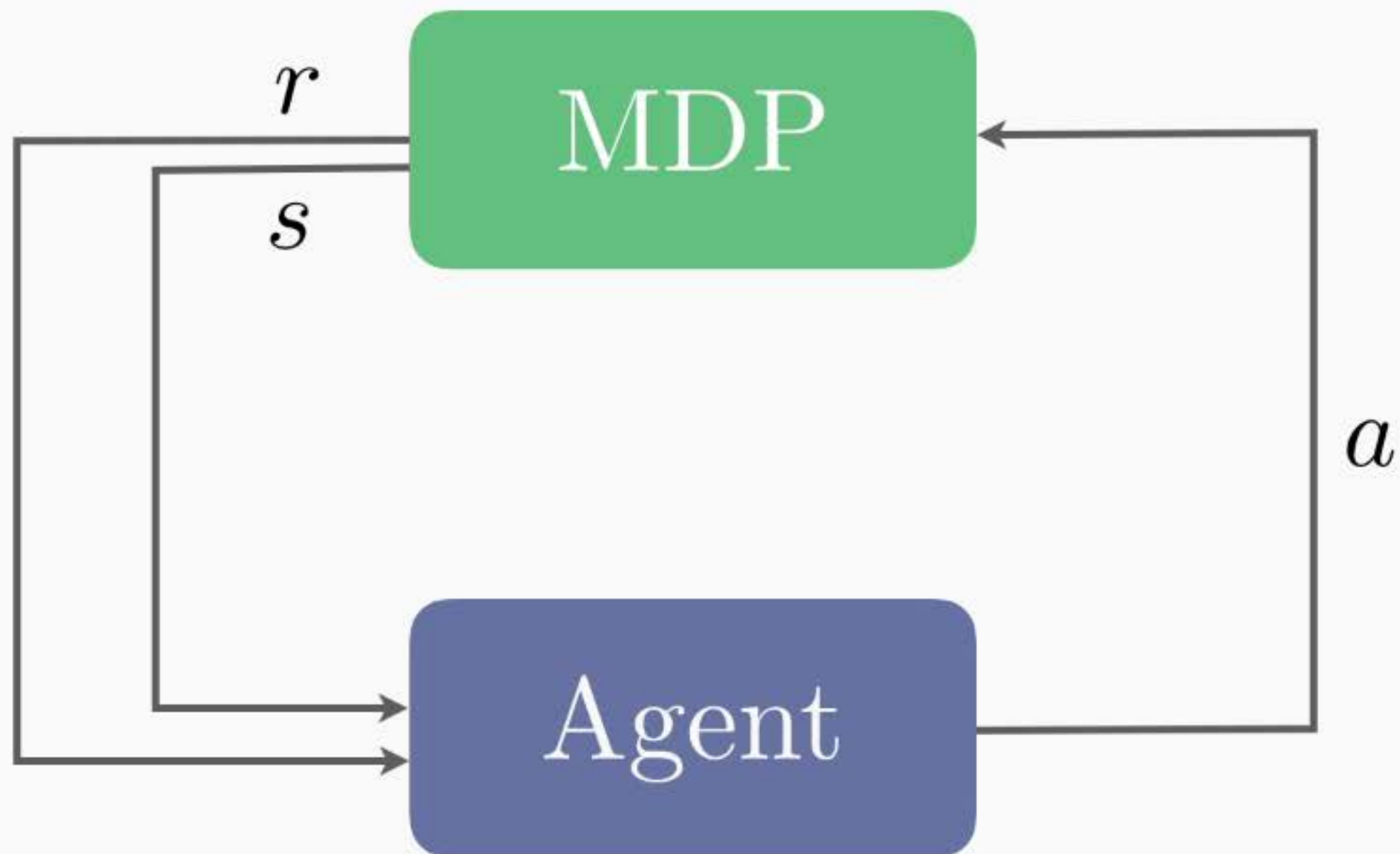


# State Abstraction as Compression

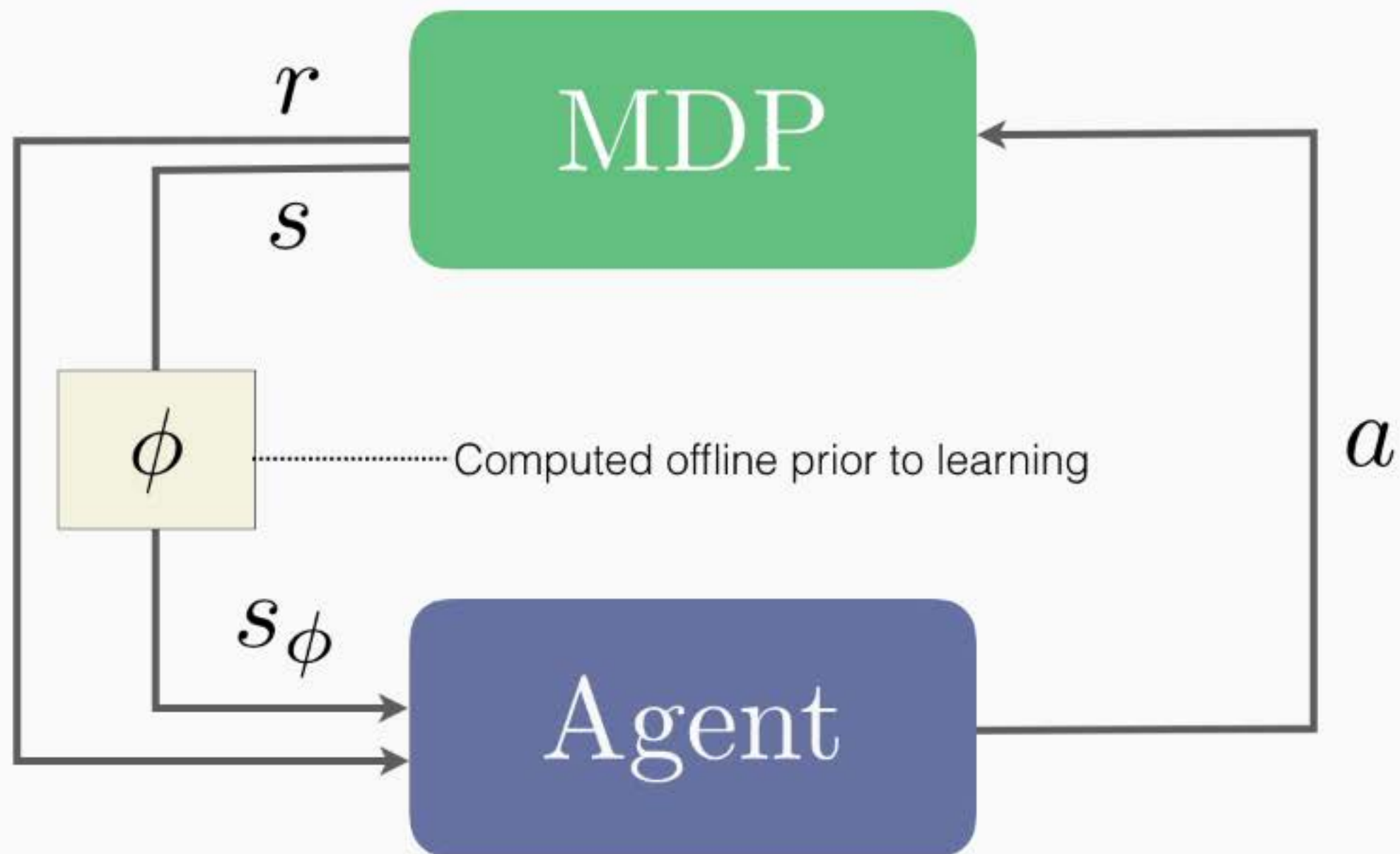


*Multitask Abstraction*

# Learning Experiments

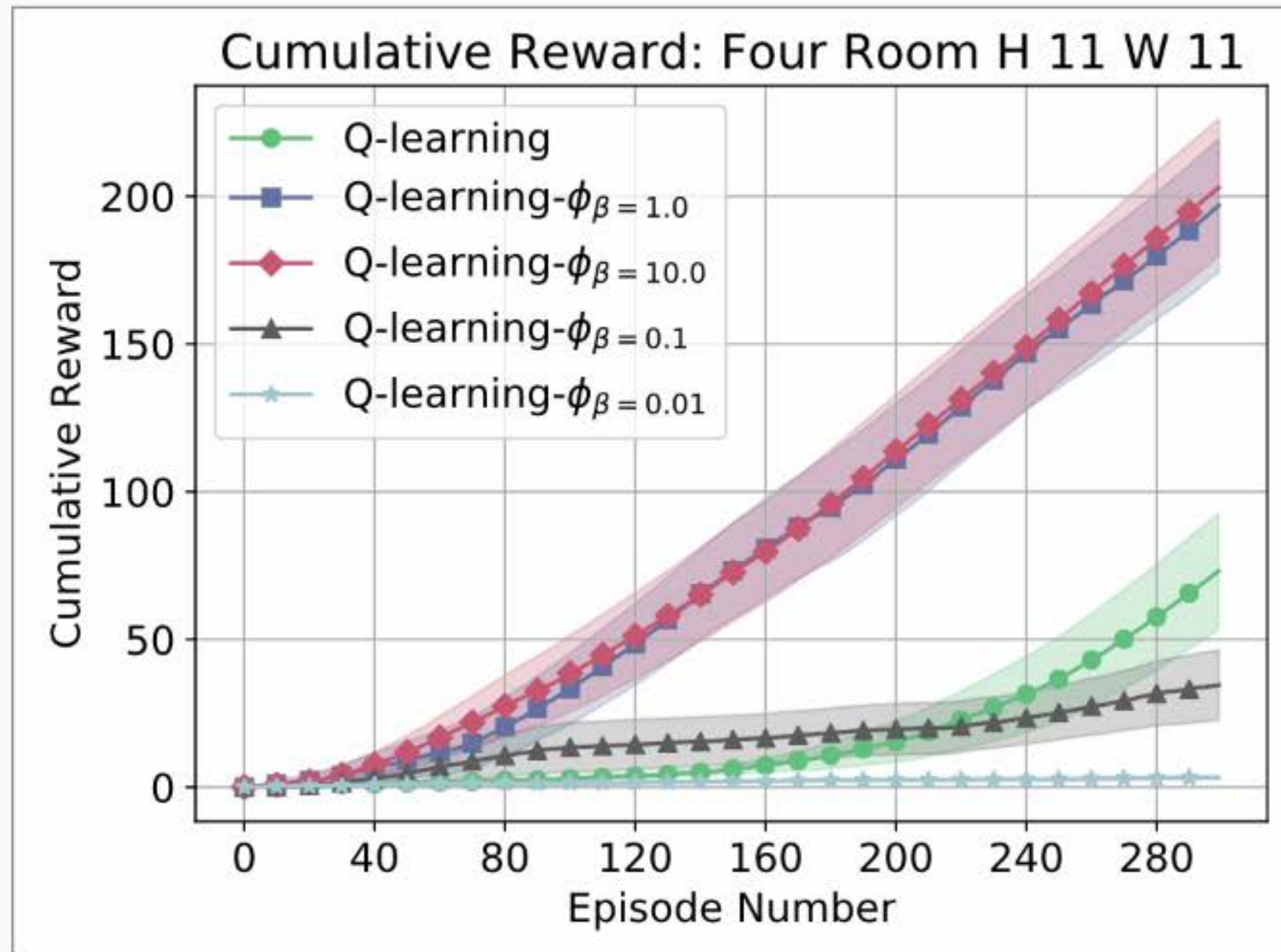


# Learning Experiments

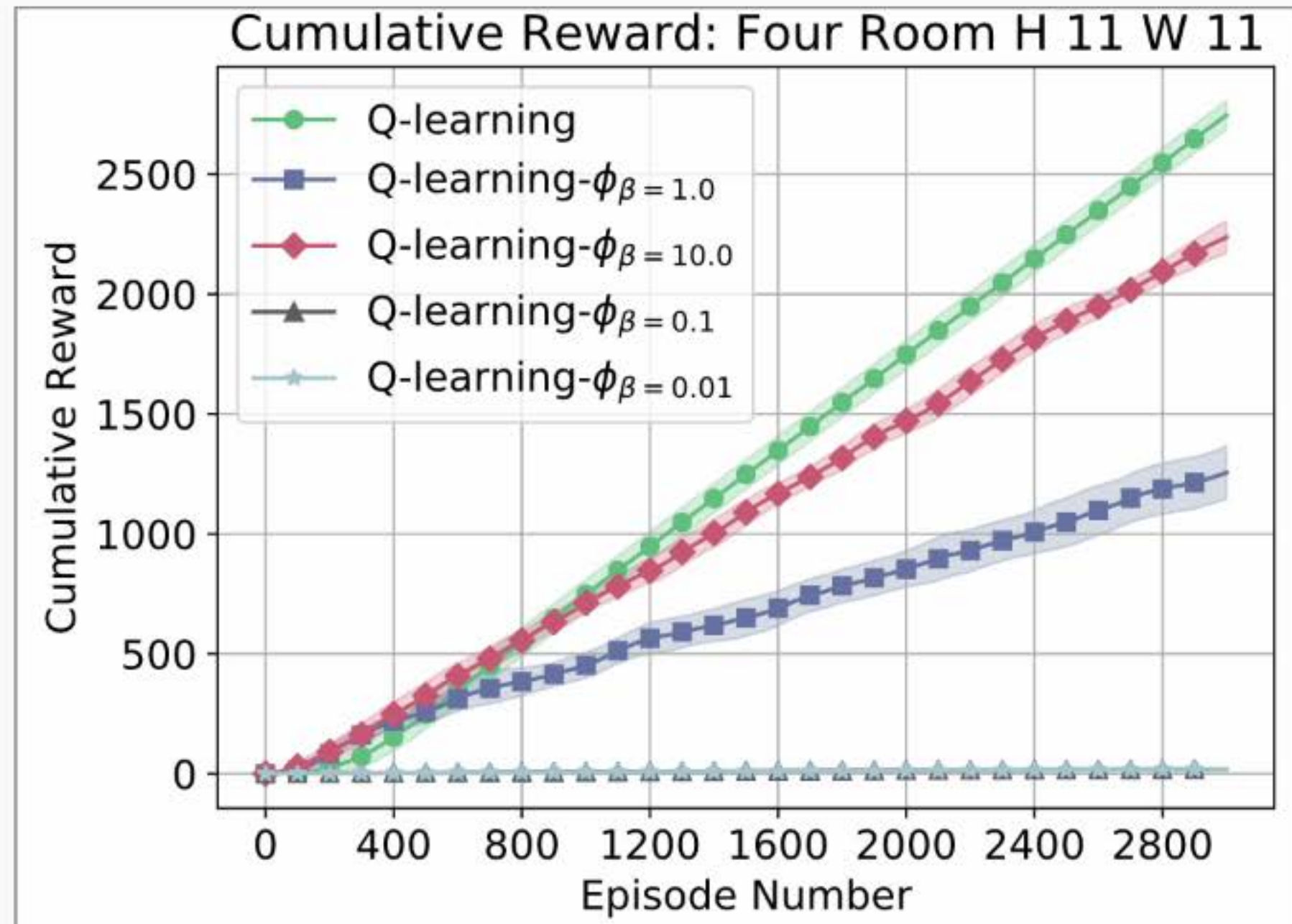




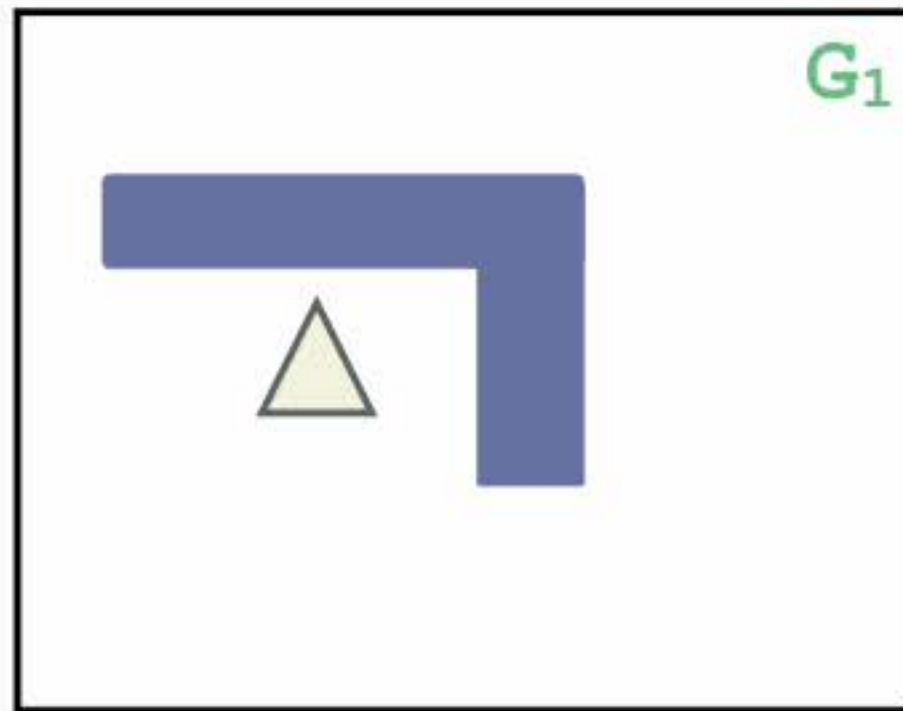
# Experiments: Four Rooms



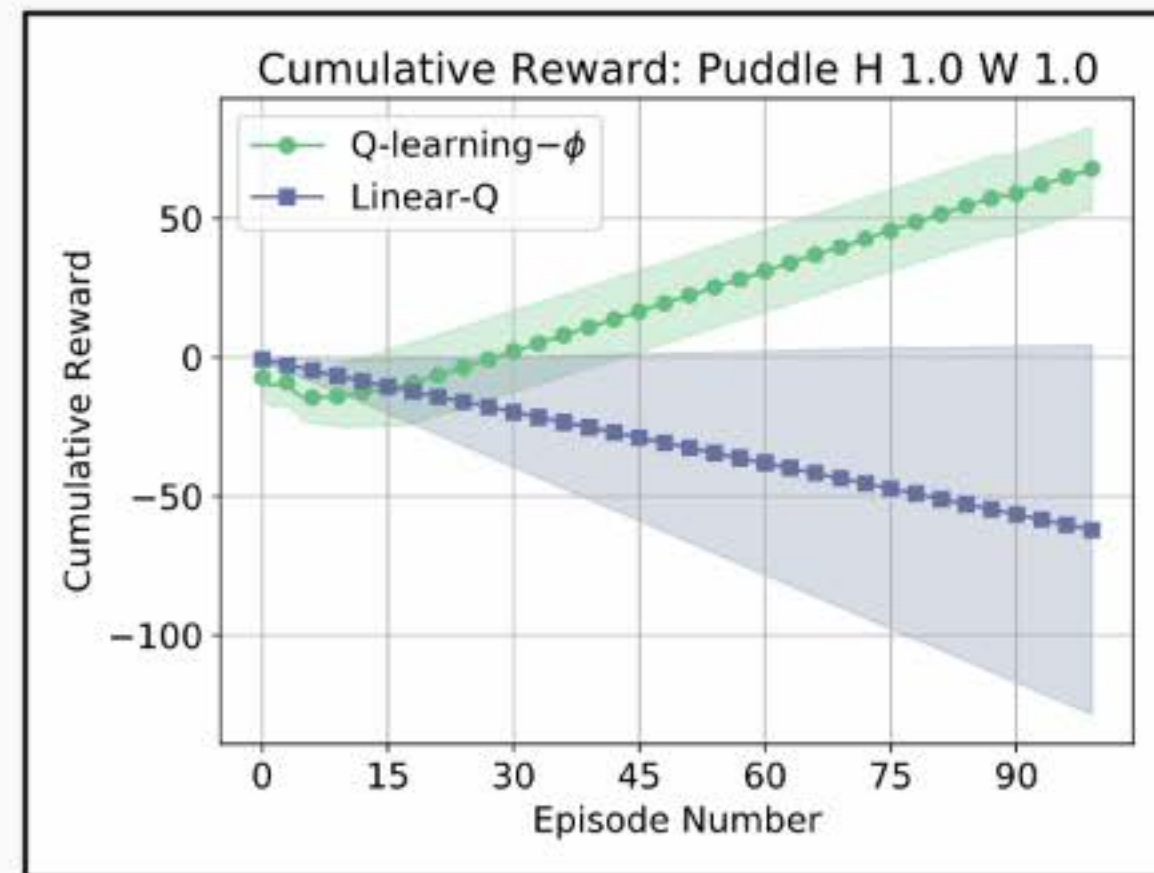
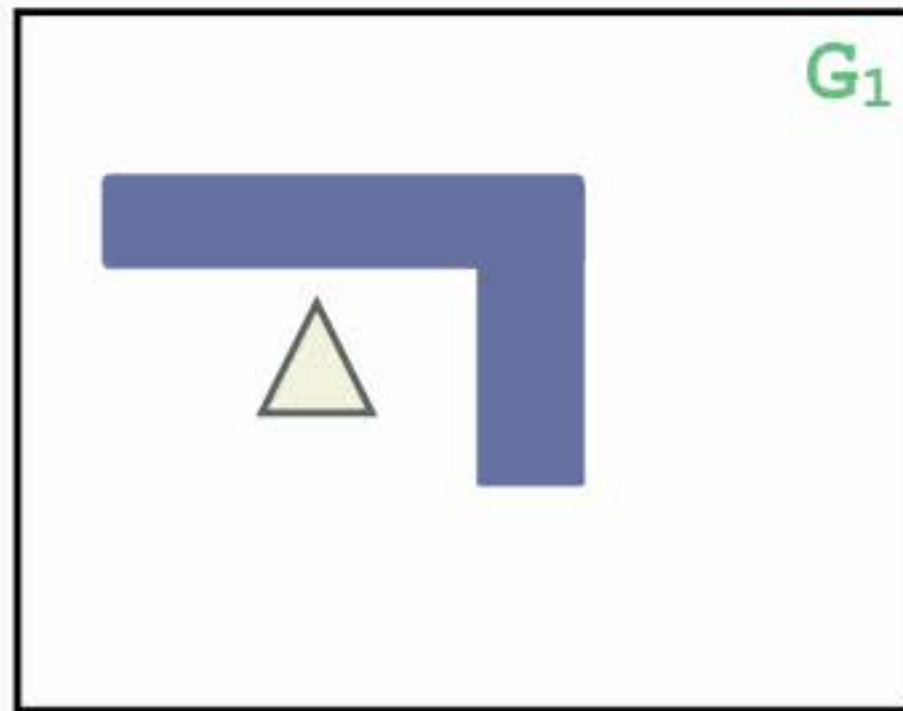
# Experiments: Four Rooms



# Continuous State

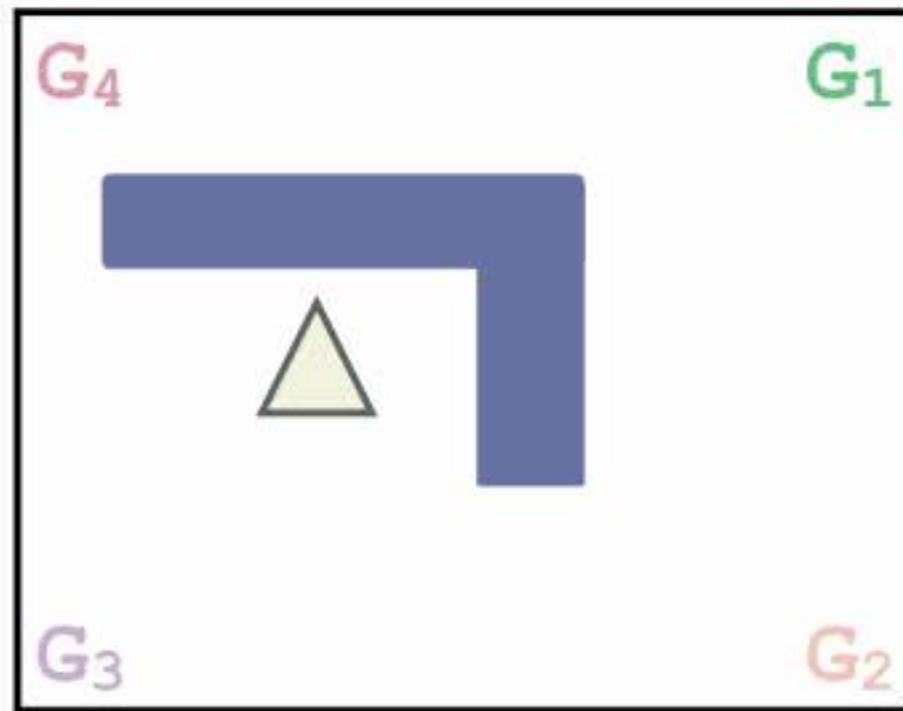


# Continuous State

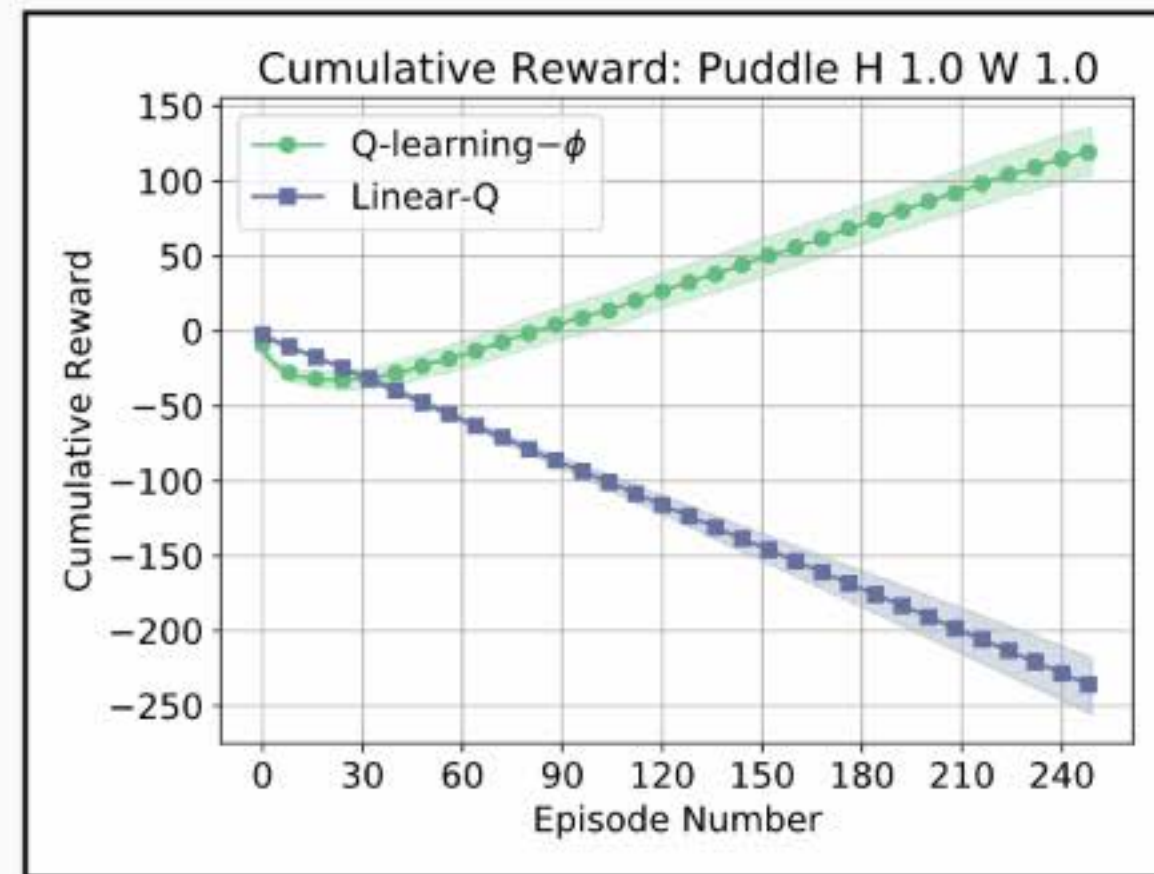
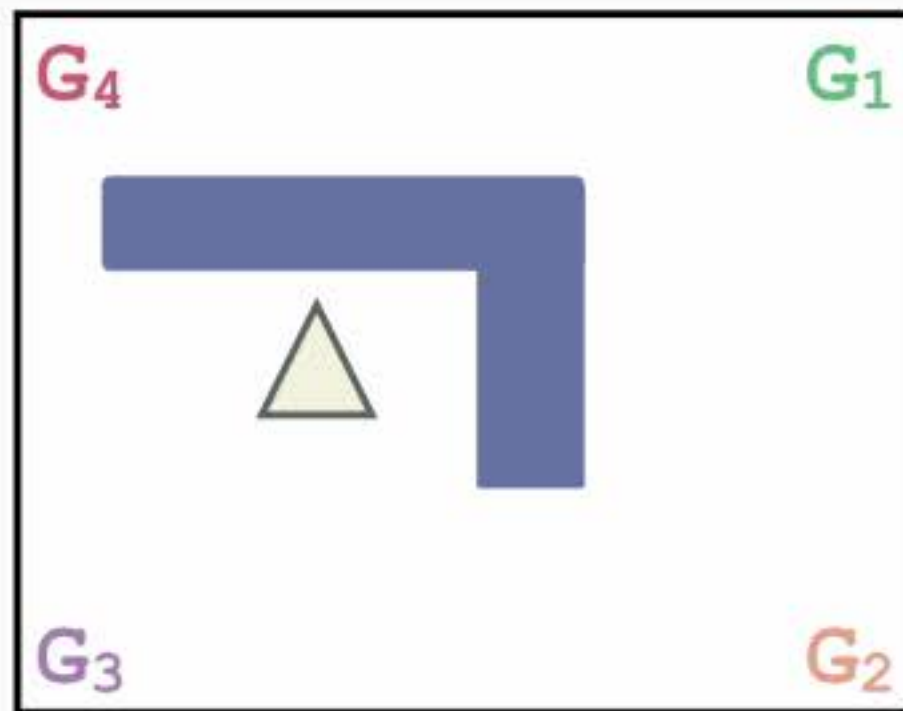


*Single Task*

# Continuous State

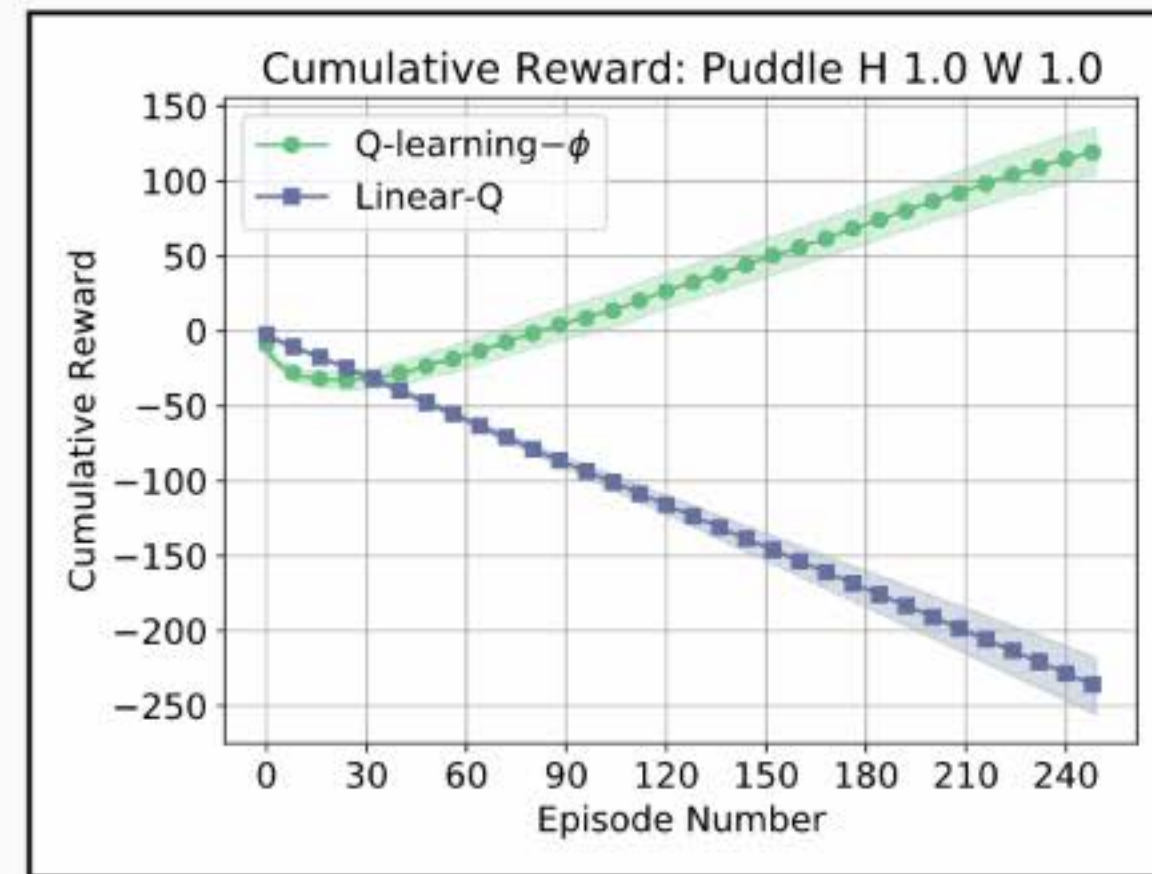
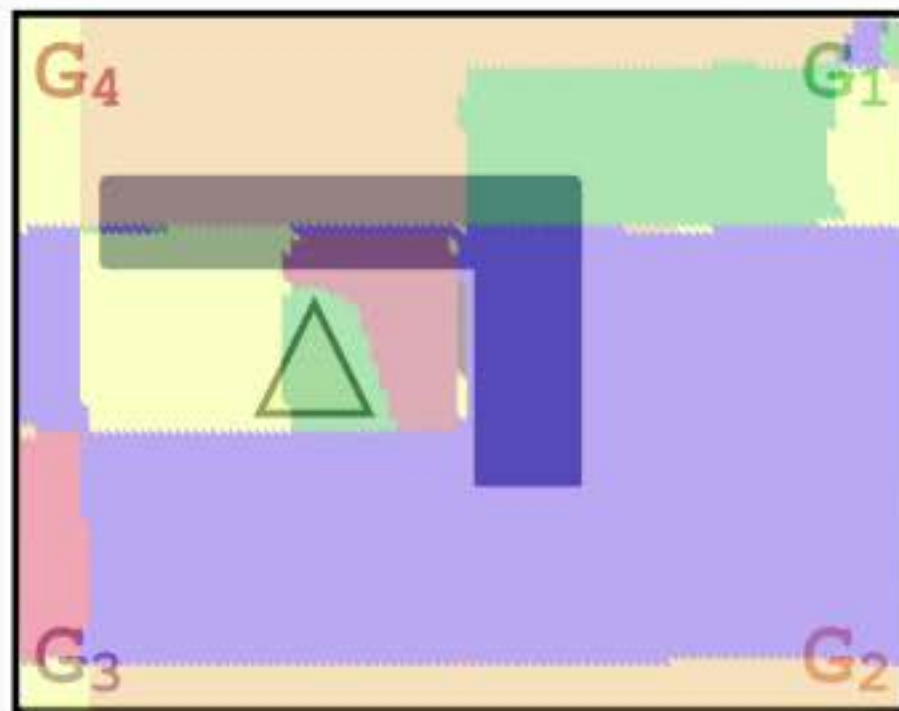


# Continuous State



*Transfer*

# Continuous State



*Transfer*

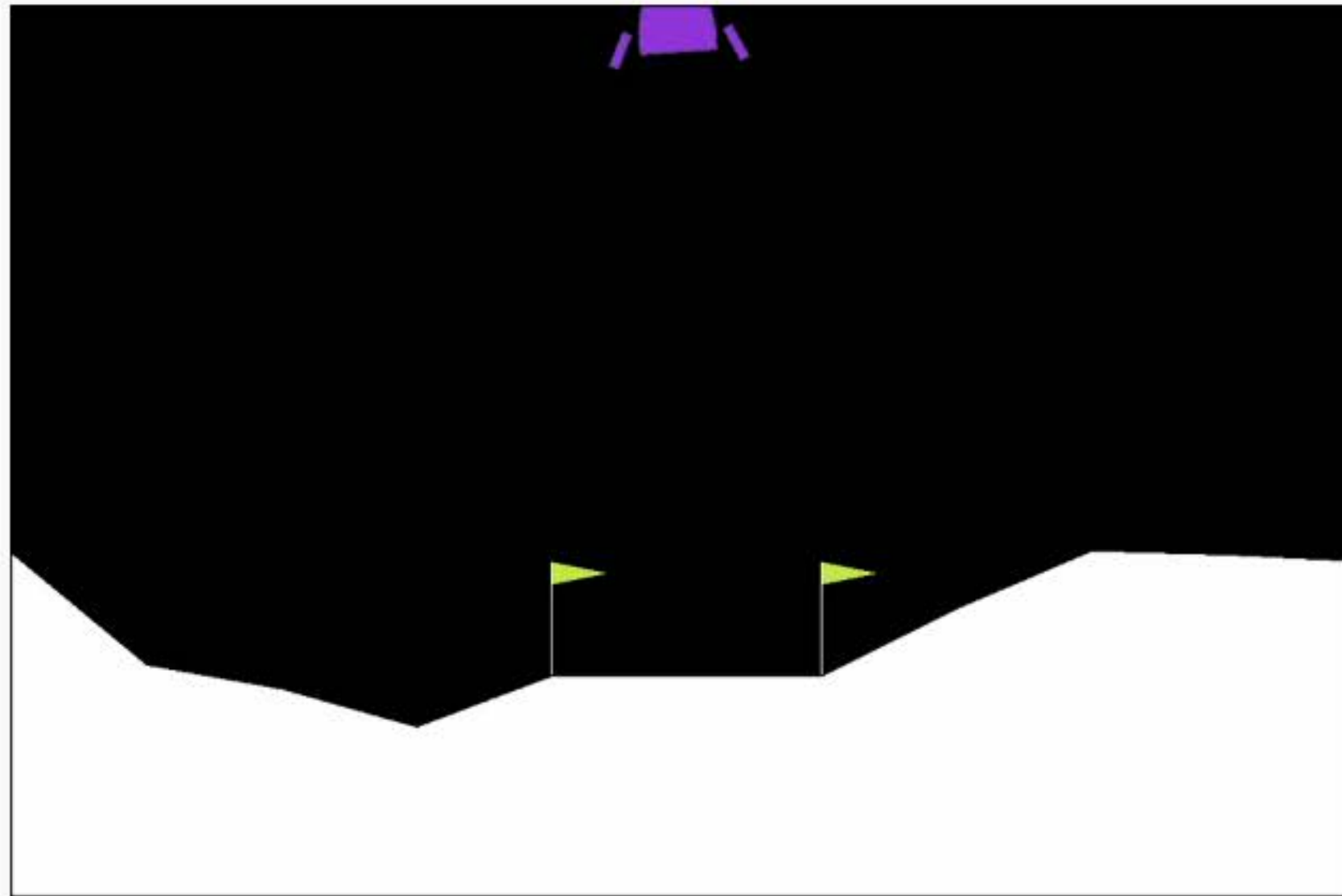
# Continuous State

**Theorem.** *With probability at least  $1 - \delta$ , for any  $\delta \in (0, 1)$ :*

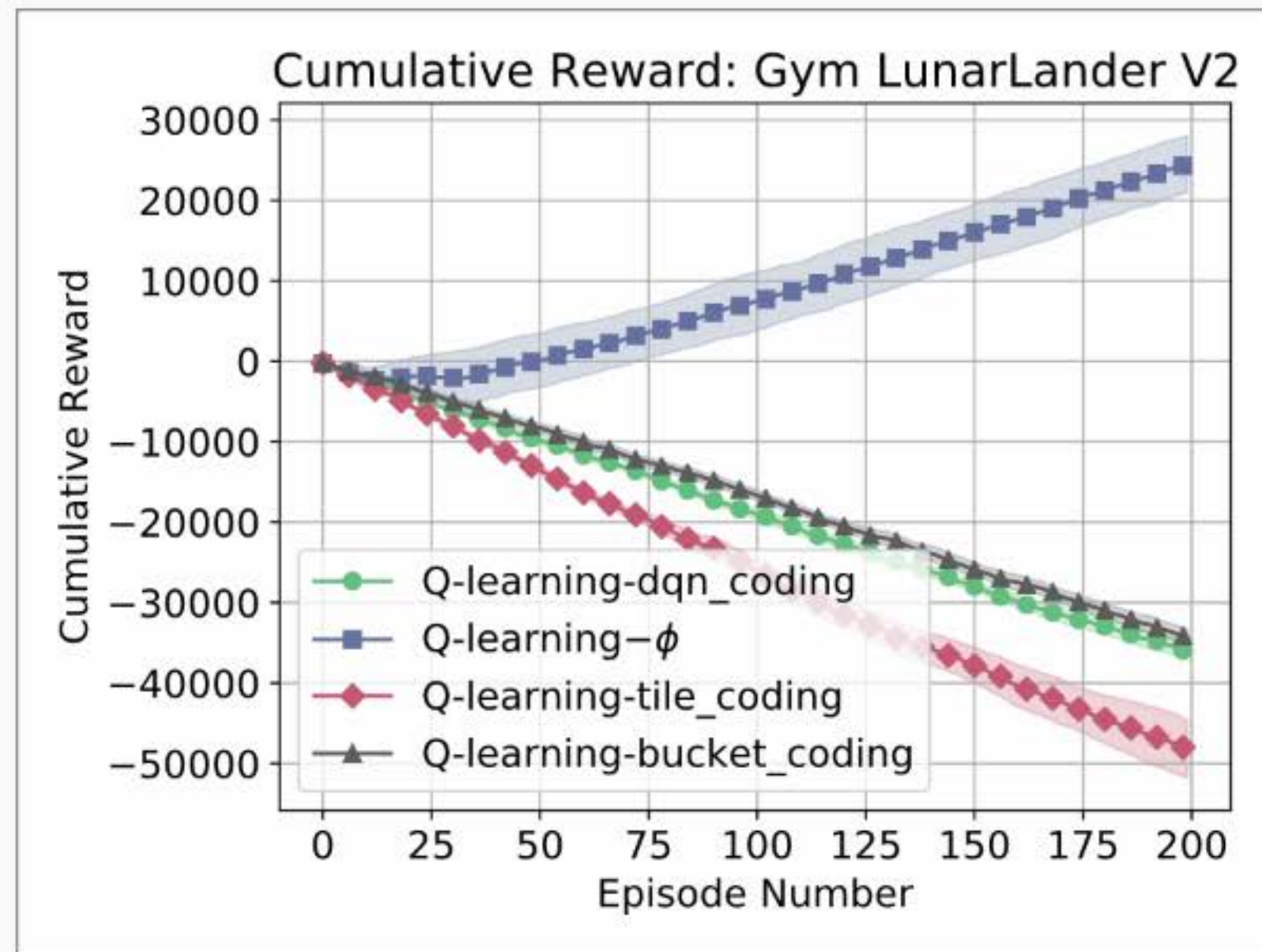
$$\mathbb{E}_s \left\| \left( \pi^*(\cdot | s) - \pi_{\phi}^*(\cdot | \phi(s)) \right) \right\|_1 \leq \frac{\Delta}{2} + 2\sqrt{2}\text{Rad}(\Phi) + \sqrt{\frac{2 \ln \frac{1}{\delta}}{n}}$$



# Lunar Lander



# Experiments: Lunar Lander



Kavosh  
Asadi

# Results Summary

**Question:** *Can we find state abstractions that minimize  $|\mathcal{S}_\phi|$  while still representing good policies?*


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**Question:** *Can we find state abstractions that minimize  $|\mathcal{S}_\phi|$  while still representing good policies?*

**Answer:** *Yes! But, perhaps state space size isn't the full story.*

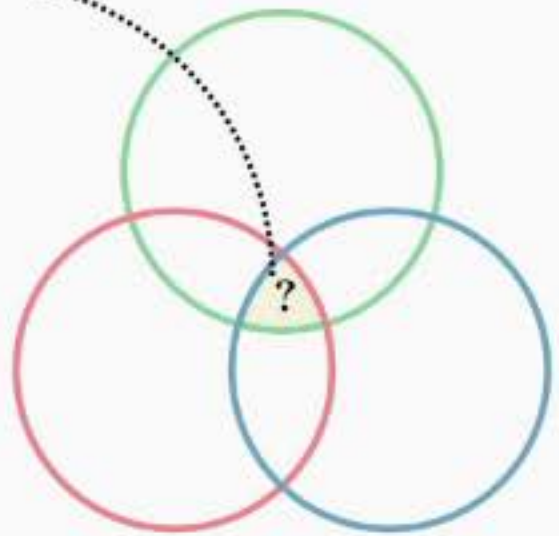
# Results Summary

**1 State Abstraction**



[AAAJLW AAAI '19]

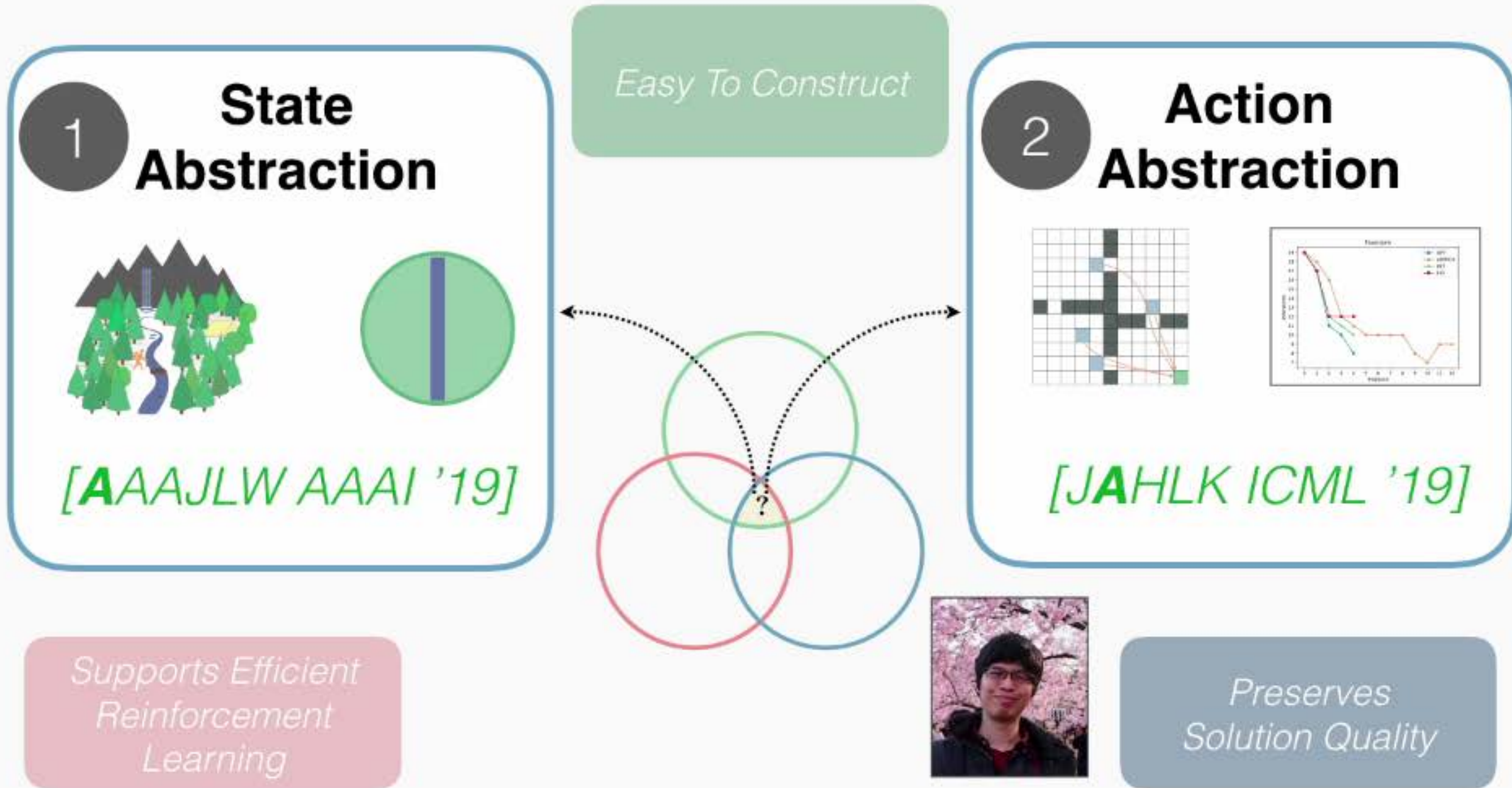
*Easy To Construct*



*Supports Efficient Reinforcement Learning*

*Preserves Solution Quality*

# Results Summary



# Action Abstraction

*[Sutton, Precup, Singh 1999]*

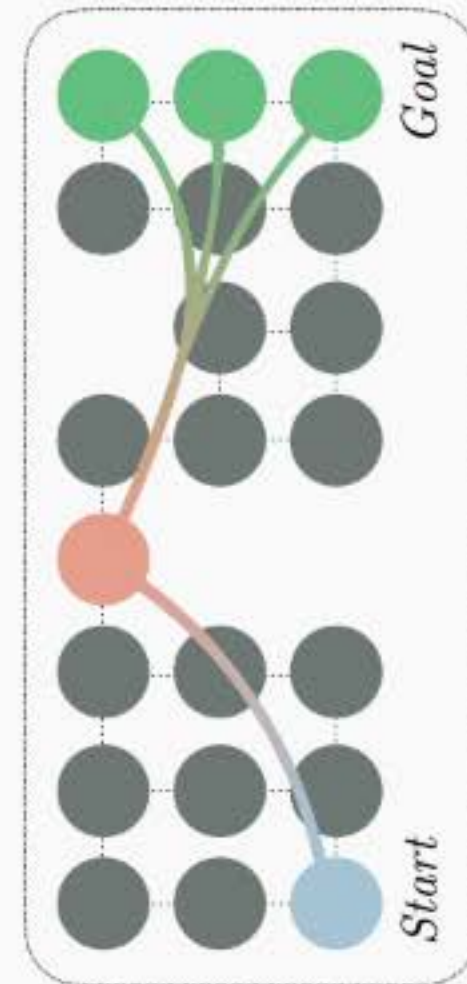
**Definition** (Option): A start condition, end condition, and a policy.

# Action Abstraction

*Example:*

$$o_1 = (\text{blue}, \text{red}, \pi)$$

$$o_2 = (\text{red}, \text{green}, \pi)$$

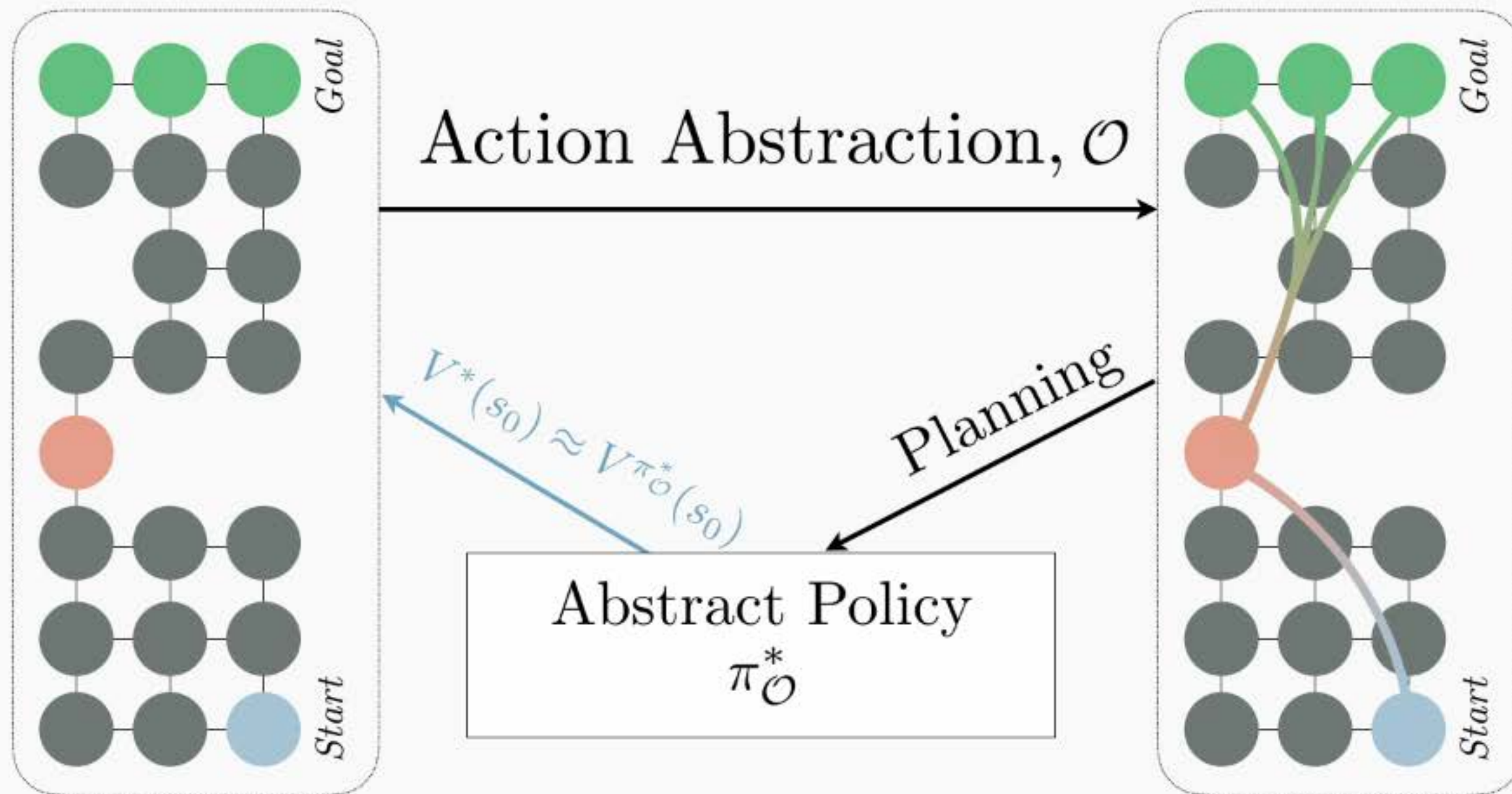


*[Sutton, Precup, Singh 1999]*

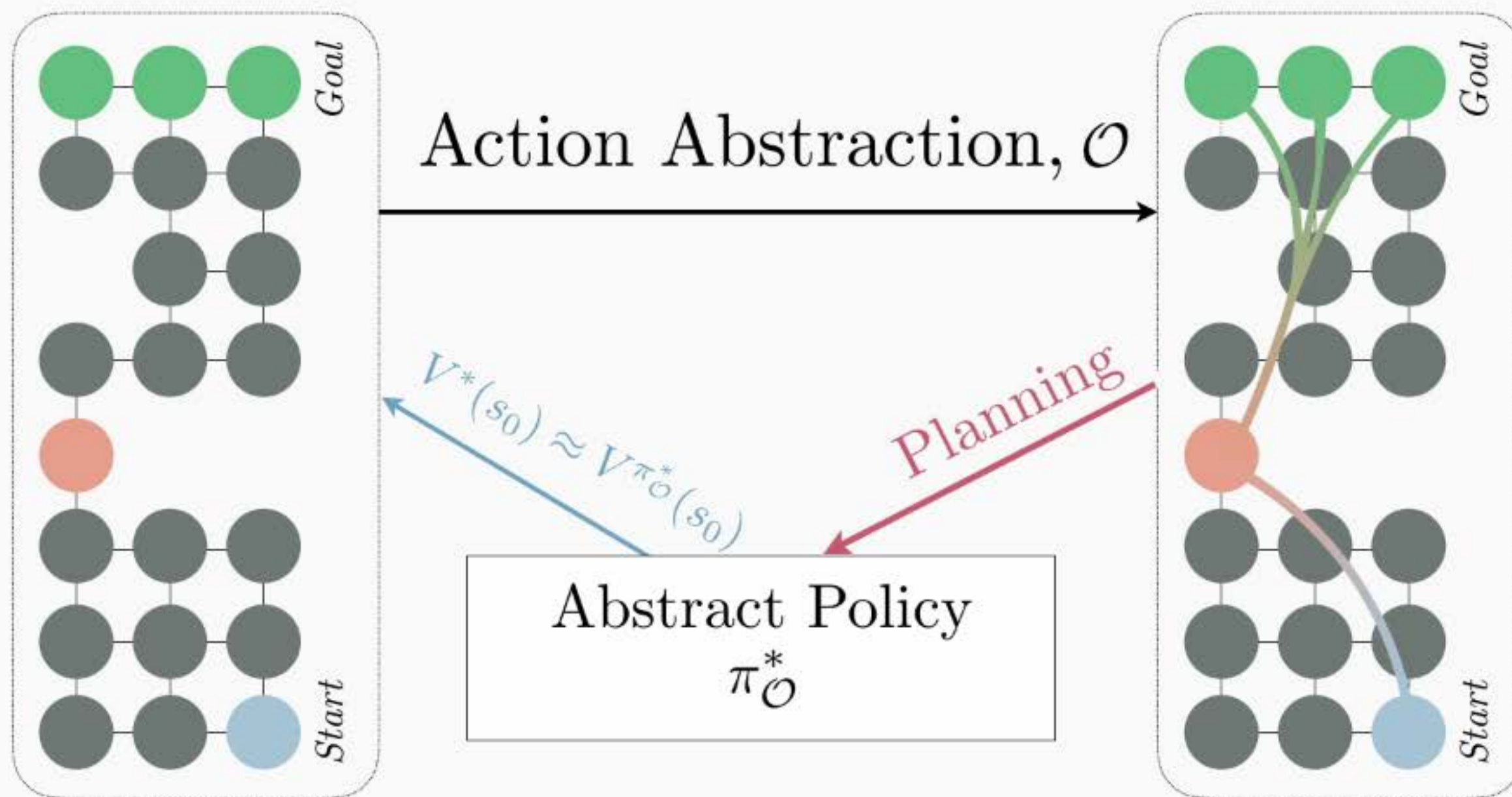
**Definition (Option):** A start condition, end condition, and a policy.



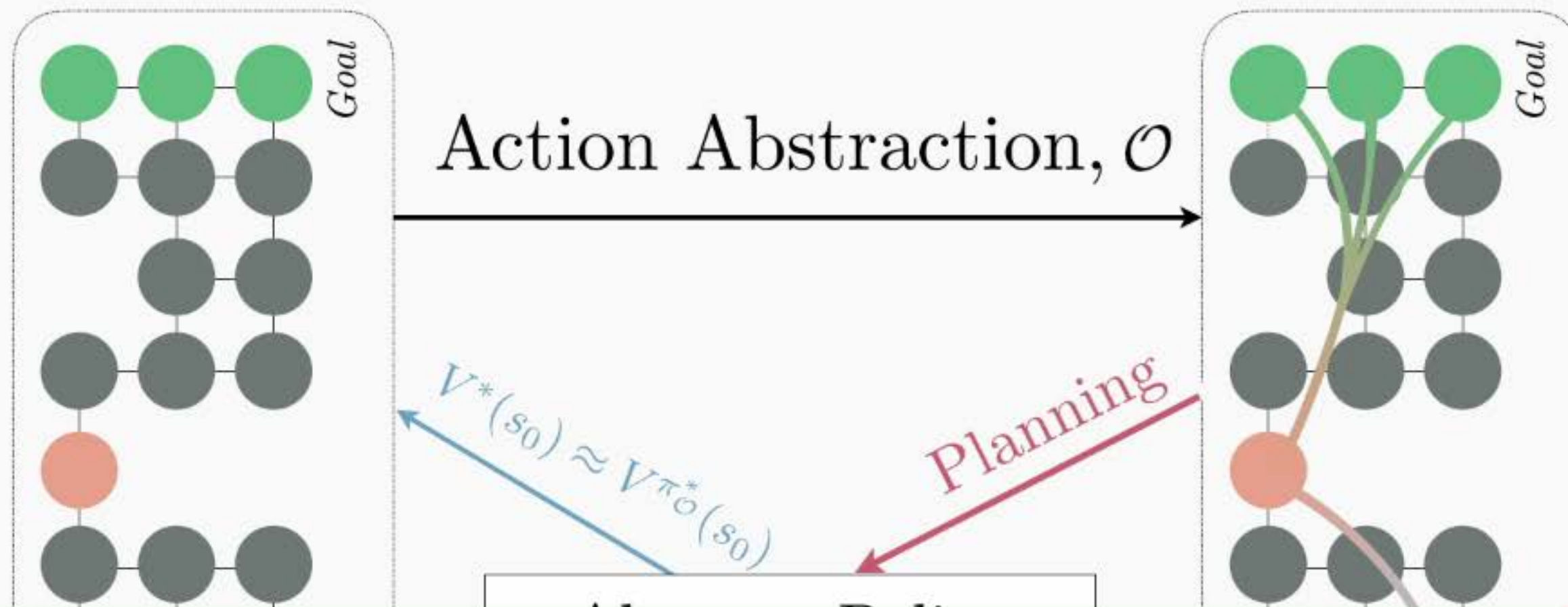
# Finding Options for Planning



# Finding Options for Planning



# Finding Options for Planning



**Question:** Can we find the set of options that make planning as fast as possible?

# Finding Options for Planning

**Question:** *Can we find the set of options that make planning as fast as possible?*

# Finding Options for Planning

**Definition** (Value-Planning Problem): **Given** an MDP  $M$  and  $\varepsilon \in \mathbb{R}_{>0}$ , **return** a value function,  $V$  such that  $|V(s) - V^*(s)| < \varepsilon$  for all  $s \in \mathcal{S}$ .

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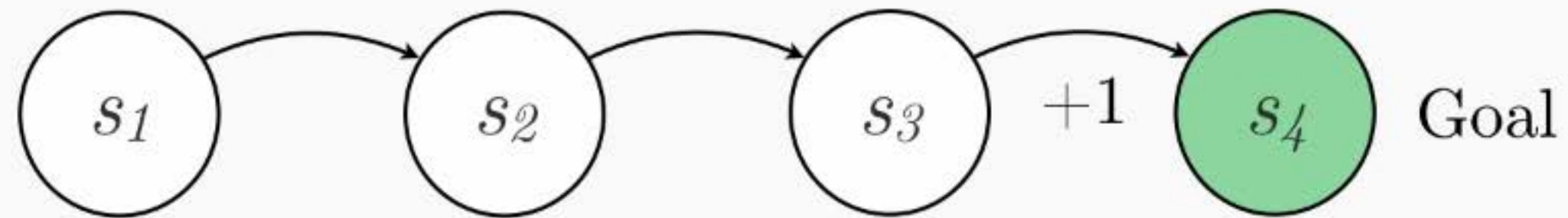
# Iterations needed for value iteration to converge

**Question:** Can we find the set of options that make **planning** as **fast** as possible?

Thanks to Yuu Jinnai  
for the example

# Value Iteration

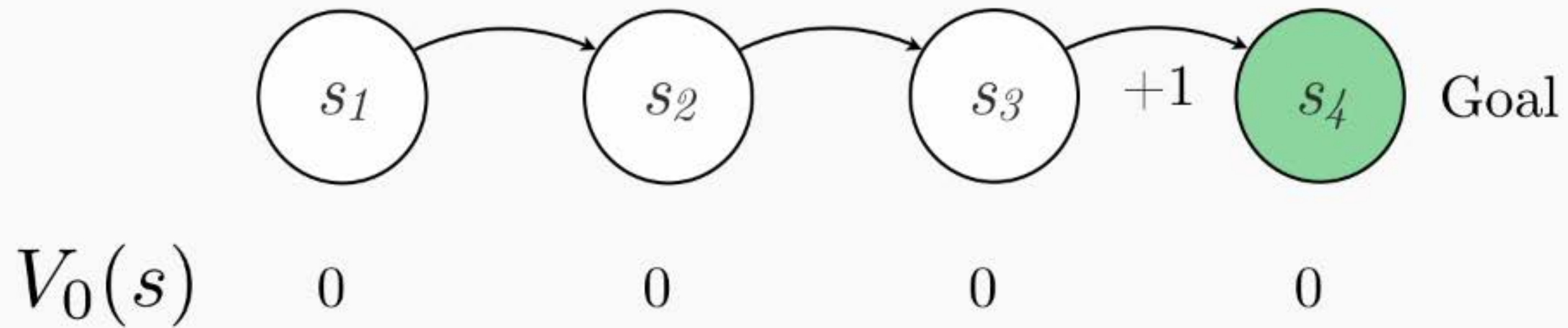
[Bellman 1957]



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# Value Iteration

[Bellman 1957]

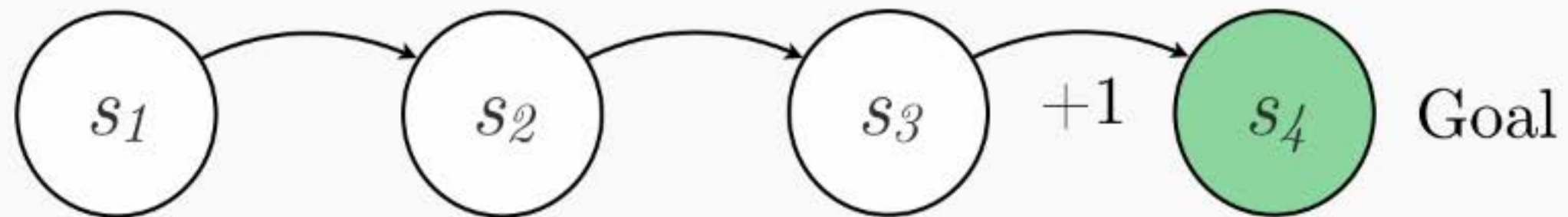




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# Value Iteration

[Bellman 1957]

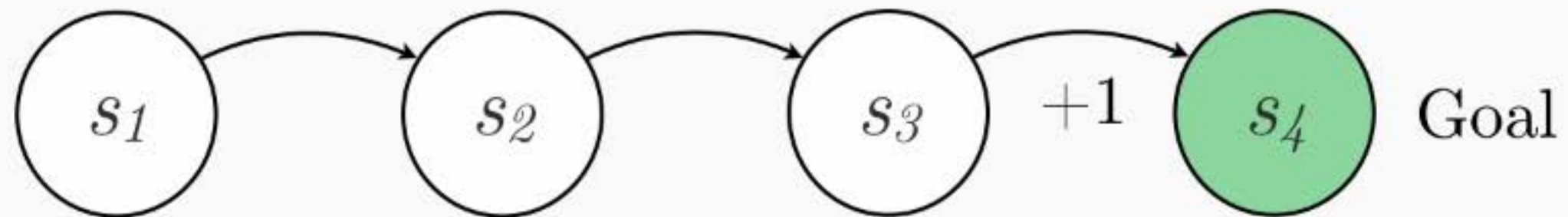


$V_0(s)$	0	0	0	0
$V_1(s)$	0	0	1	0

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# Value Iteration

[Bellman 1957]

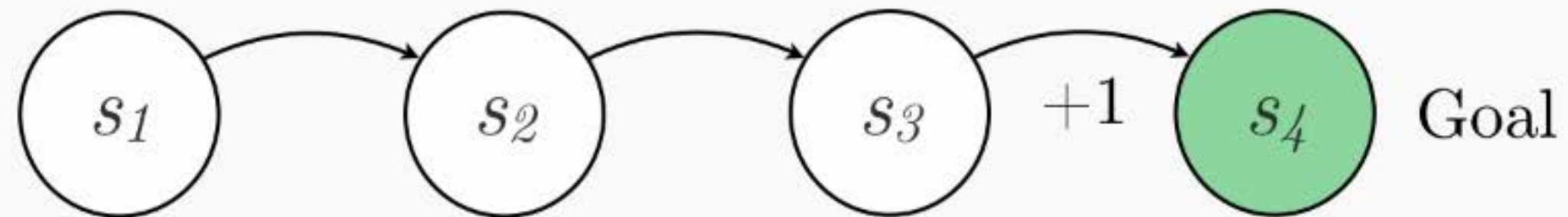


$V_0(s)$	0	0	0	0
$V_1(s)$	0	0	1	0
$V_2(s)$	0	$\gamma$	1	0

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# Value Iteration

[Bellman 1957]

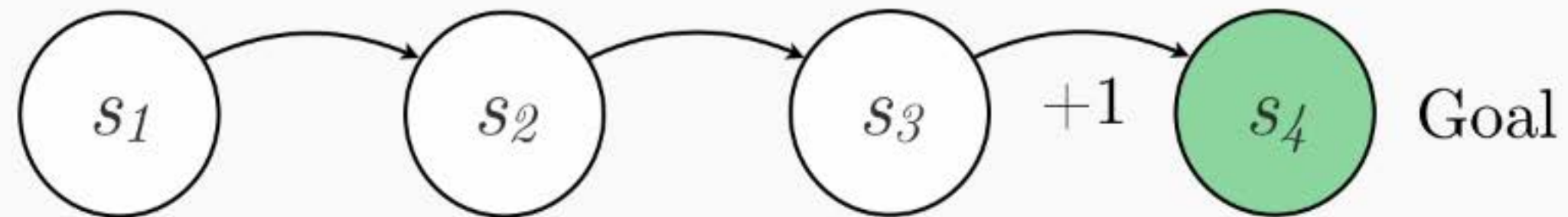


$V_0(s)$	0	0	0	0
$V_1(s)$	0	0	1	0
$V_2(s)$	0	$\gamma$	1	0
$V_3(s)$	$\gamma^2$	$\gamma$	1	0

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# Value Iteration

[Bellman 1957]

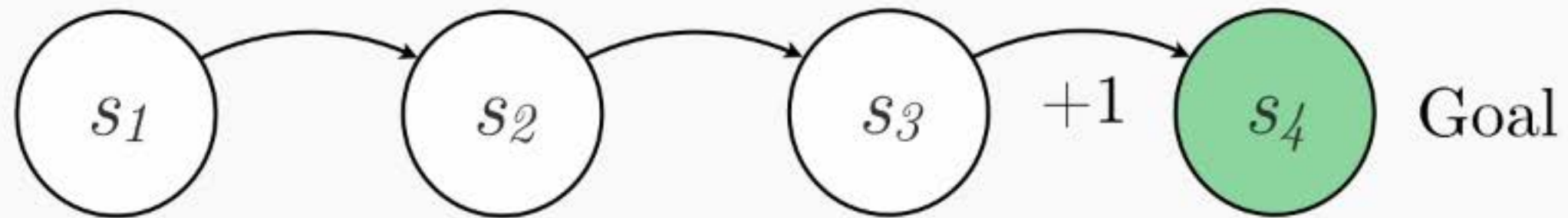


$V_0(s)$	0	0	0	0
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$V_2(s)$	0	$\gamma$	1	0
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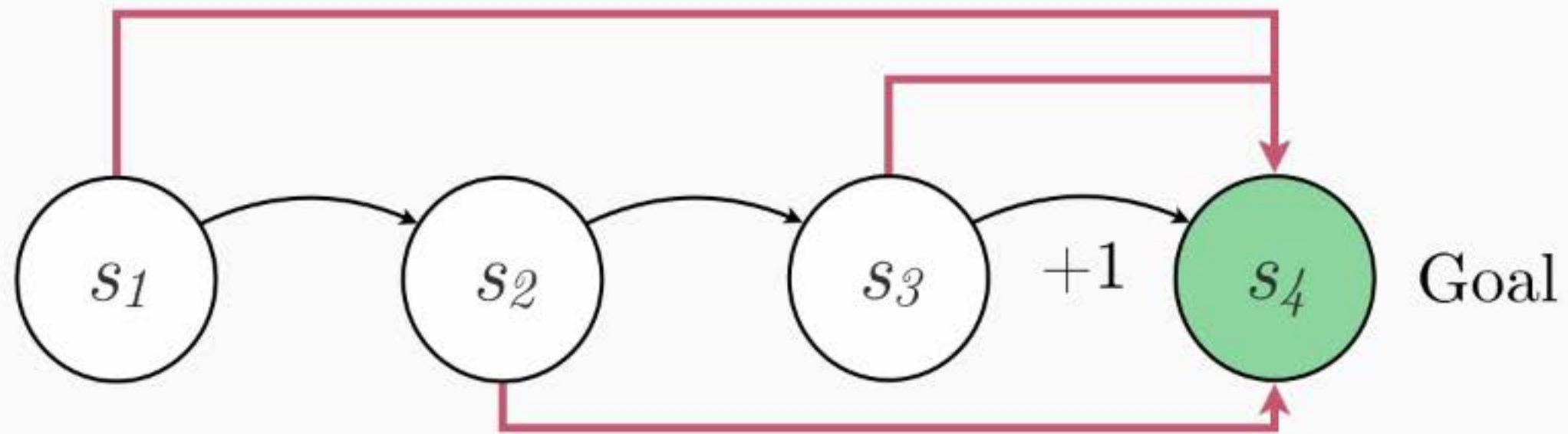
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# Value Iteration with Options



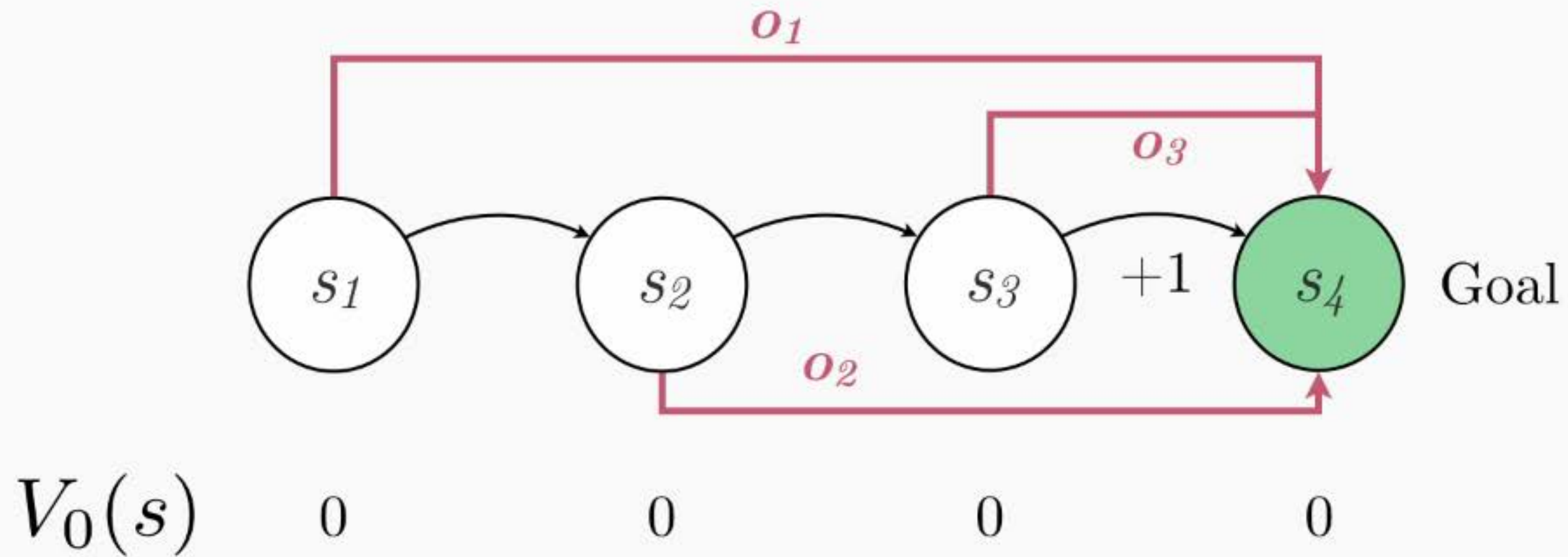
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# Value Iteration with Options



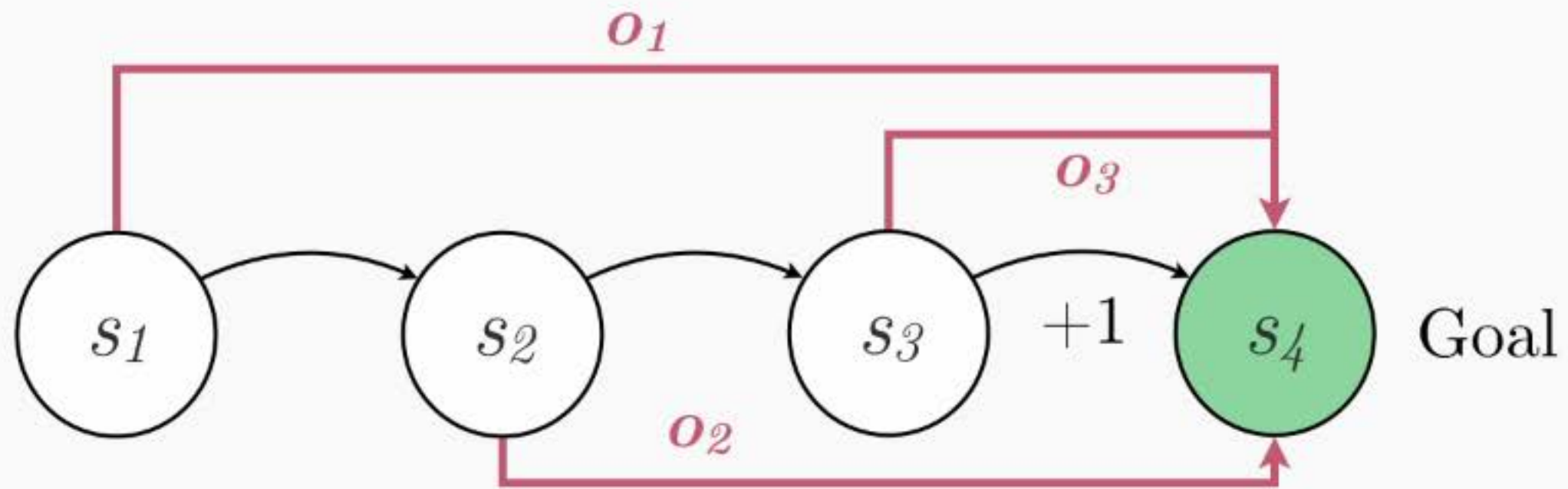
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# Value Iteration with Options

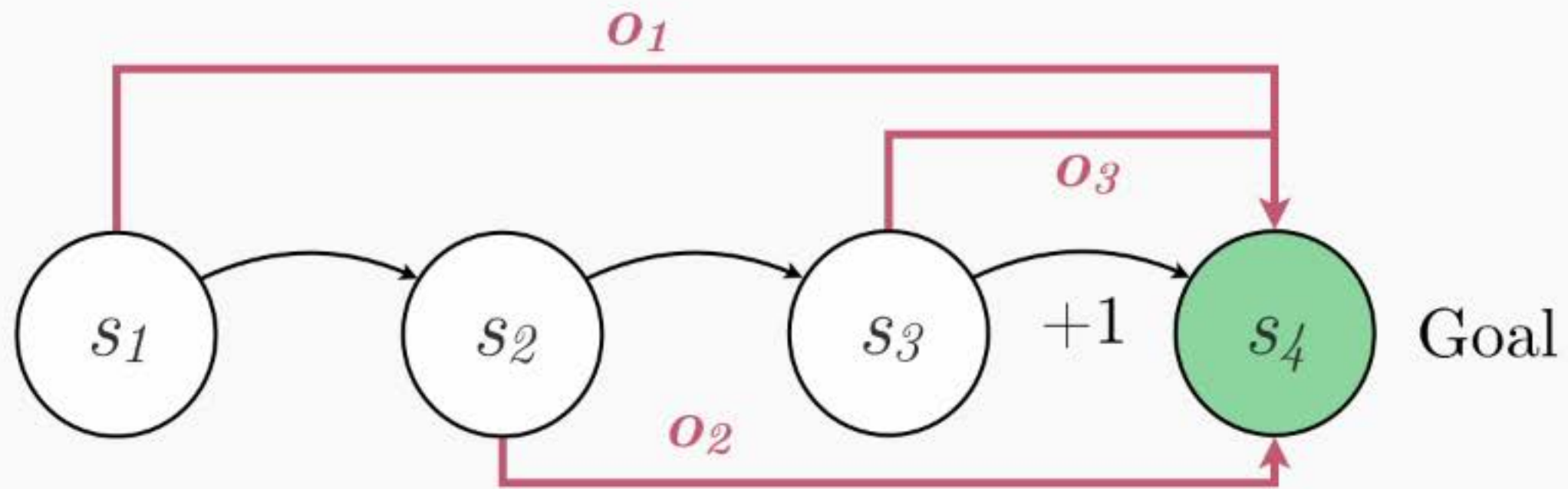


$V_0(s)$	0	0	0	0
$V_1(s)$	$\gamma^2$	$\gamma$	1	0



Thanks to Yuu Jinnai  
for the example

# Value Iteration with Options



$V_0(s)$	0	0	0	0
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$V_1(s)$	$\gamma^2$	$\gamma$	1	0
----------	------------	----------	---	---



# Value Iteration with Options

$$V_{i+1}(s) = \max_{o \in \mathcal{A} \cup \mathcal{O}} \left( R_\gamma(s, o) + \sum_{s' \in \mathcal{S}} T_\gamma(s' | s, o) V_i(s') \right)$$

# Value Iteration with Options

$$V_{i+1}(s) = \max_{o \in \mathcal{AO}} \left( R_\gamma(s, o) + \sum_{s' \in \mathcal{S}} T_\gamma(s' | s, o) V_i(s') \right)$$

Planning with options  
*and* primitives

[Ciosek and Silver 2015; Sutton, Precup, Singh 1999]

# Value Iteration with Options

$$V_{i+1}(s) = \max_{o \in \mathcal{A} \cup \mathcal{O}} \left( R_\gamma(s, o) + \sum_{s' \in \mathcal{S}} T_\gamma(s' | s, o) V_i(s') \right)$$

Planning with options  
*and* primitives

Multi-time model

[Ciosek and Silver 2015; Sutton, Precup, Singh 1999]

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# Iterations needed for value iteration to converge

**Question:** Can we find the set of options that make **planning** as **fast** as possible?

# Finding Options for Planning

**Theorem.** Finding the set of options that minimizes planning time is:

- 1) NP-hard in general.
- 2)  $2^{\log^{1-\varepsilon} n}$ -hard to approximate.<sup>1</sup>

<sup>1</sup>Unless  $NP \subseteq DTIME(n^{\text{poly log } n})$  [Dinitz et al. 2012]

**Question:** Can we find the set of options that make planning as fast as possible?

# Finding Options for Planning

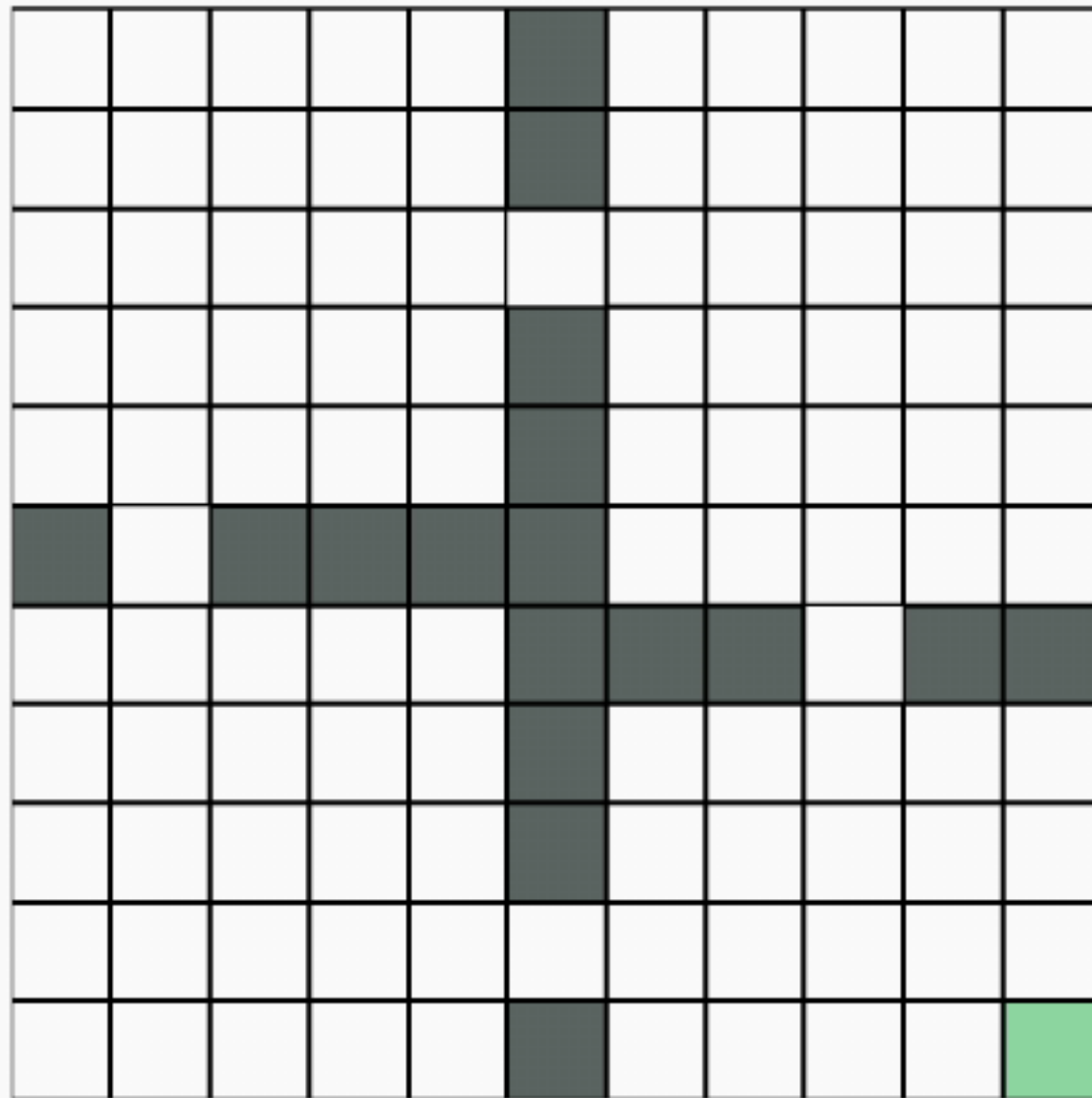
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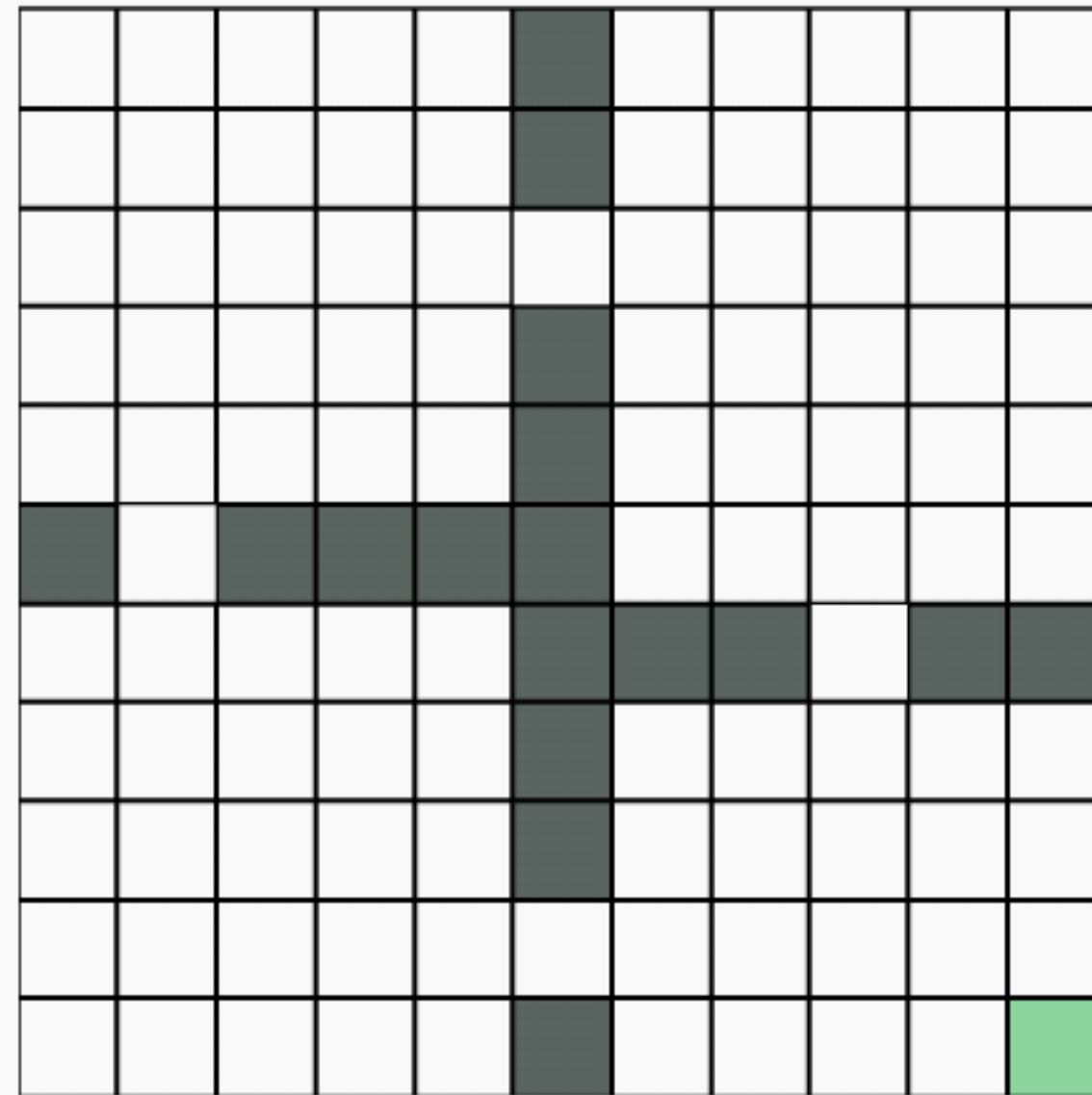
<sup>1</sup>Unless  $NP \subseteq DTIME(n^{\text{poly log } n})$  [Dinitz et al. 2012]

**Corollary.** Also holds for distributions of tasks.

# Visuals: $K = 1$



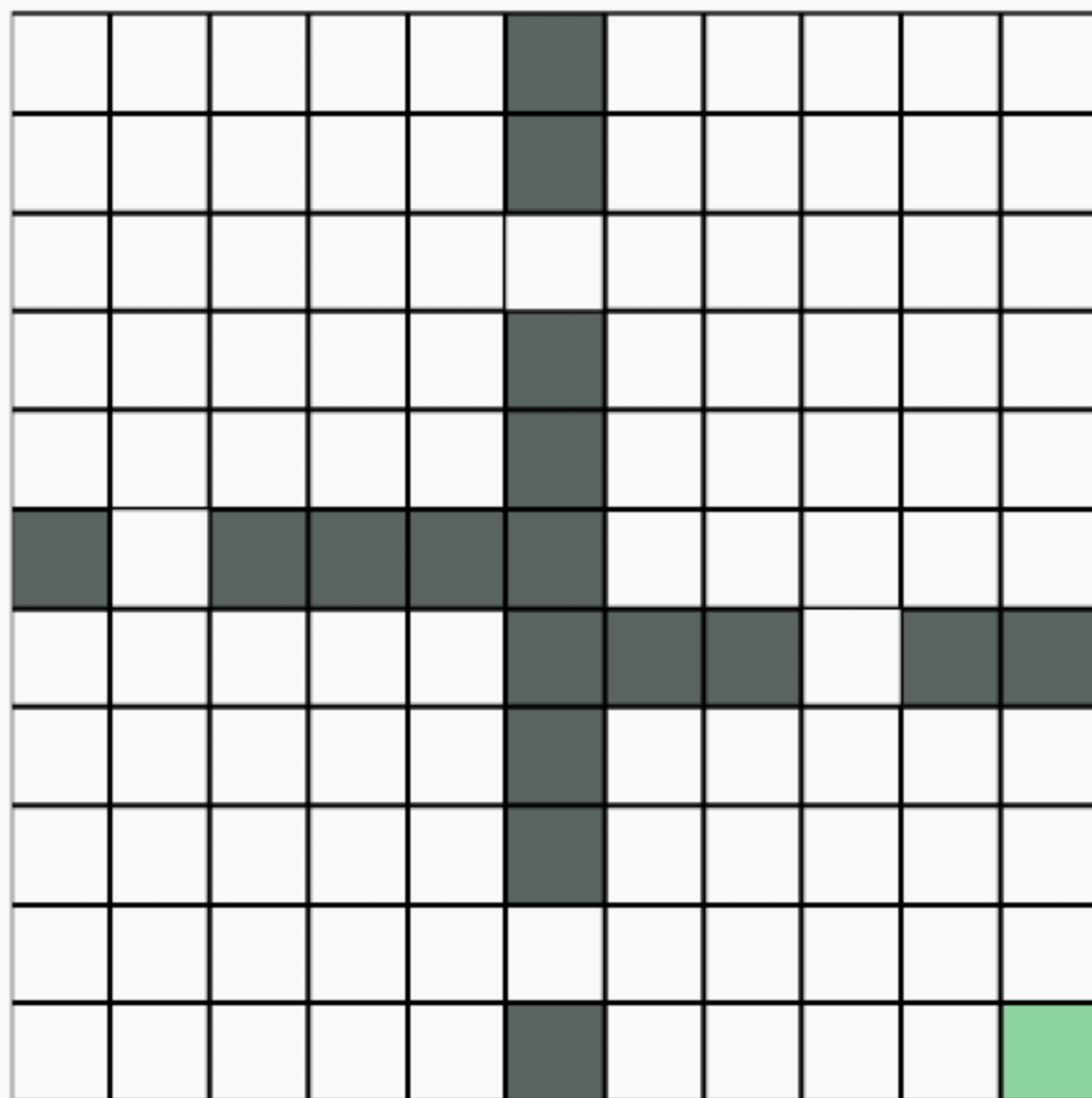
*Optimal*



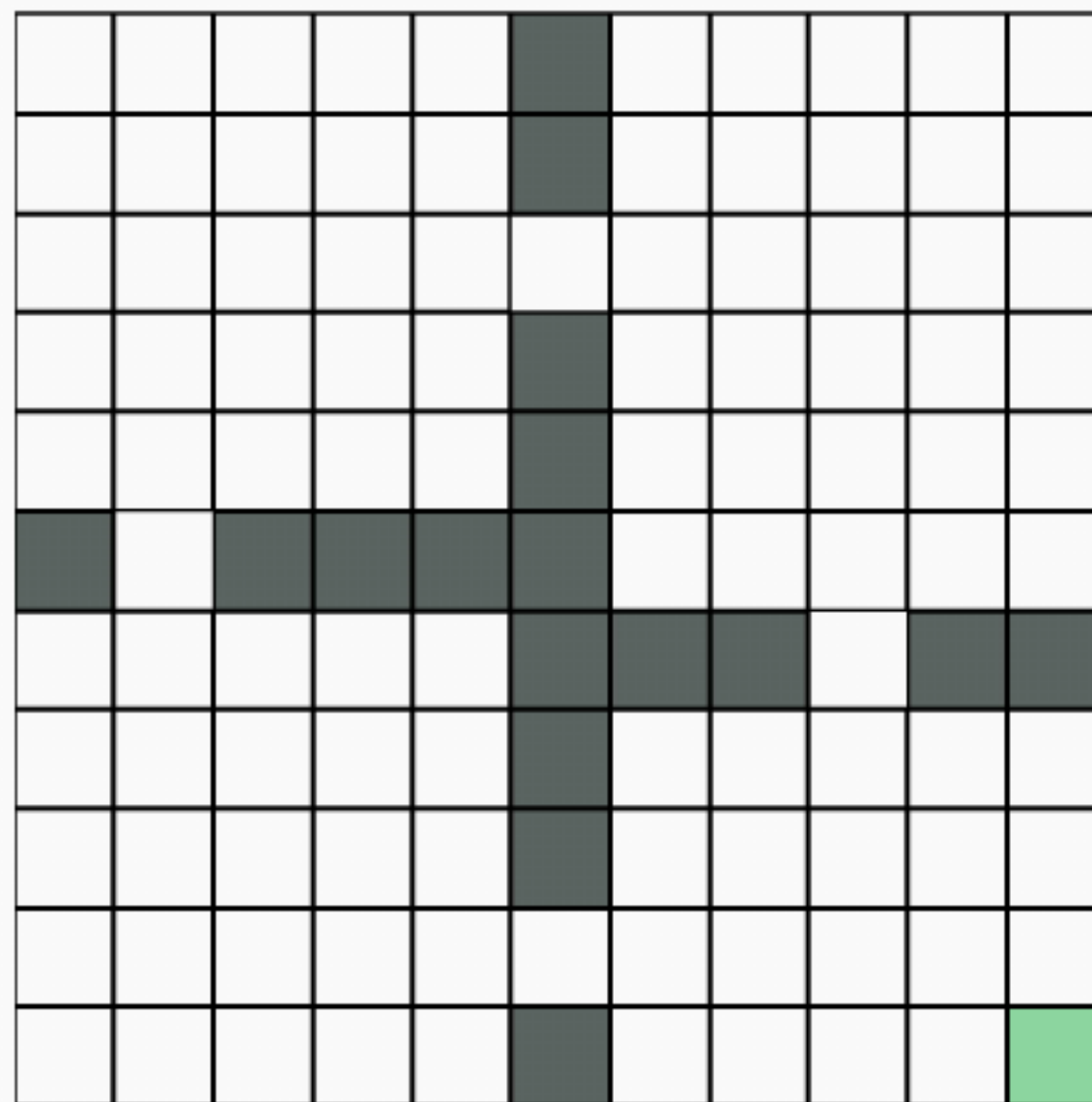
*Approximation*



# Visuals: $K = 2$

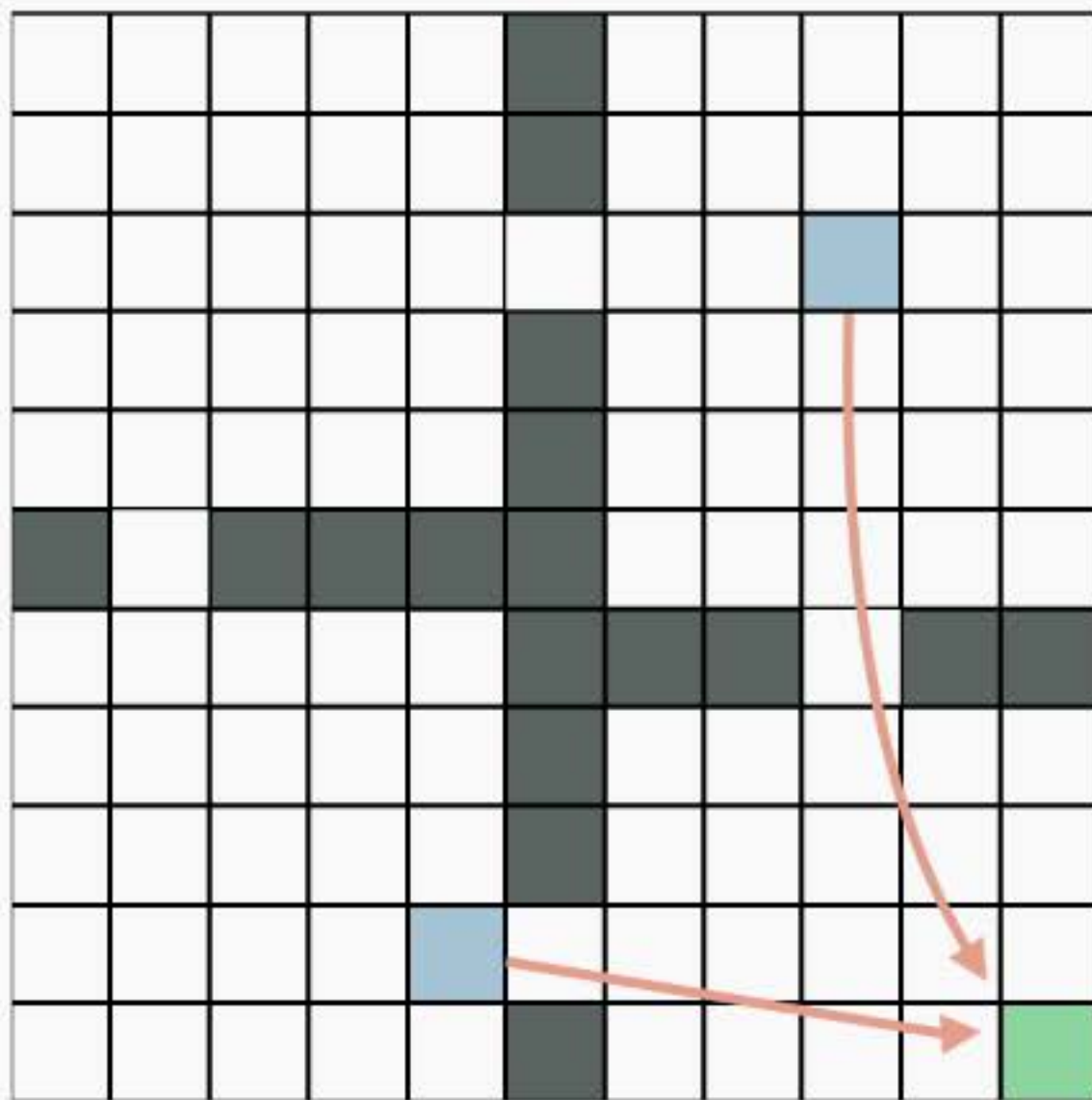


*Optimal*

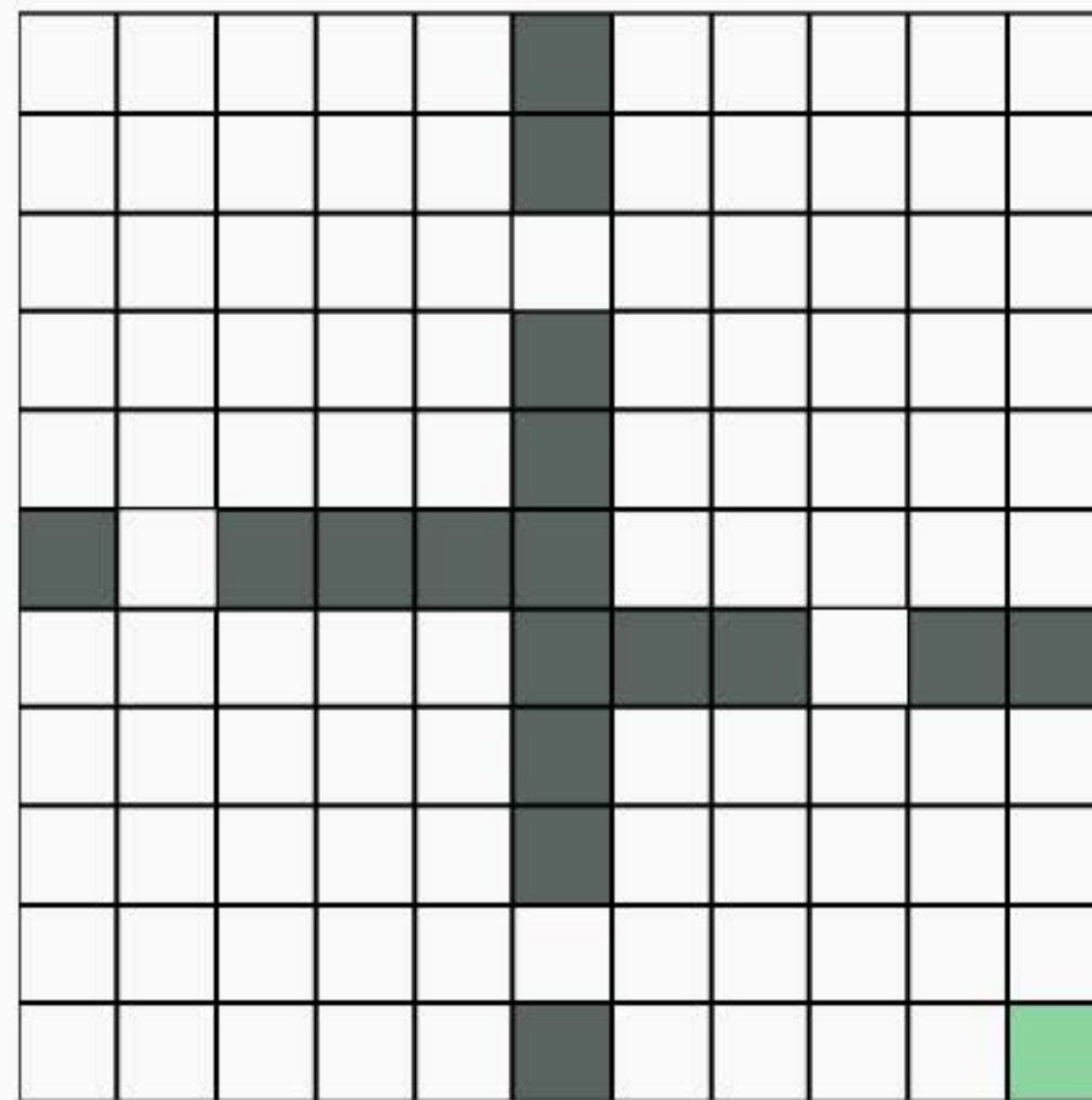


*Approximation*

# Visuals: $K = 2$

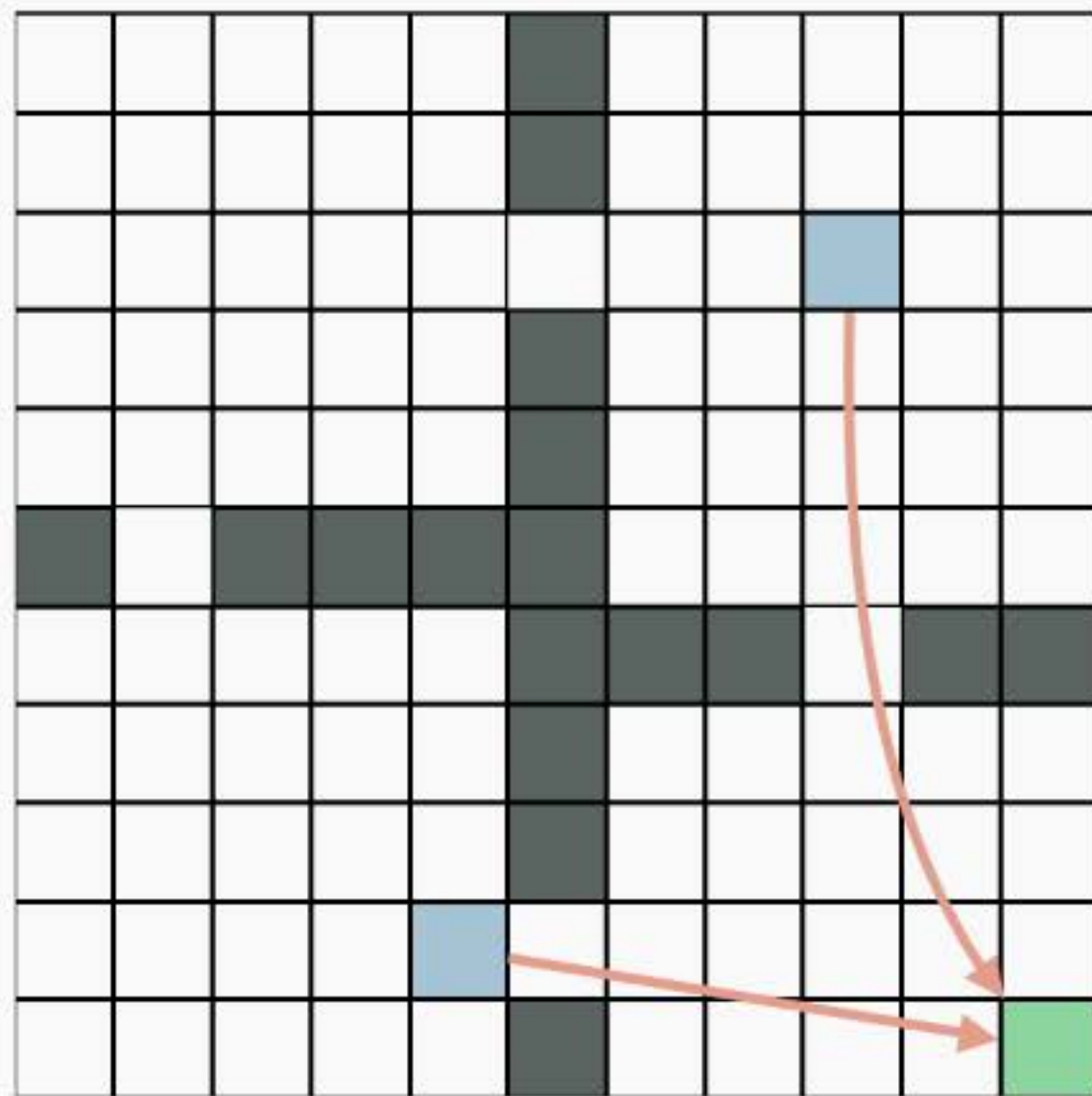


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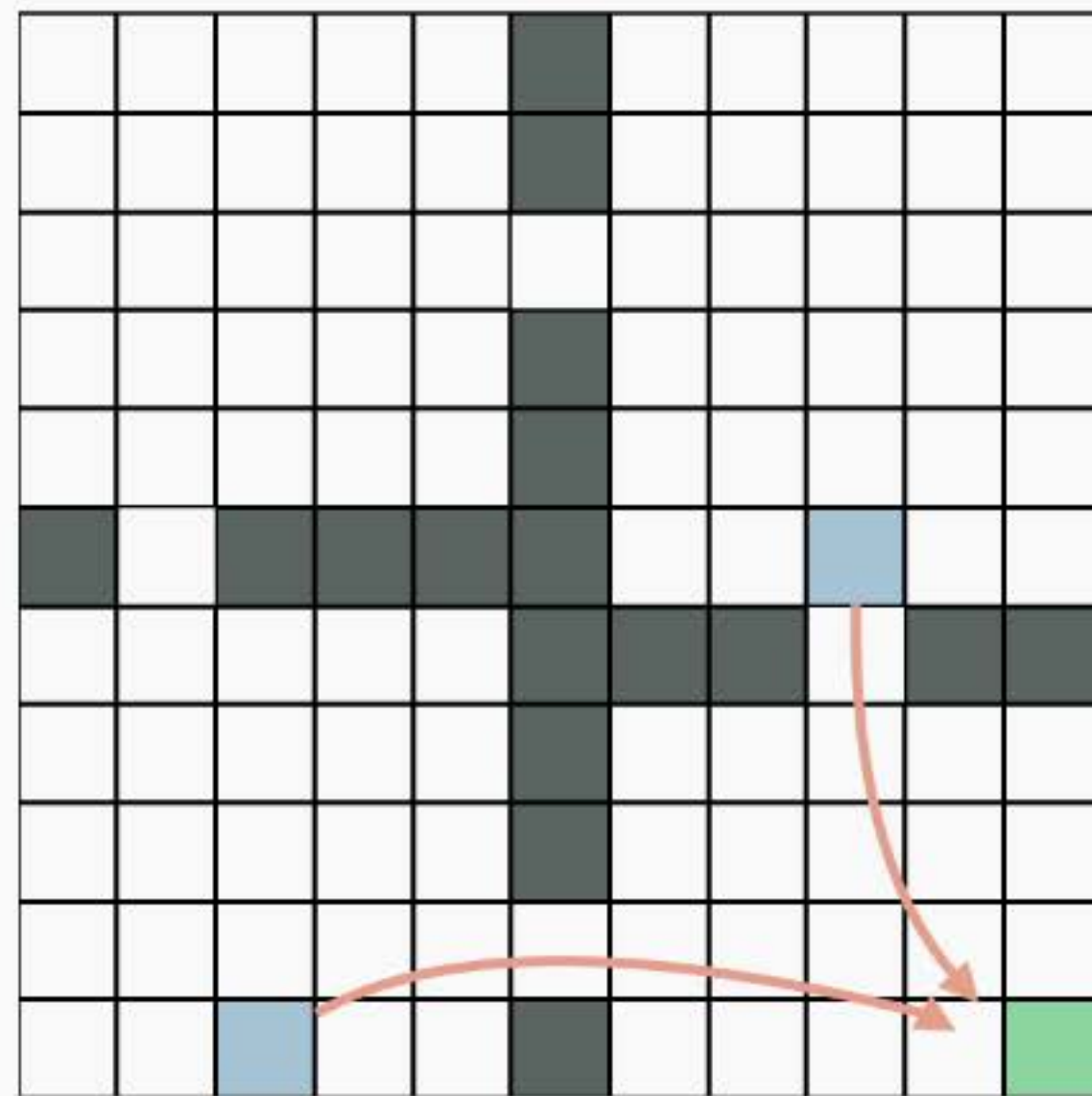


*Approximation*

# Visuals: $K = 2$

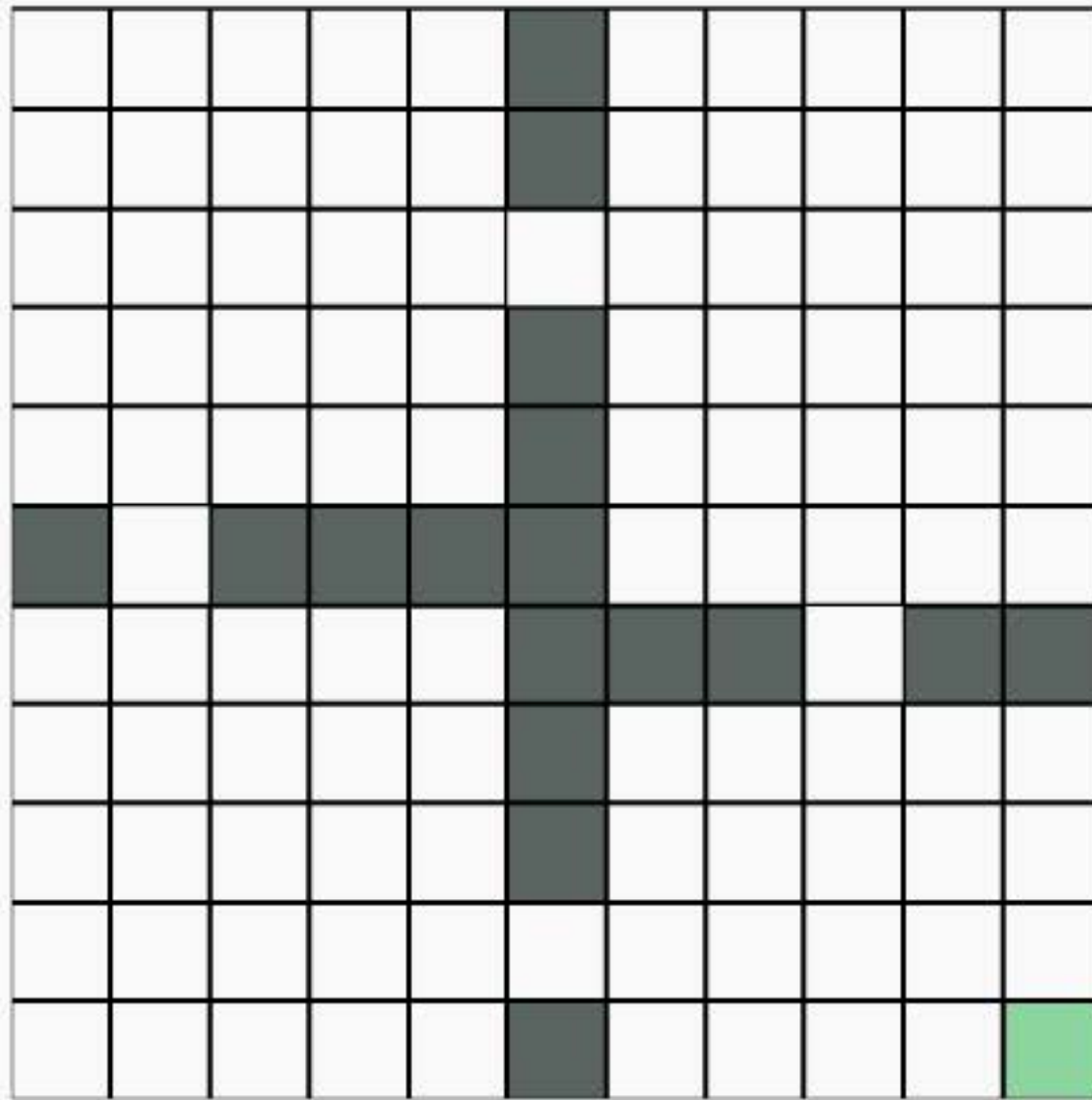


*Optimal*



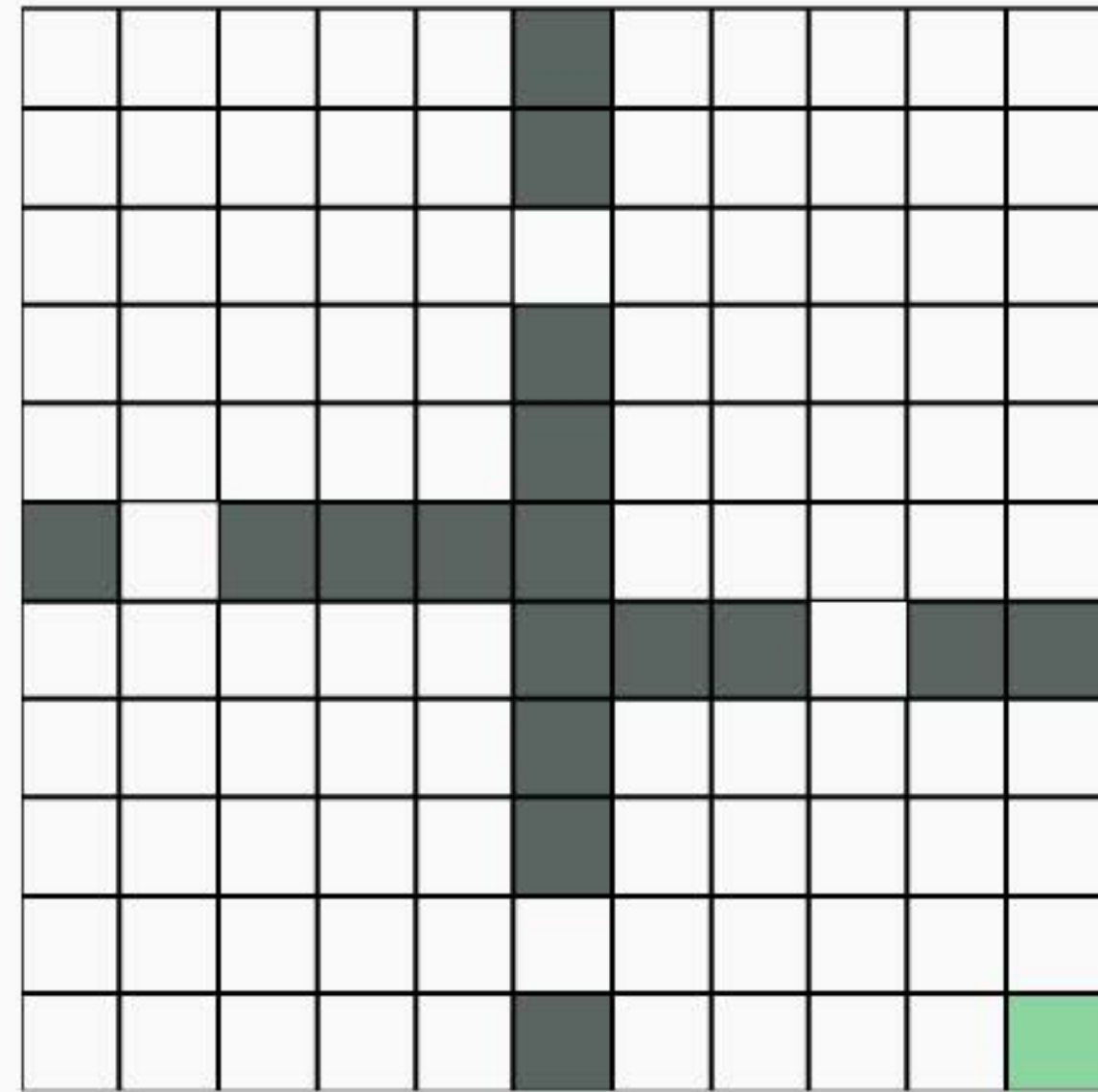
*Approximation*

# Visuals: $K = 4$



*Betweenness Options*

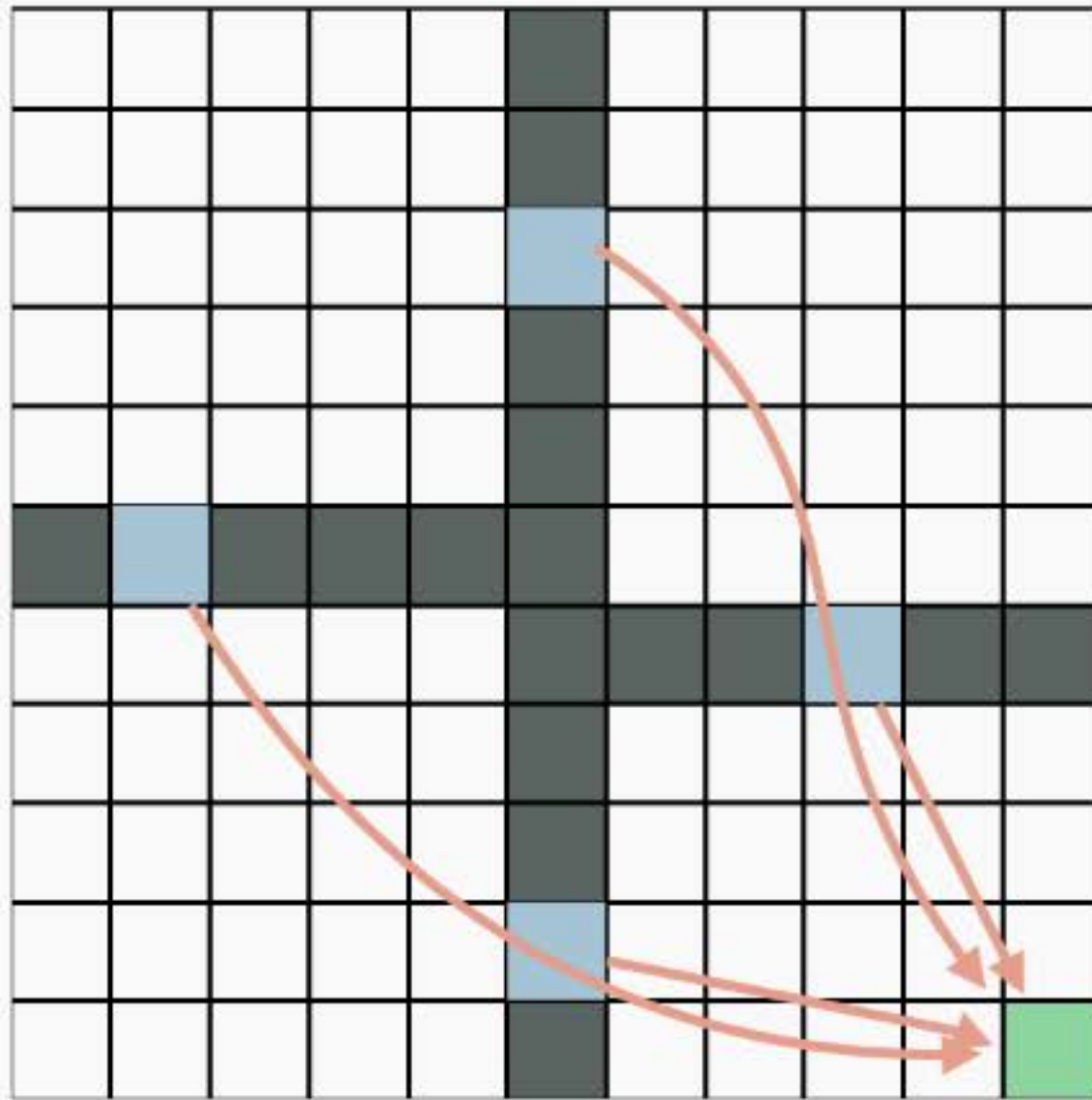
[Simsek and Barto 2005, 2008]



*Eigen Options*

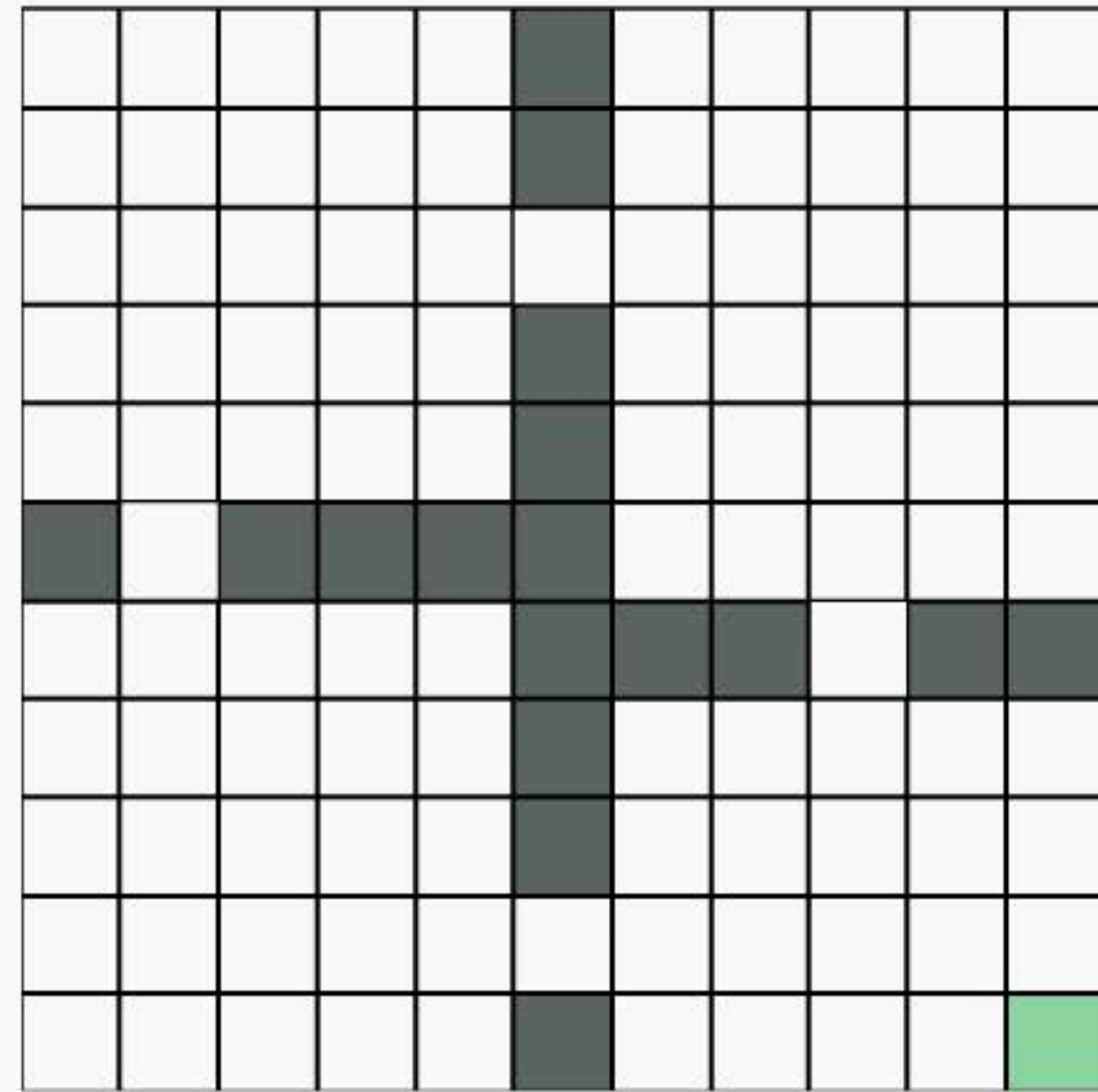
[Machado et al. 2017]

# Visuals: $K = 4$



*Betweenness Options*

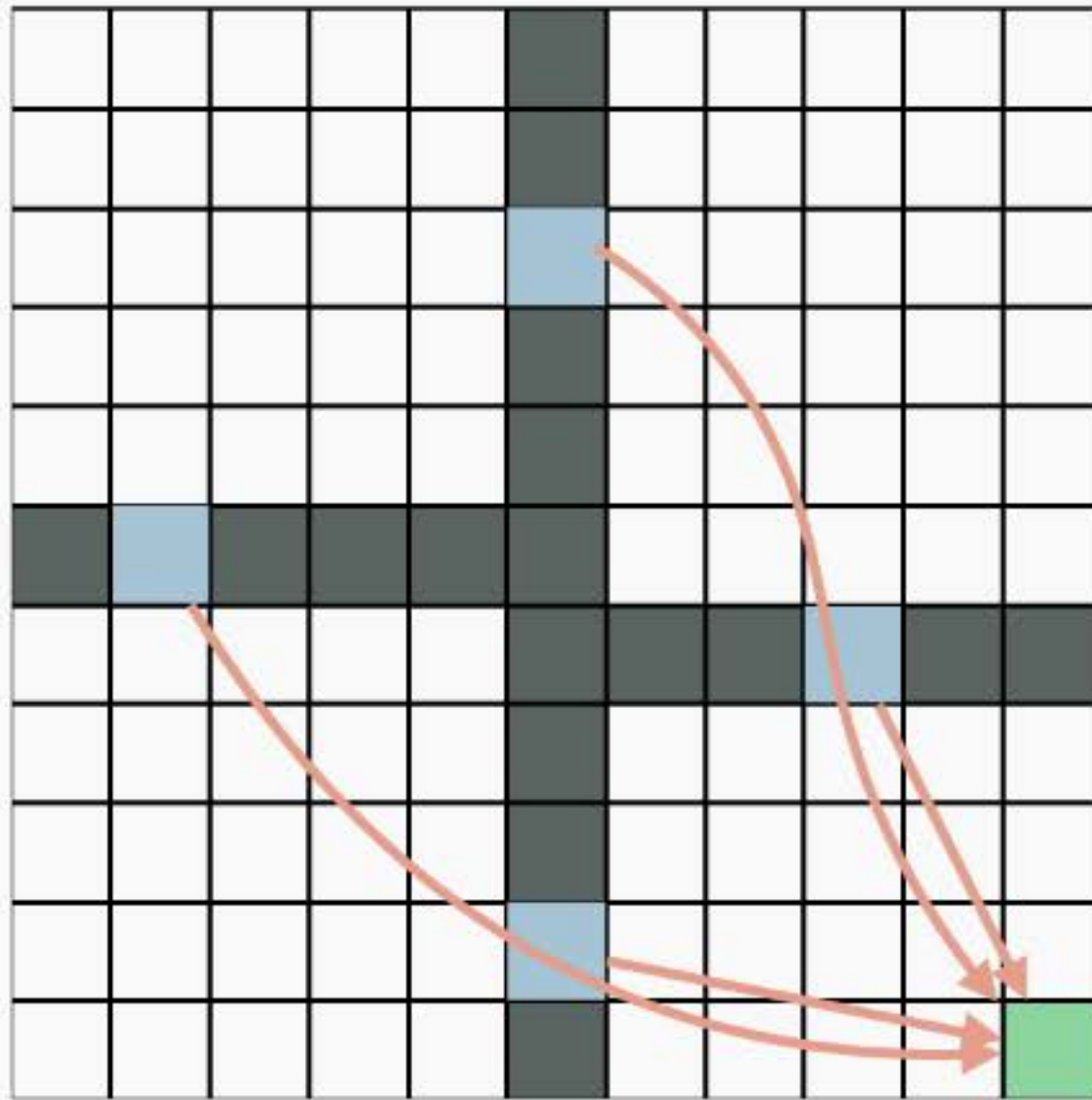
[Simsek and Barto 2005, 2008]



*Eigen Options*

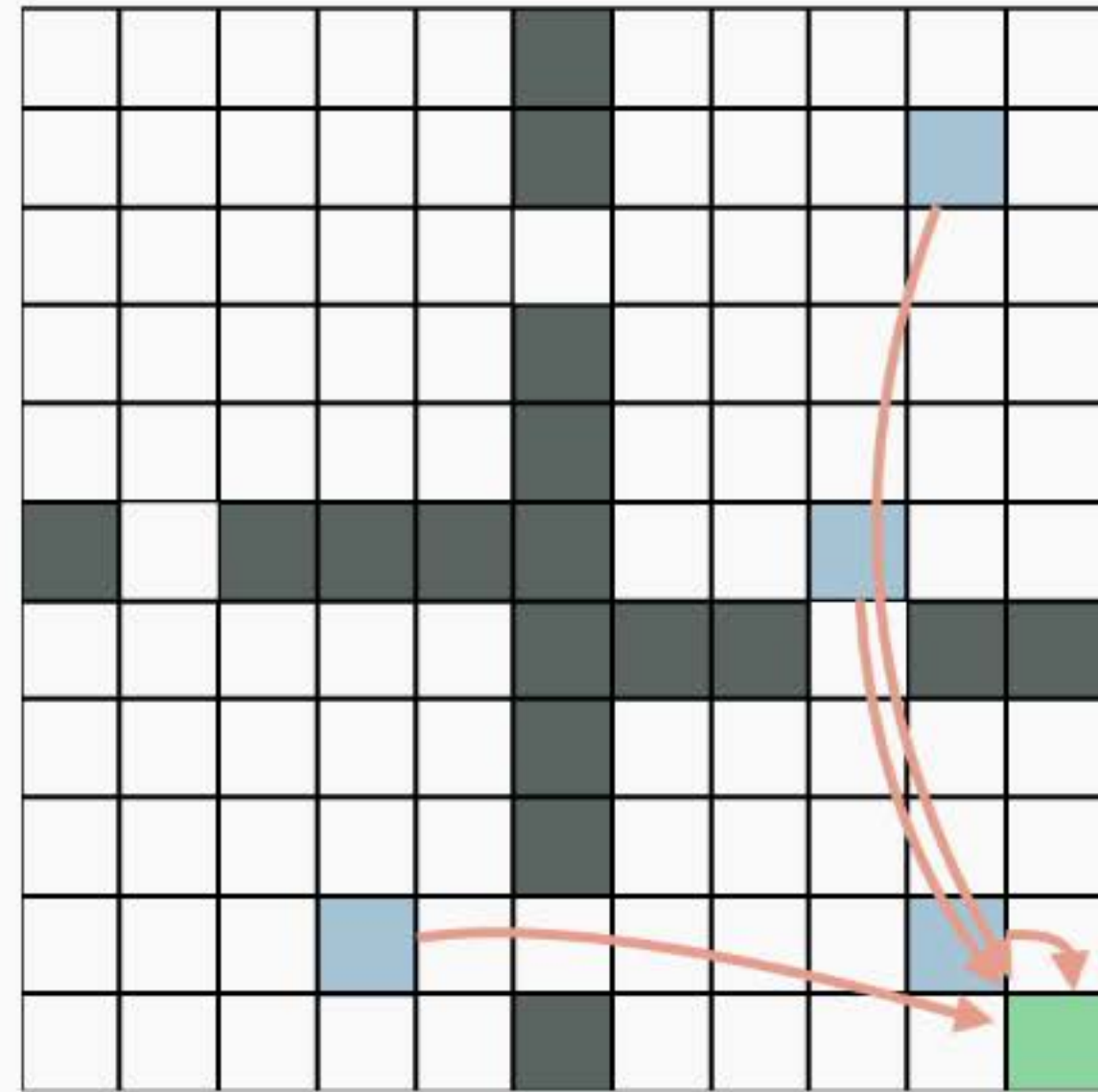
[Machado et al. 2017]

# Visuals: $K = 4$



*Betweenness Options*

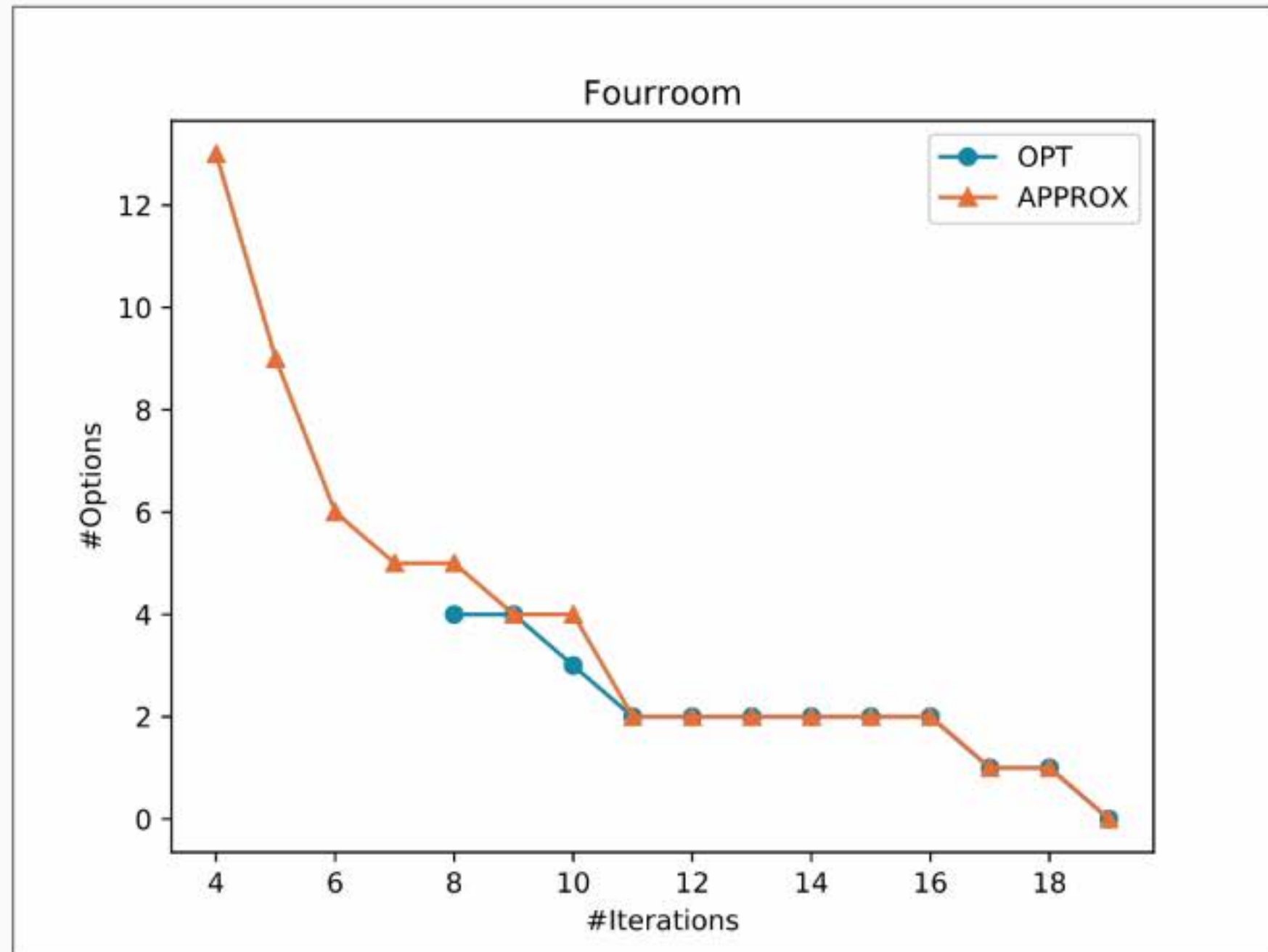
[Simsek and Barto 2005, 2008]



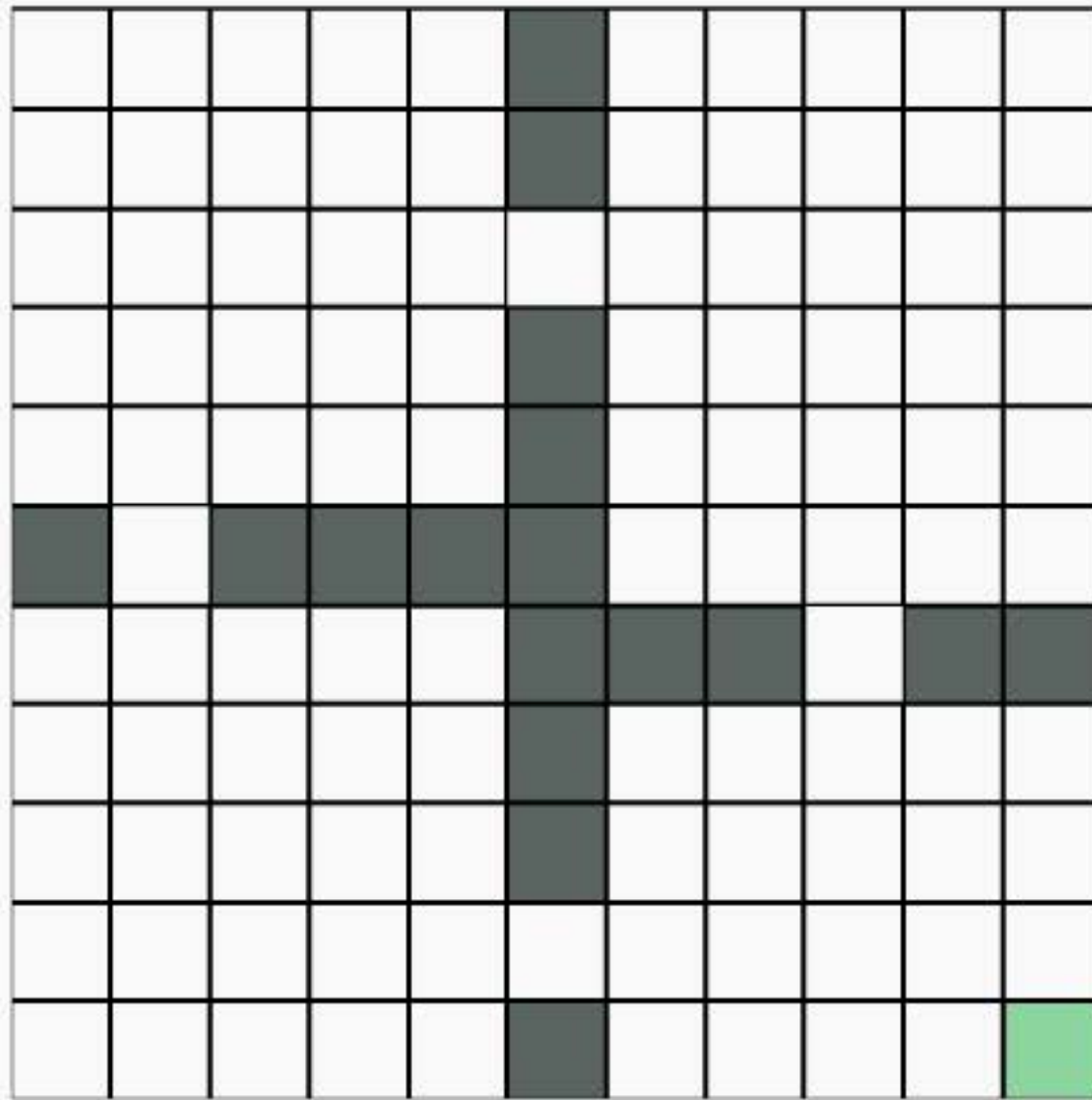
*Eigen Options*

[Machado et al. 2017]

# Evaluation

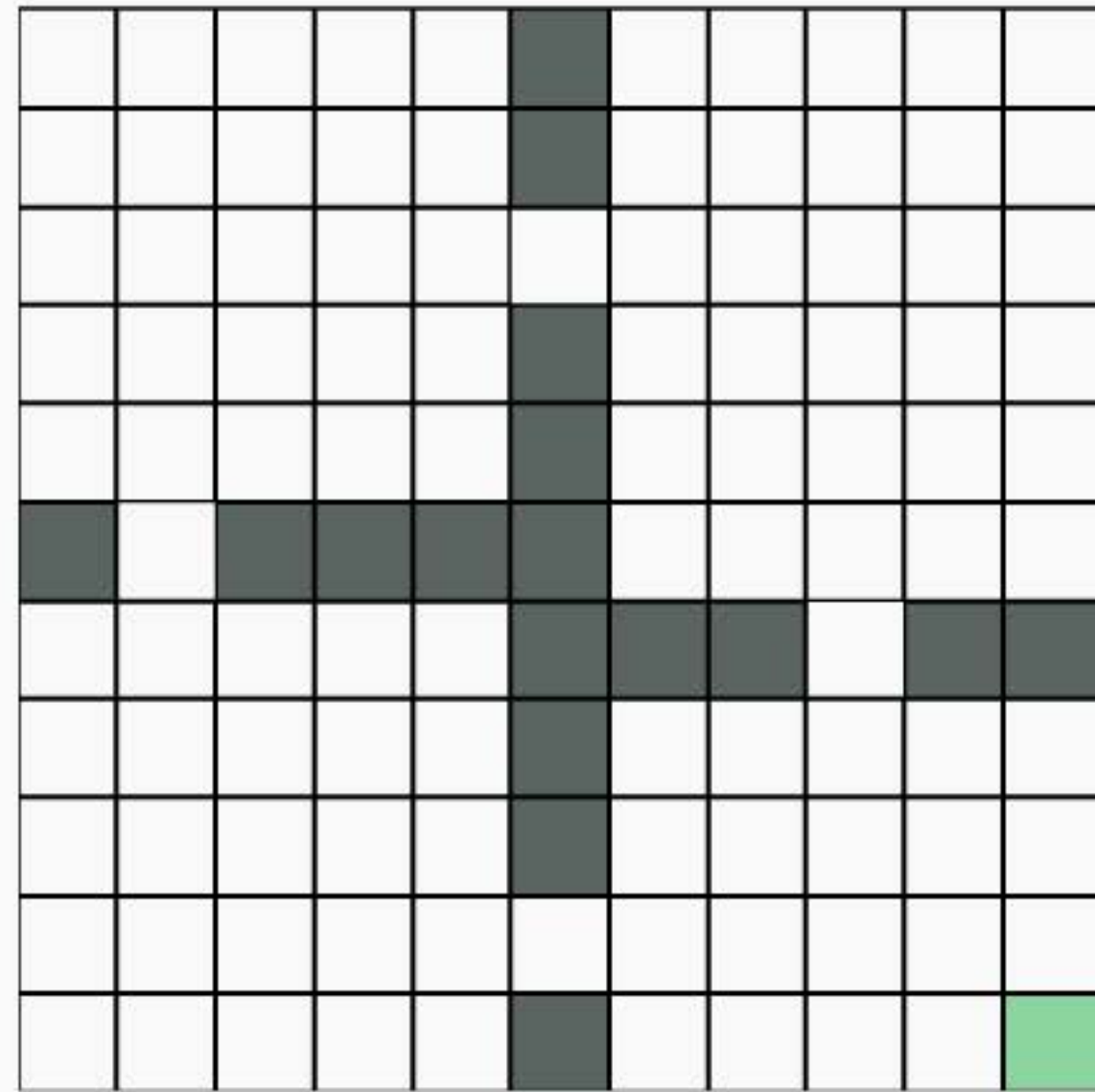


# Visuals: $K = 4$



*Betweenness Options*

*[Simsek and Barto 2005, 2008]*

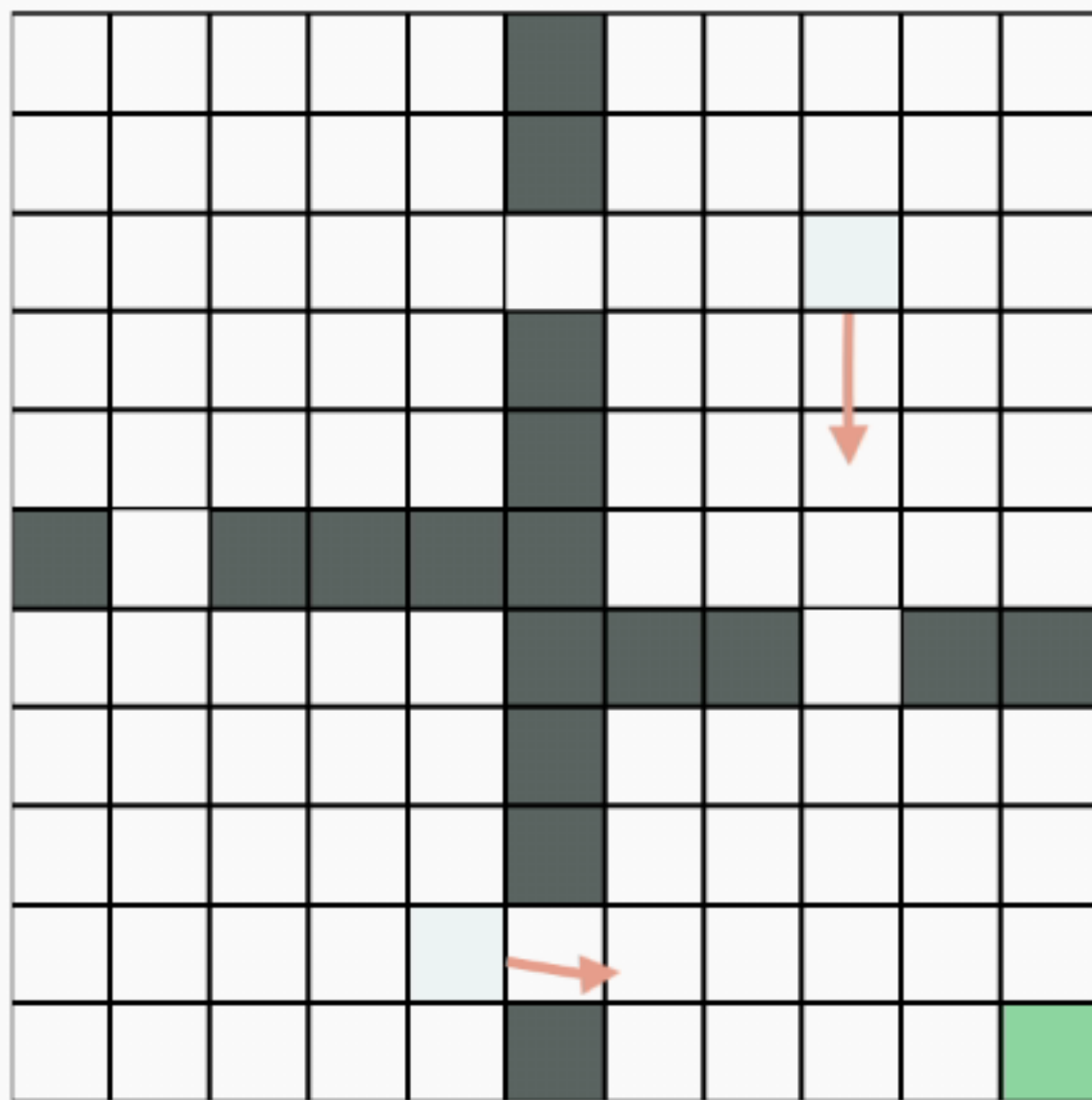


*Eigen Options*

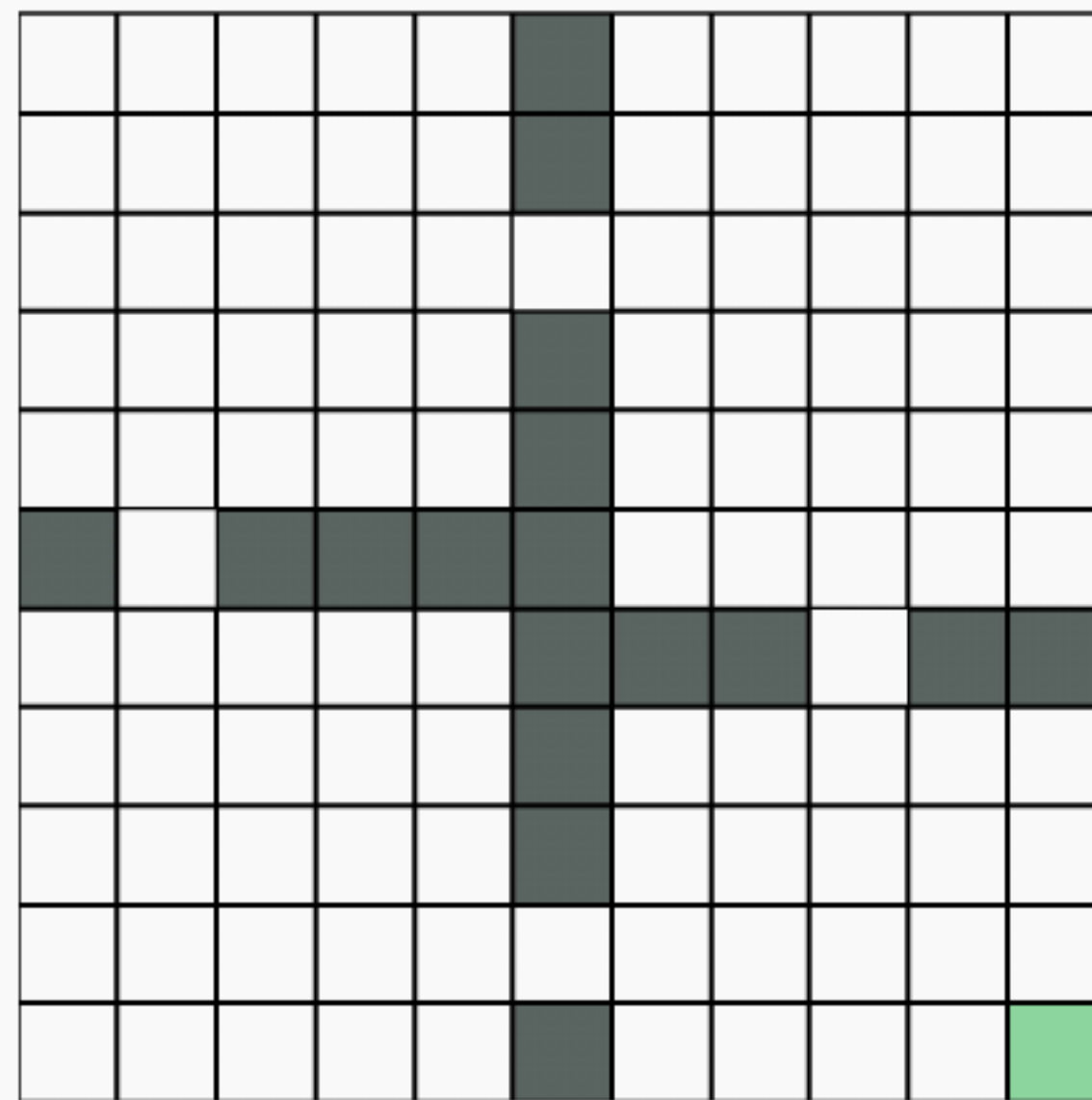
*[Machado et al. 2017]*



# Visuals: $K = 2$

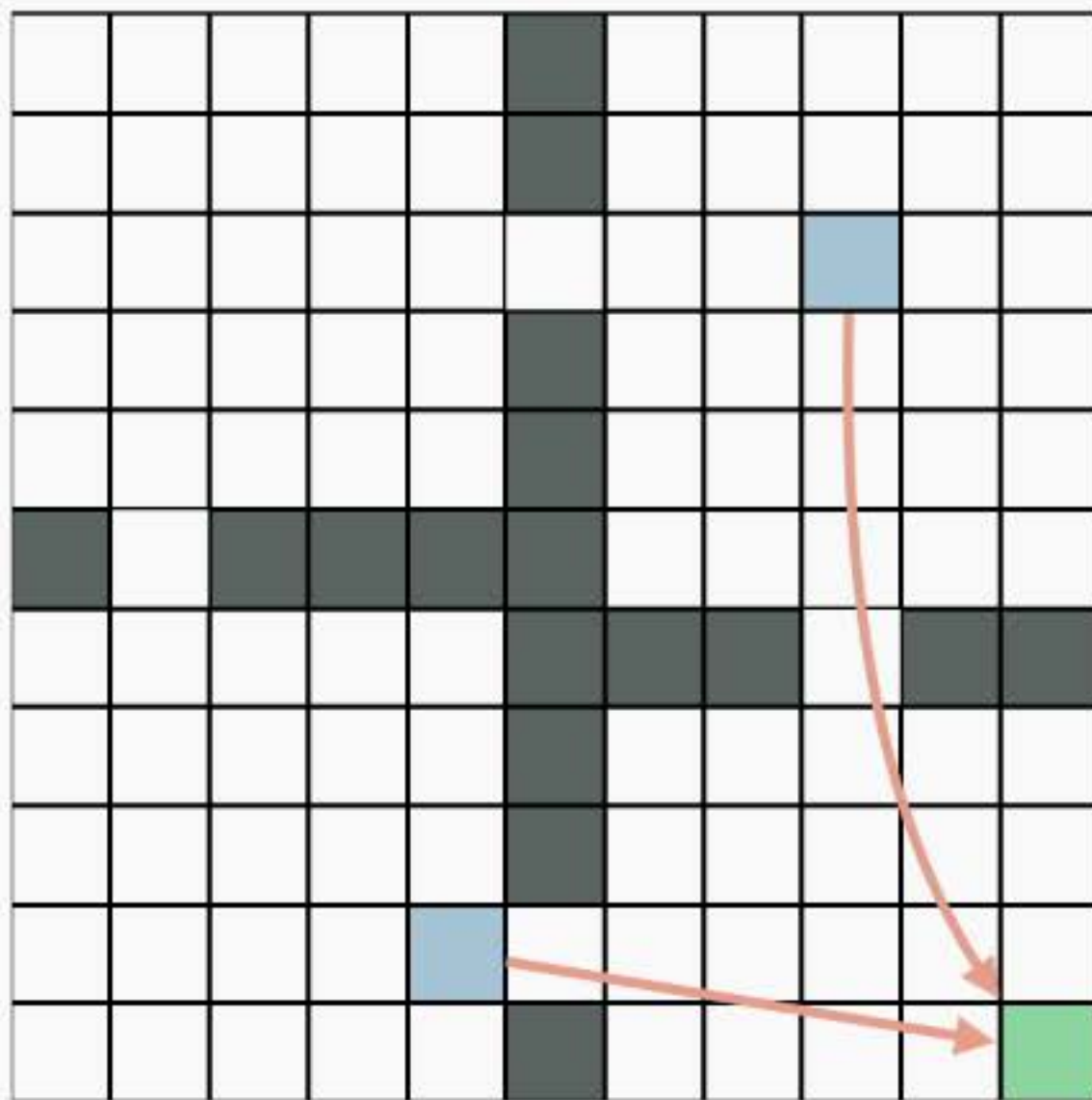


*Optimal*

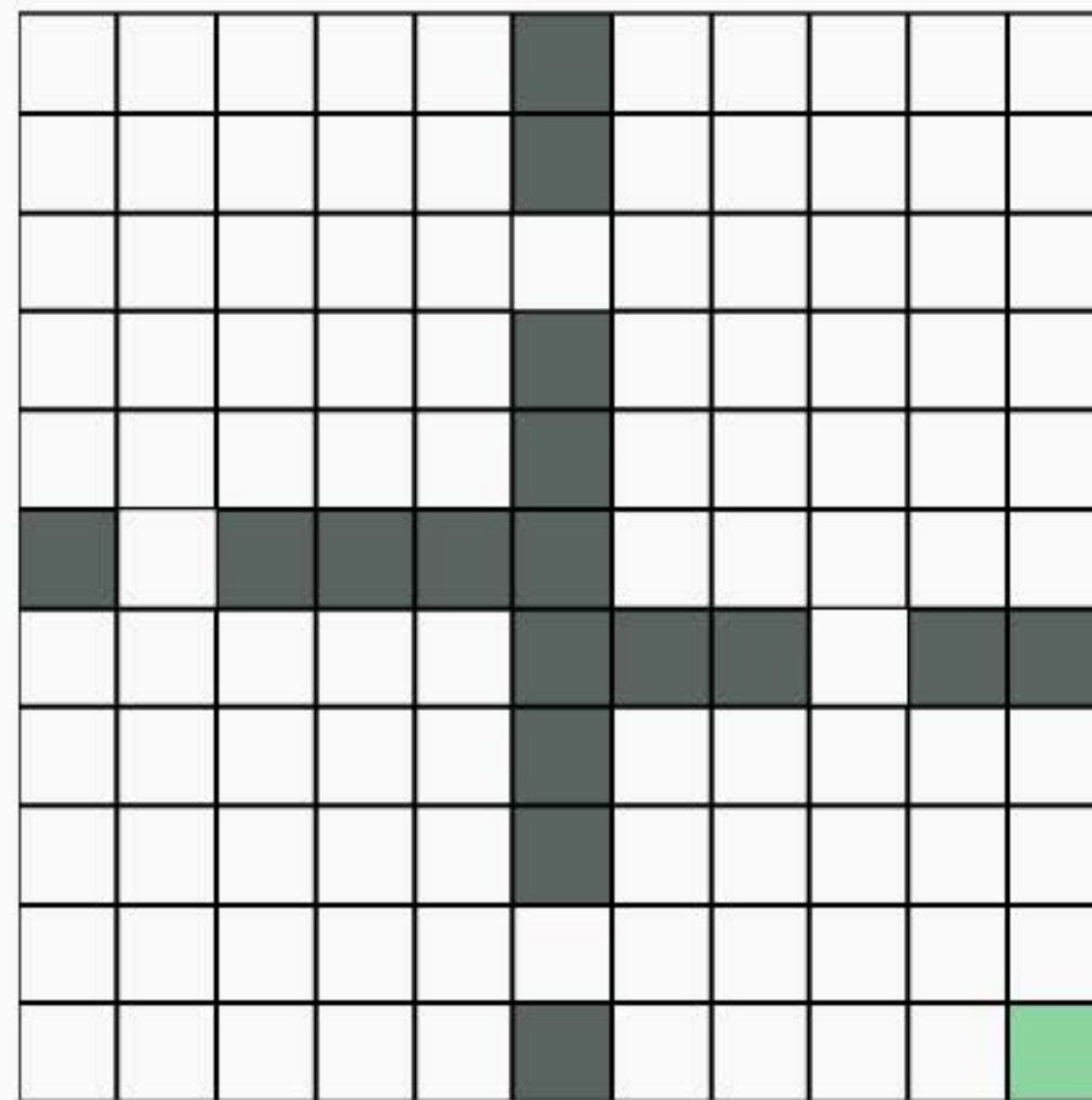


*Approximation*

# Visuals: $K = 2$

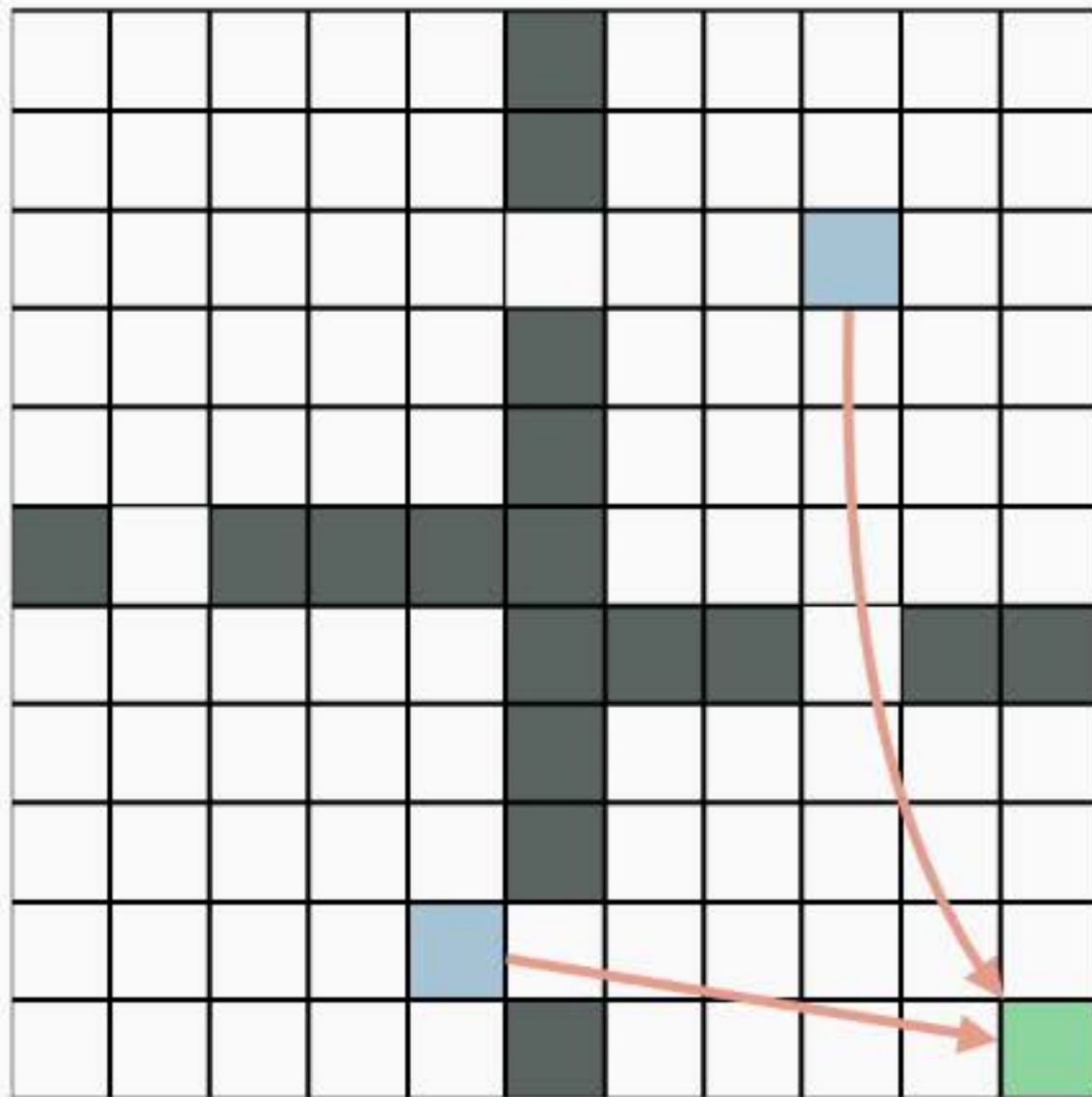


*Optimal*

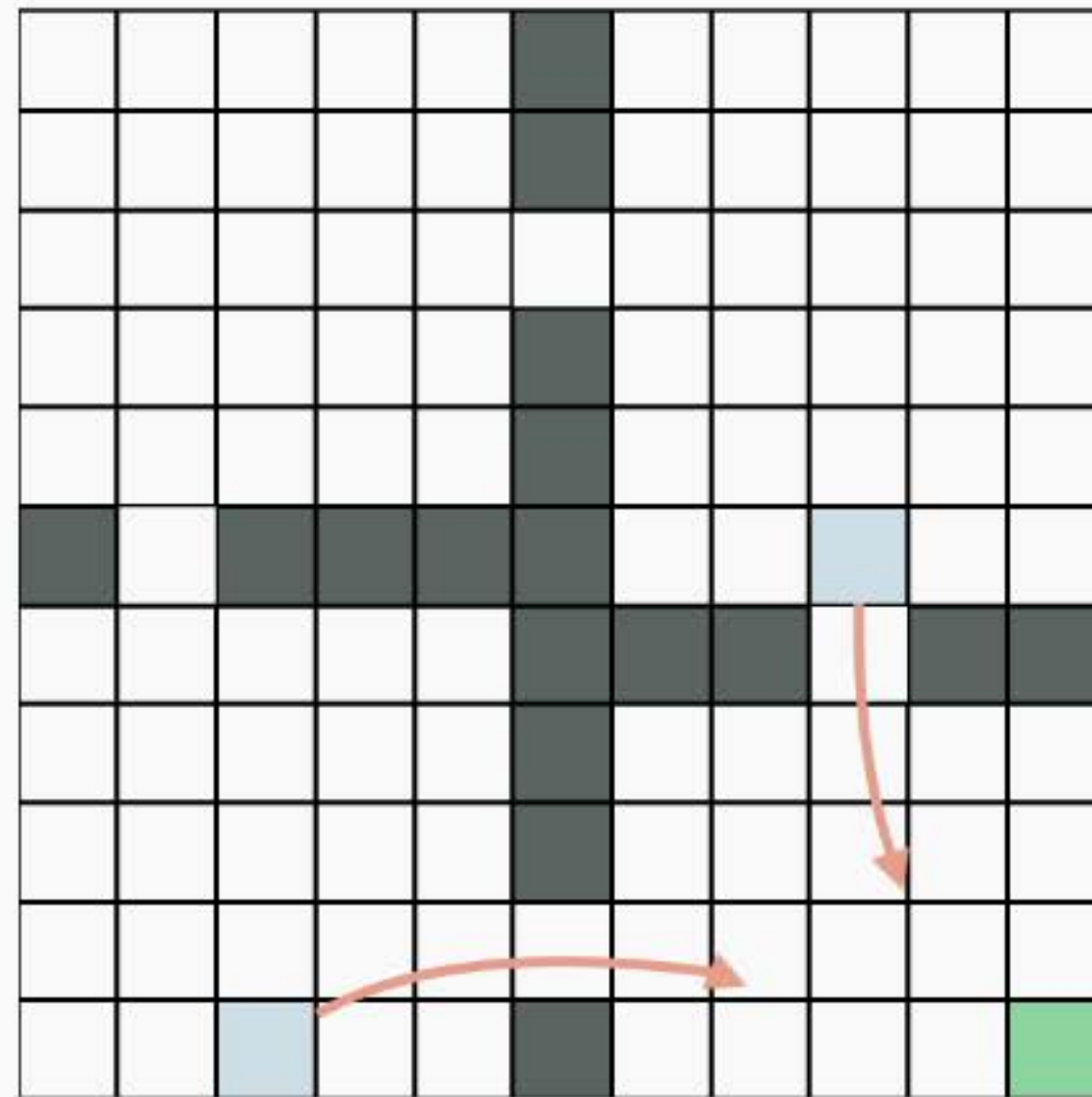


*Approximation*

# Visuals: $K = 2$

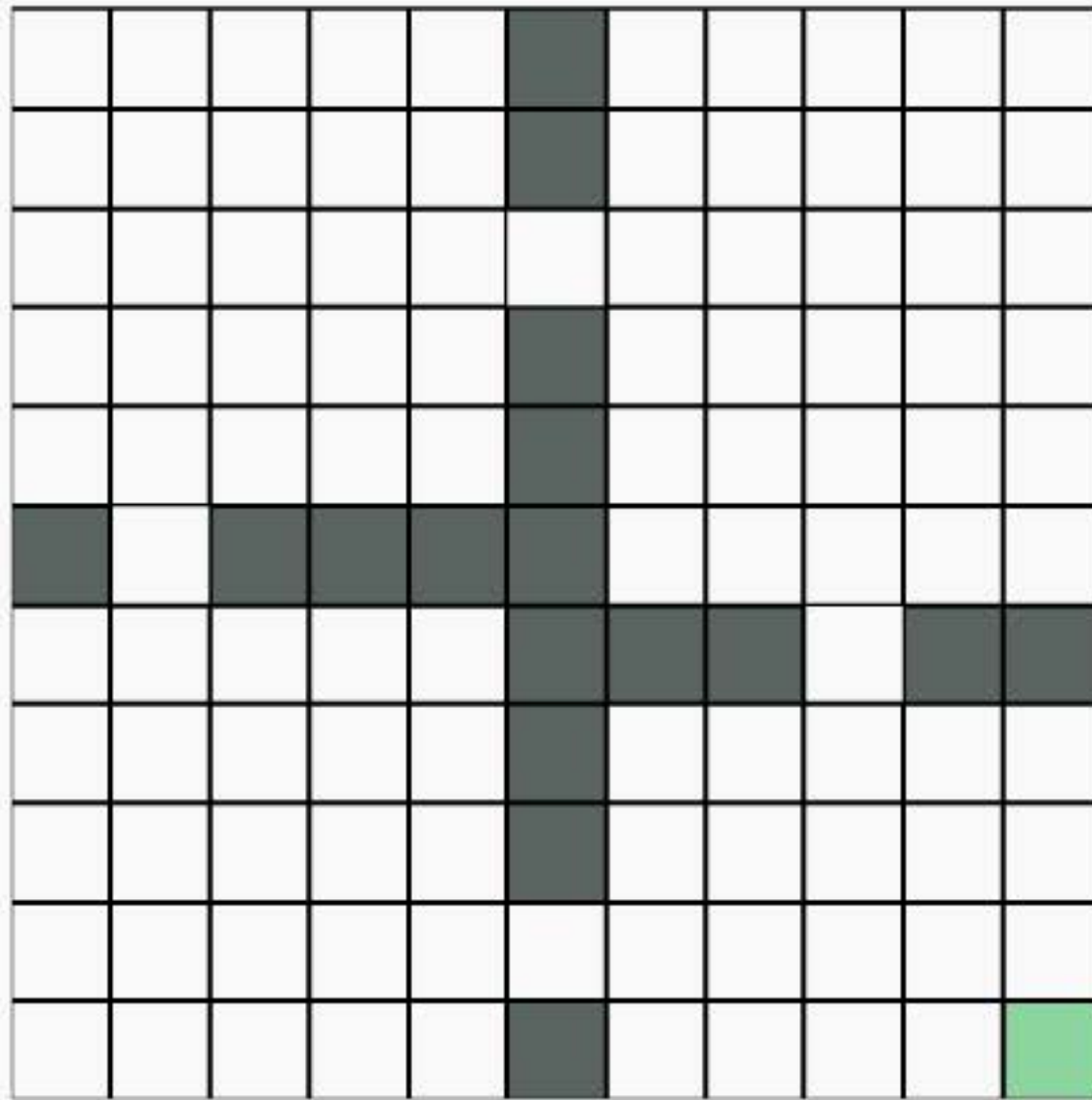


*Optimal*



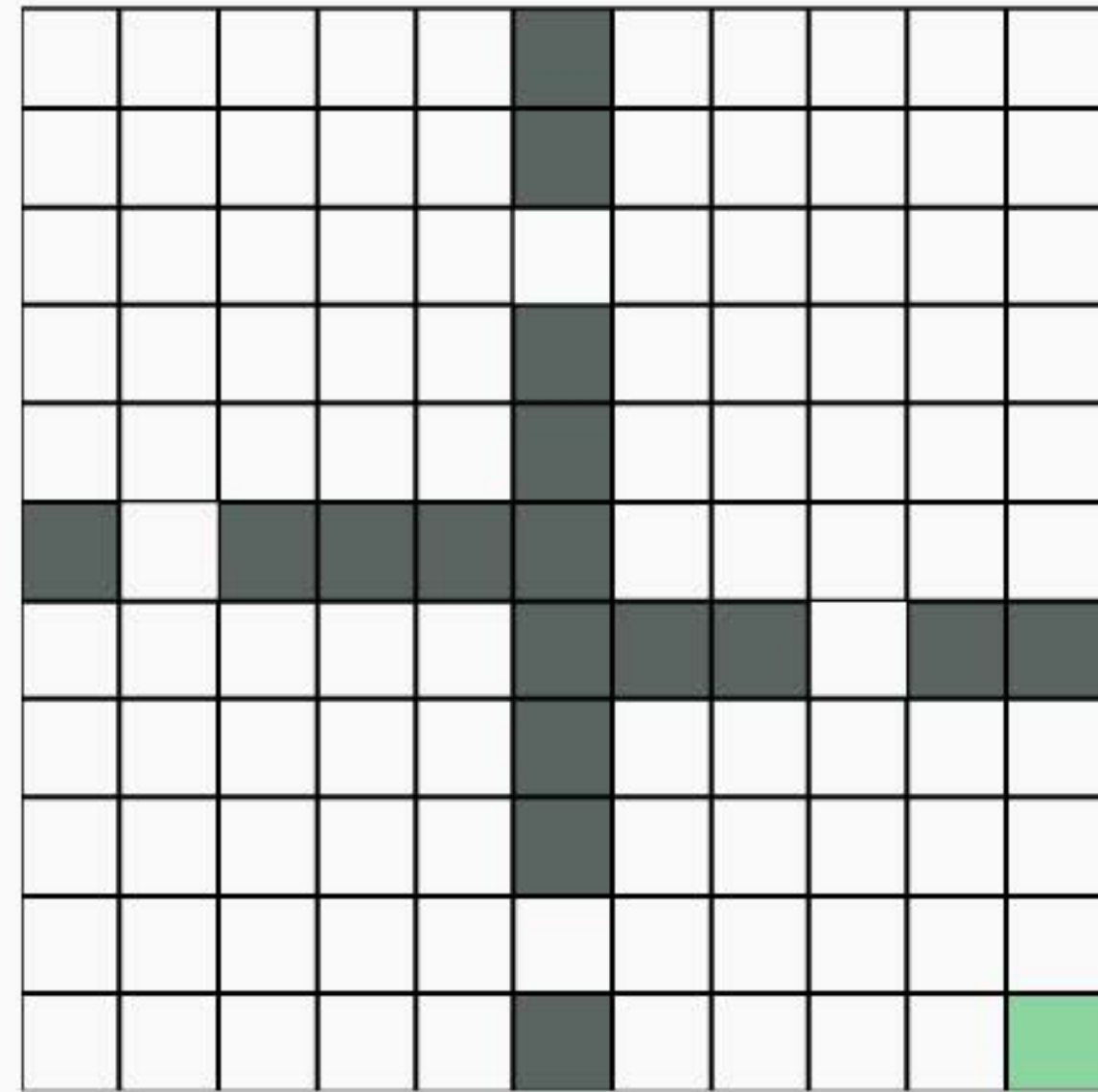
*Approximation*

# Visuals: $K = 4$



*Betweenness Options*

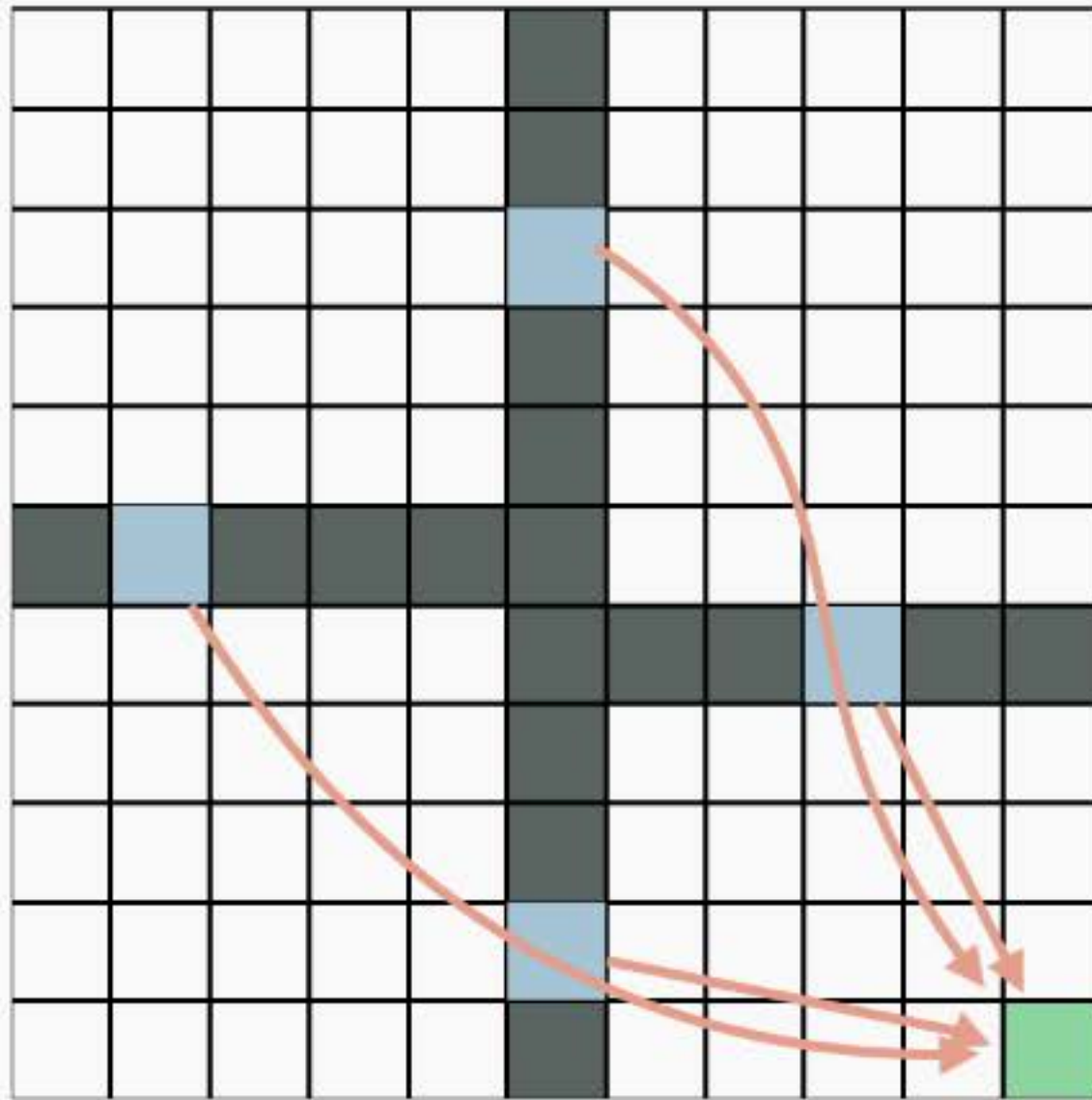
[Simsek and Barto 2005, 2008]



*Eigen Options*

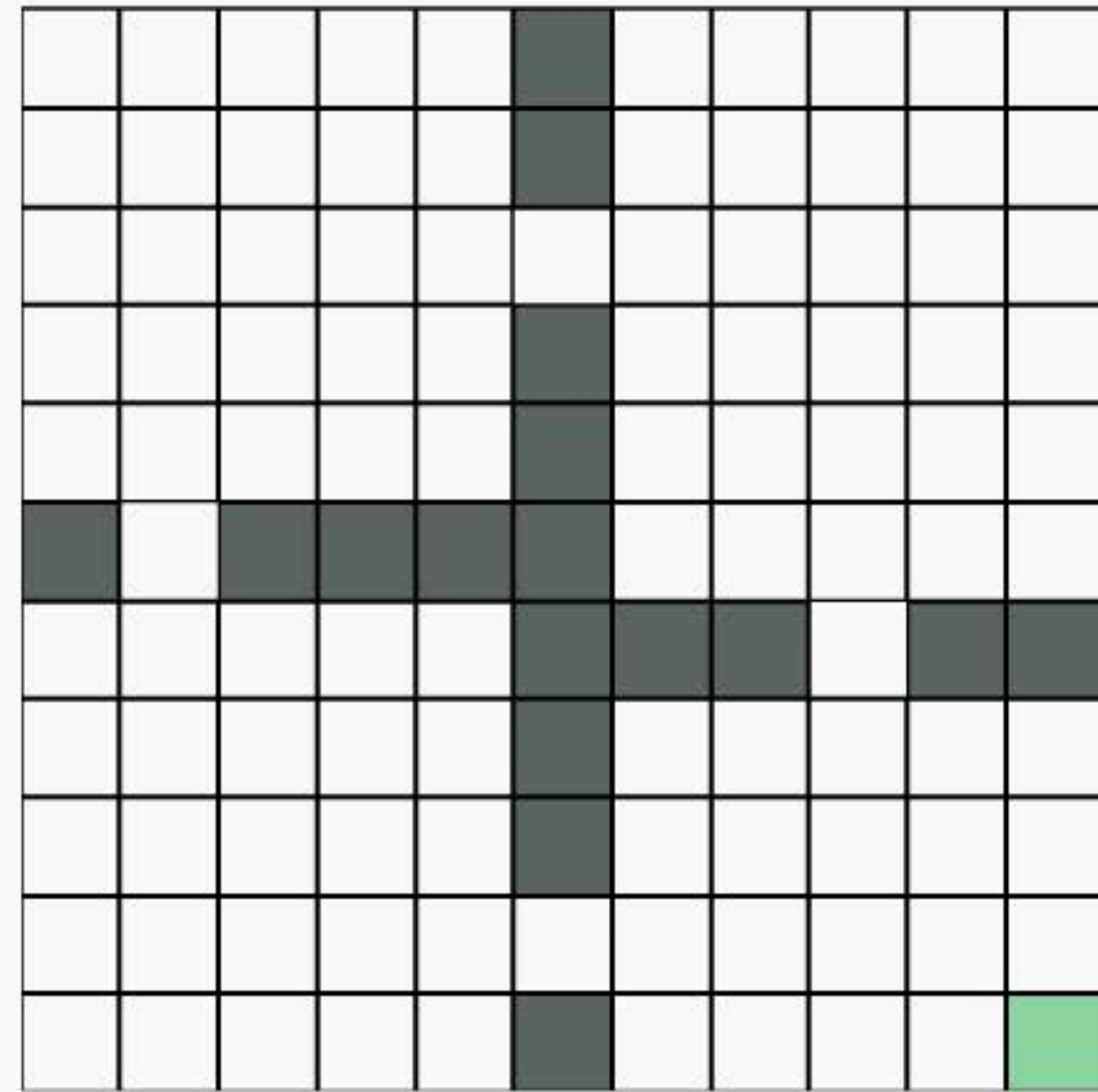
[Machado et al. 2017]

# Visuals: $K = 4$



*Betweenness Options*

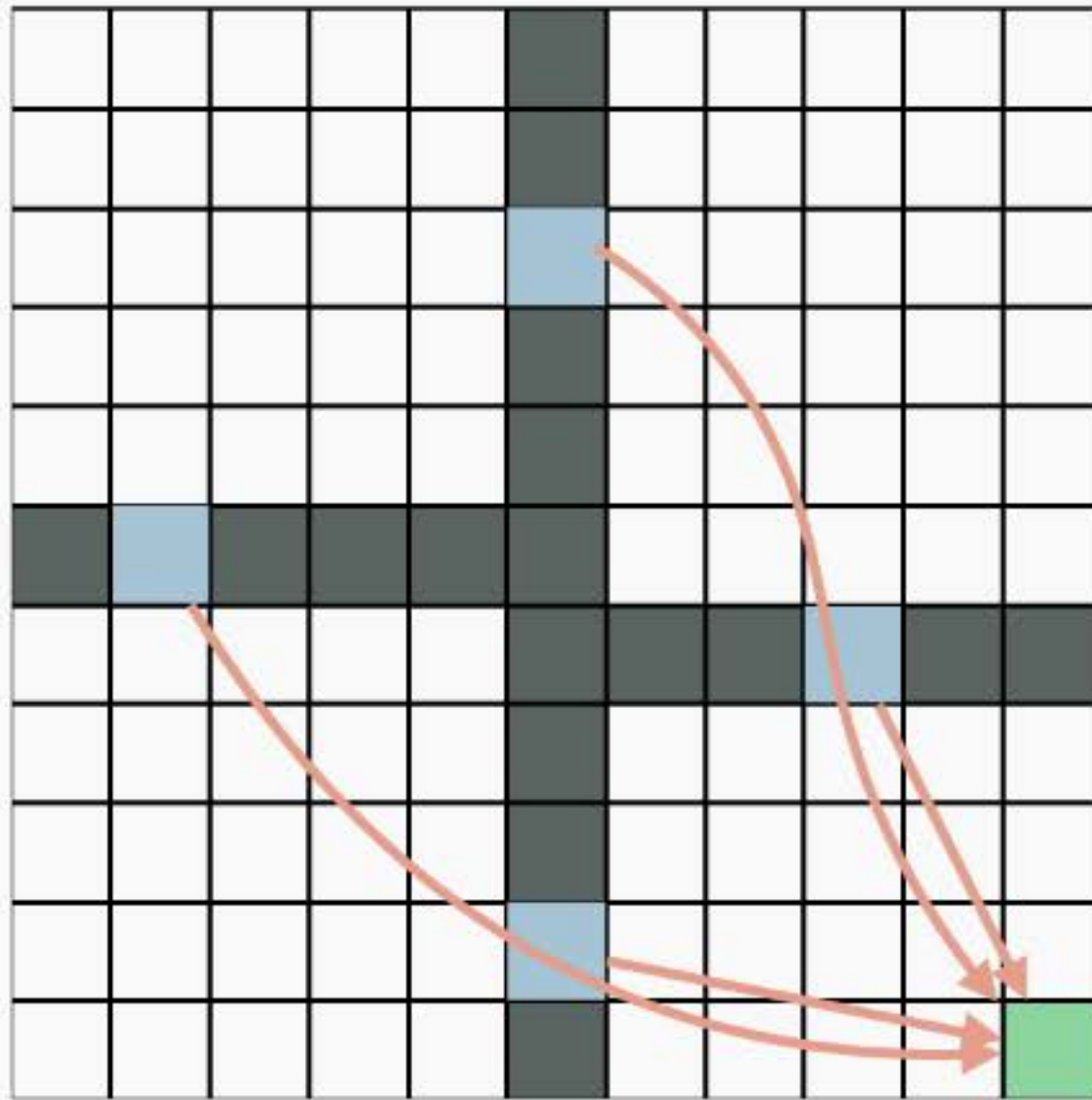
*[Simsek and Barto 2005, 2008]*



*Eigen Options*

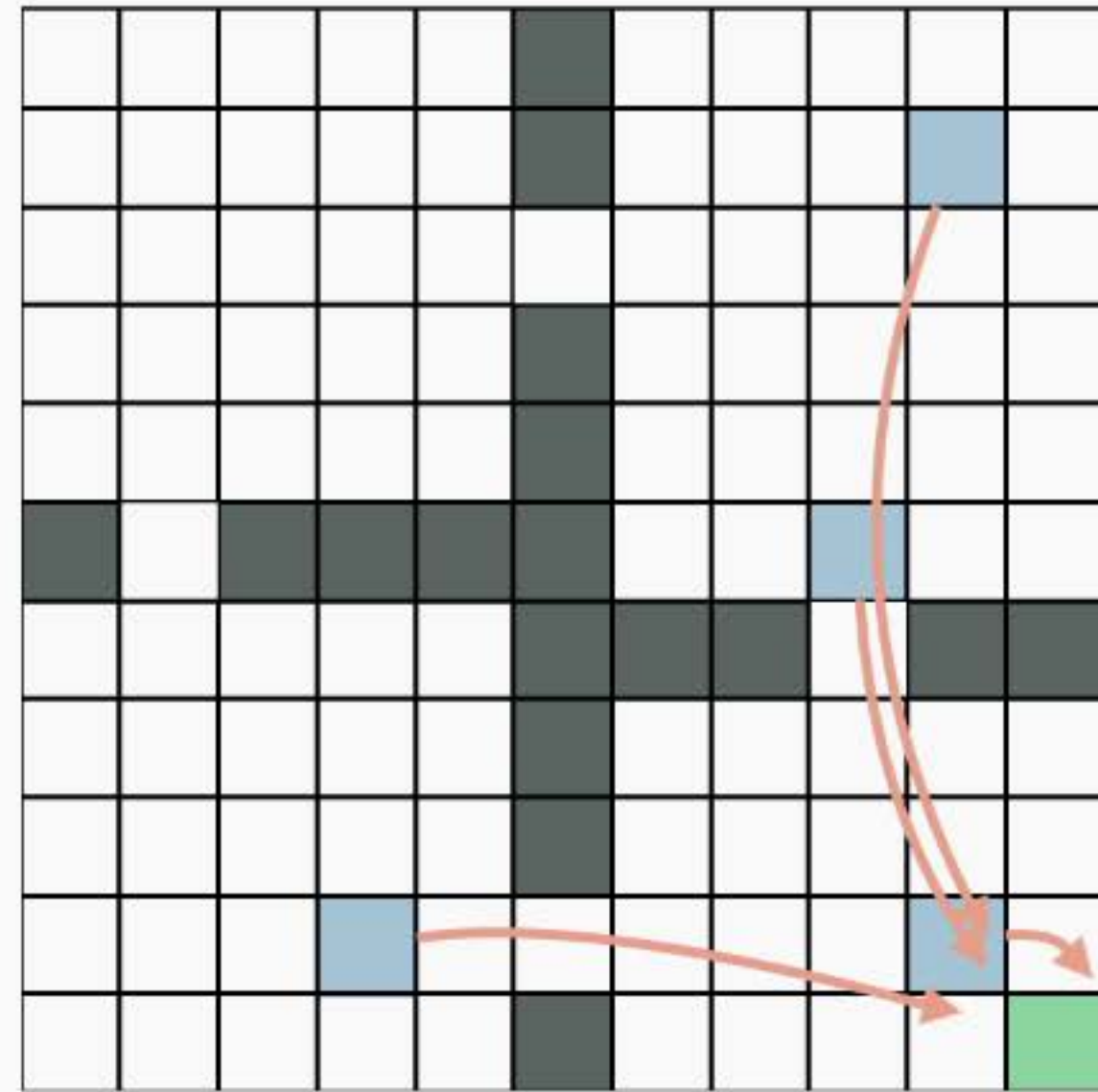
*[Machado et al. 2017]*

# Visuals: $K = 4$



*Betweenness Options*

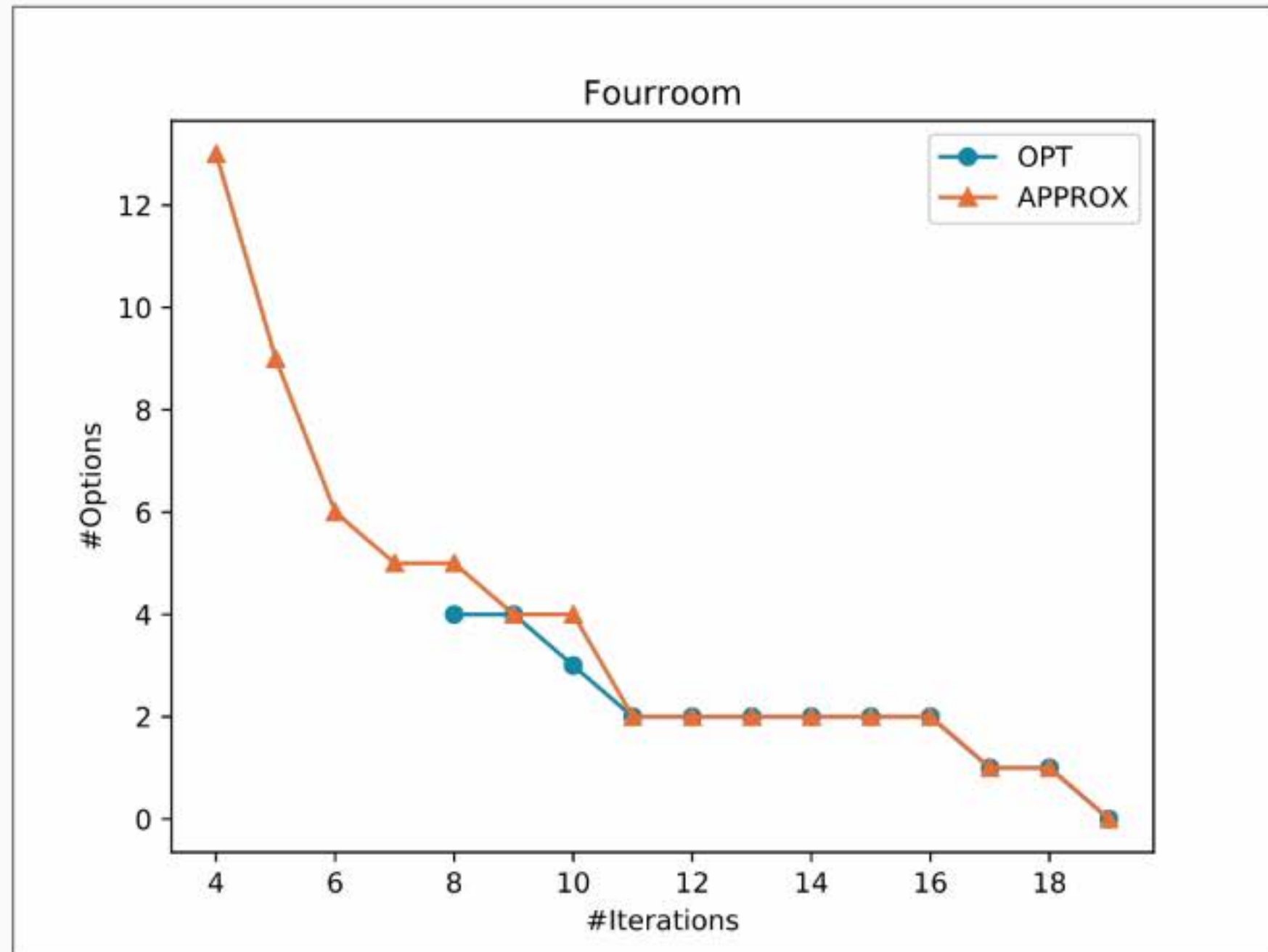
[Simsek and Barto 2005, 2008]



*Eigen Options*

[Machado et al. 2017]

# Evaluation



# Results Summary

**Question:** *Can we find the set of options that minimize the number of iterations of VI?*

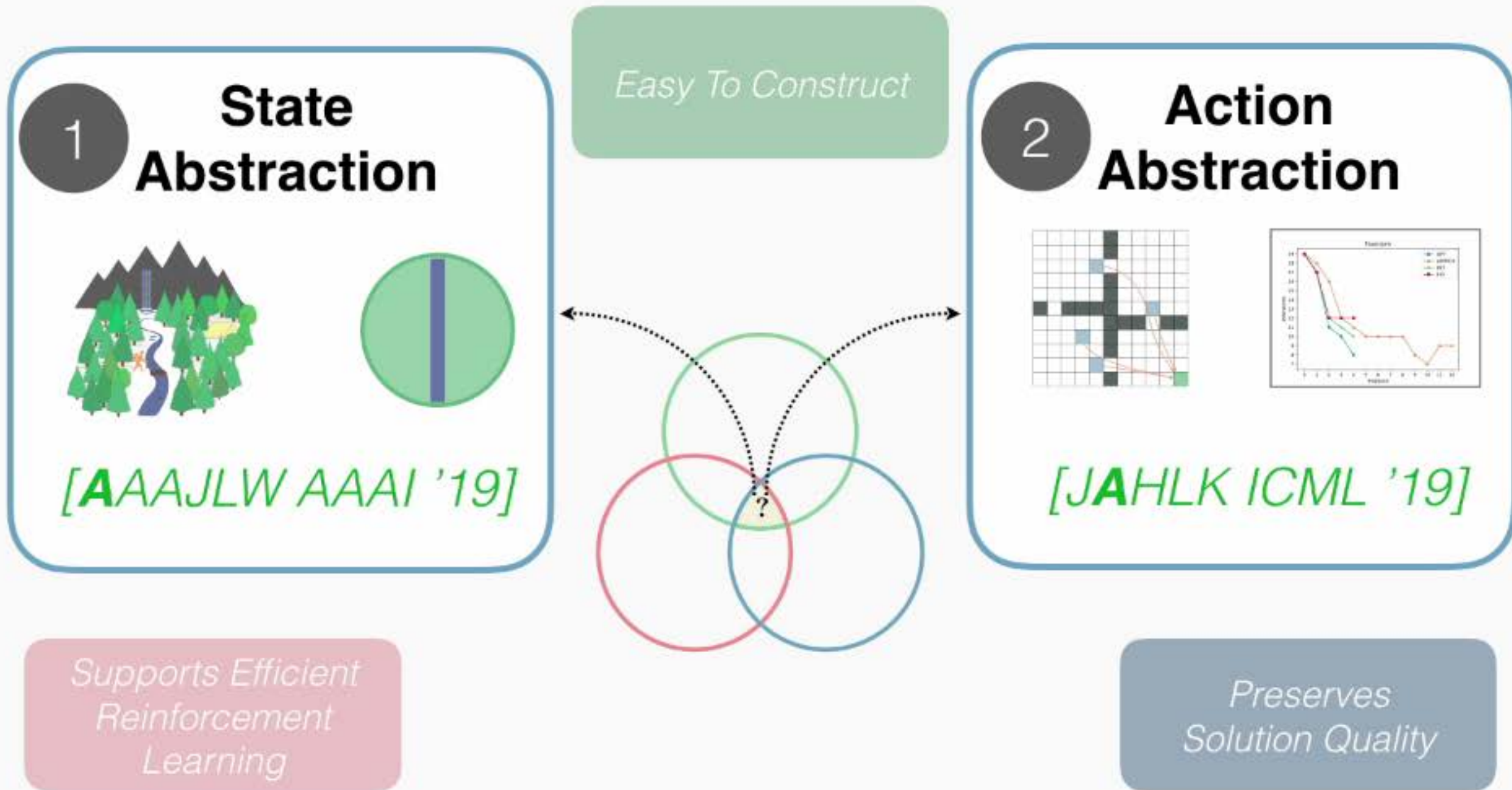


# Results Summary

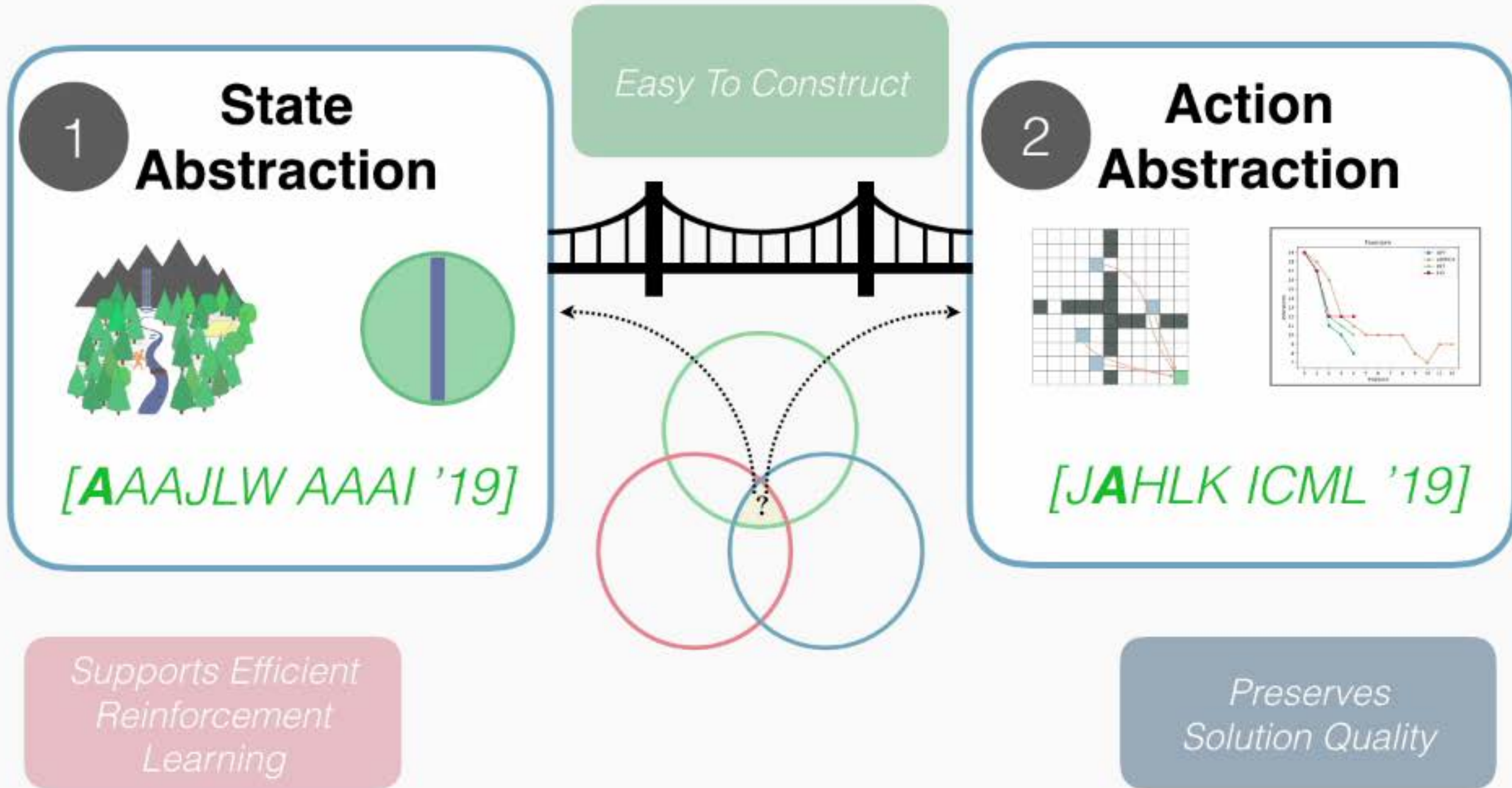
**Question:** *Can we find the set of options that minimize the number of iterations of VI?*

**Answer:** *Yes, but it takes serious computational work.*

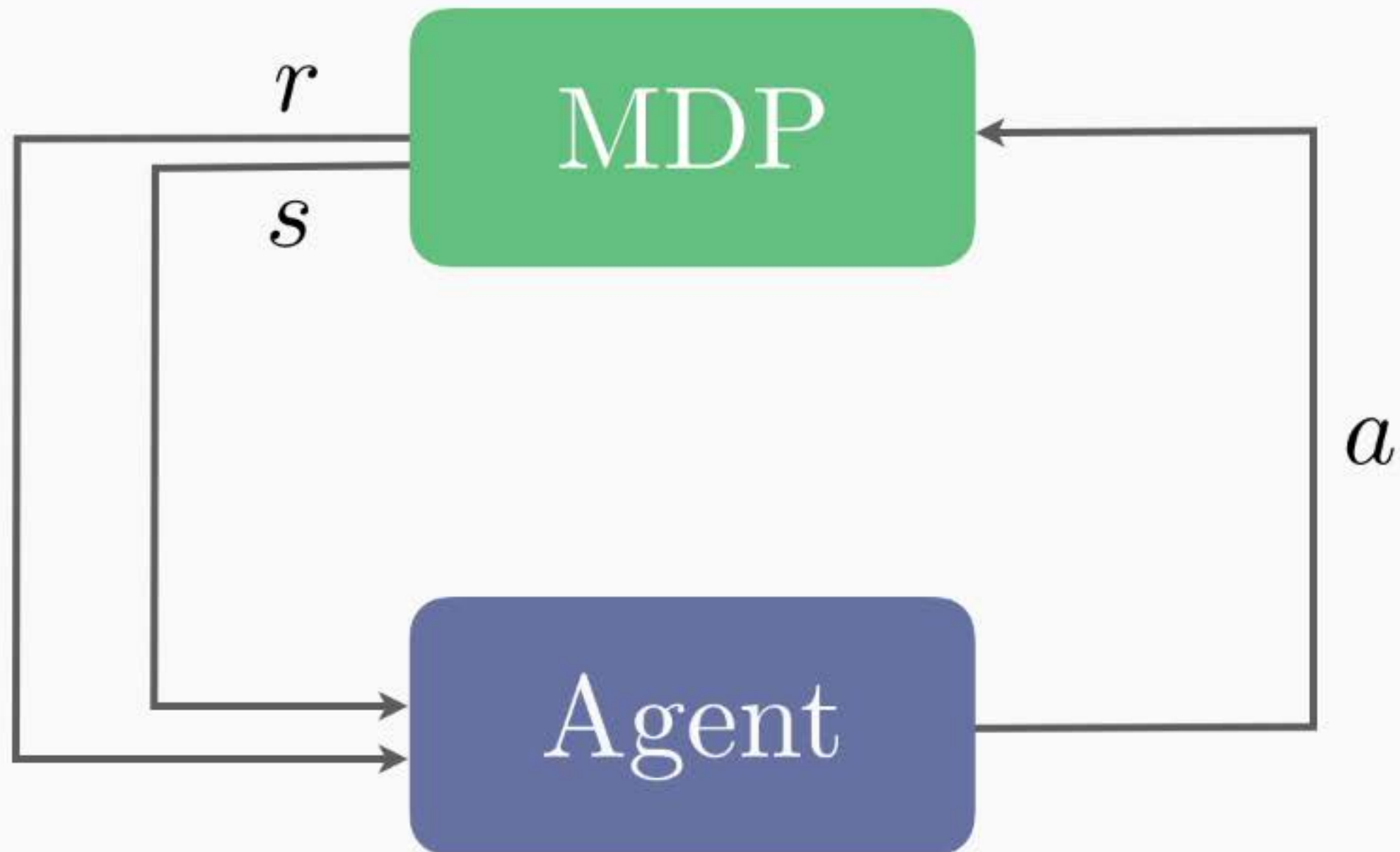
# Results Summary



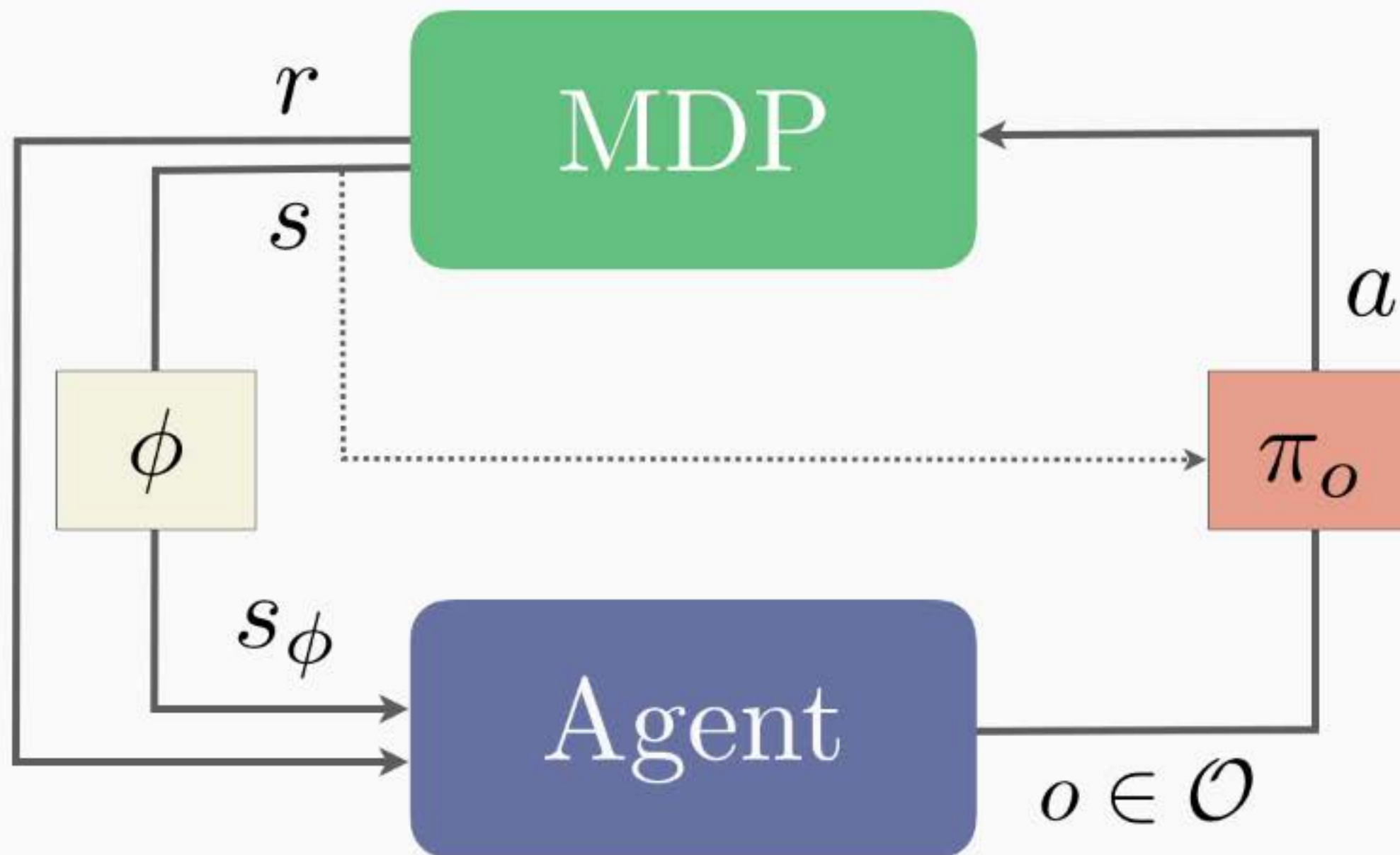
# Results Summary



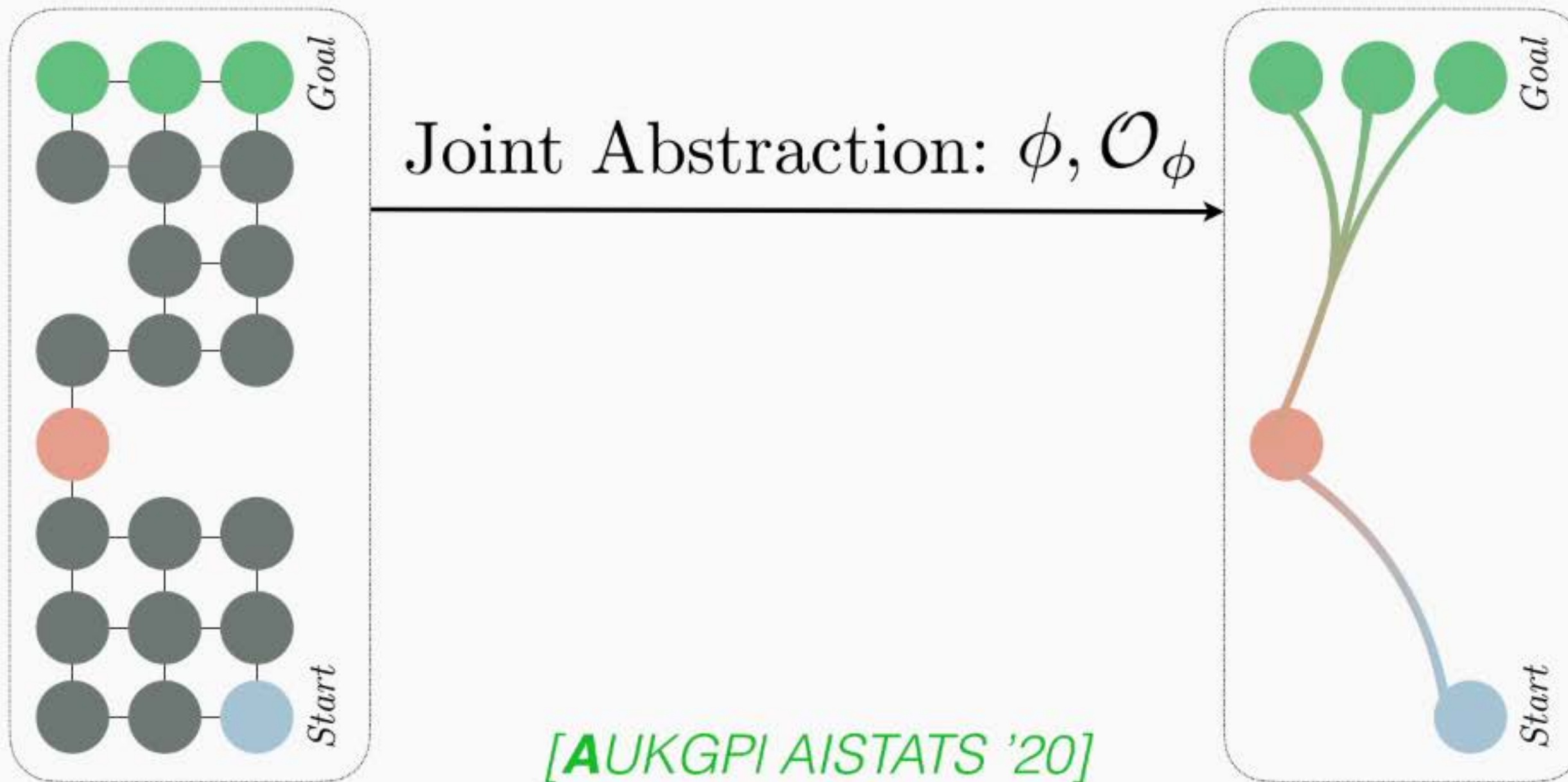
# State-Action Abstraction



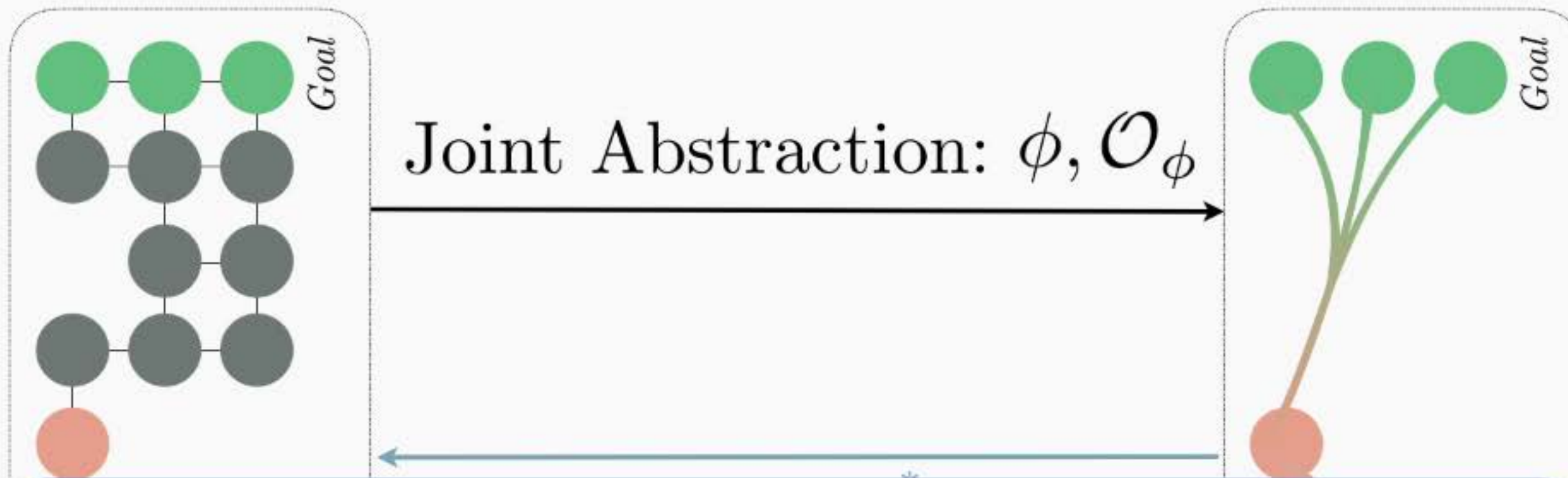
# State-Action Abstraction



# State-Action Abstraction



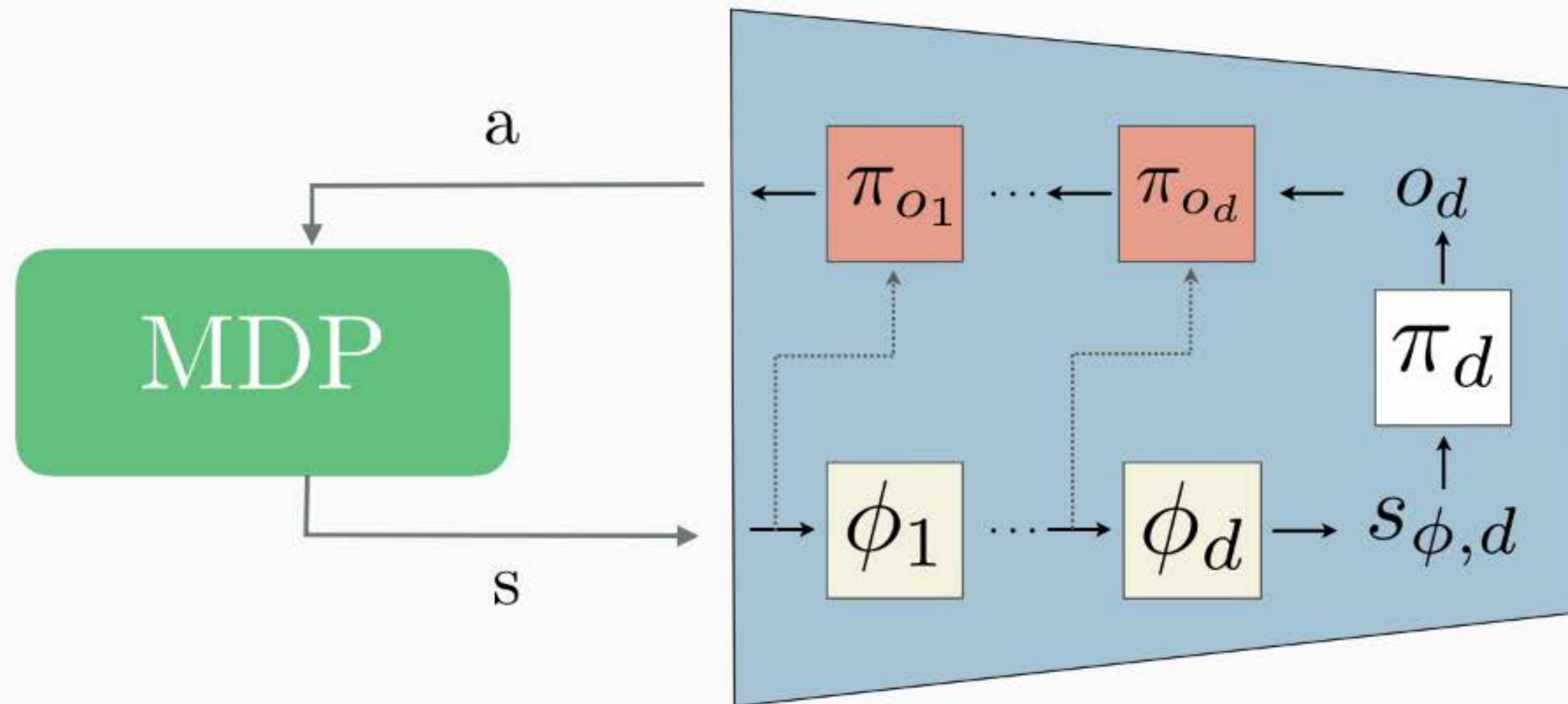
# State-Action Abstraction



**Theorem.** Well chosen  $\phi, \mathcal{O}_\phi$ , preserve value:

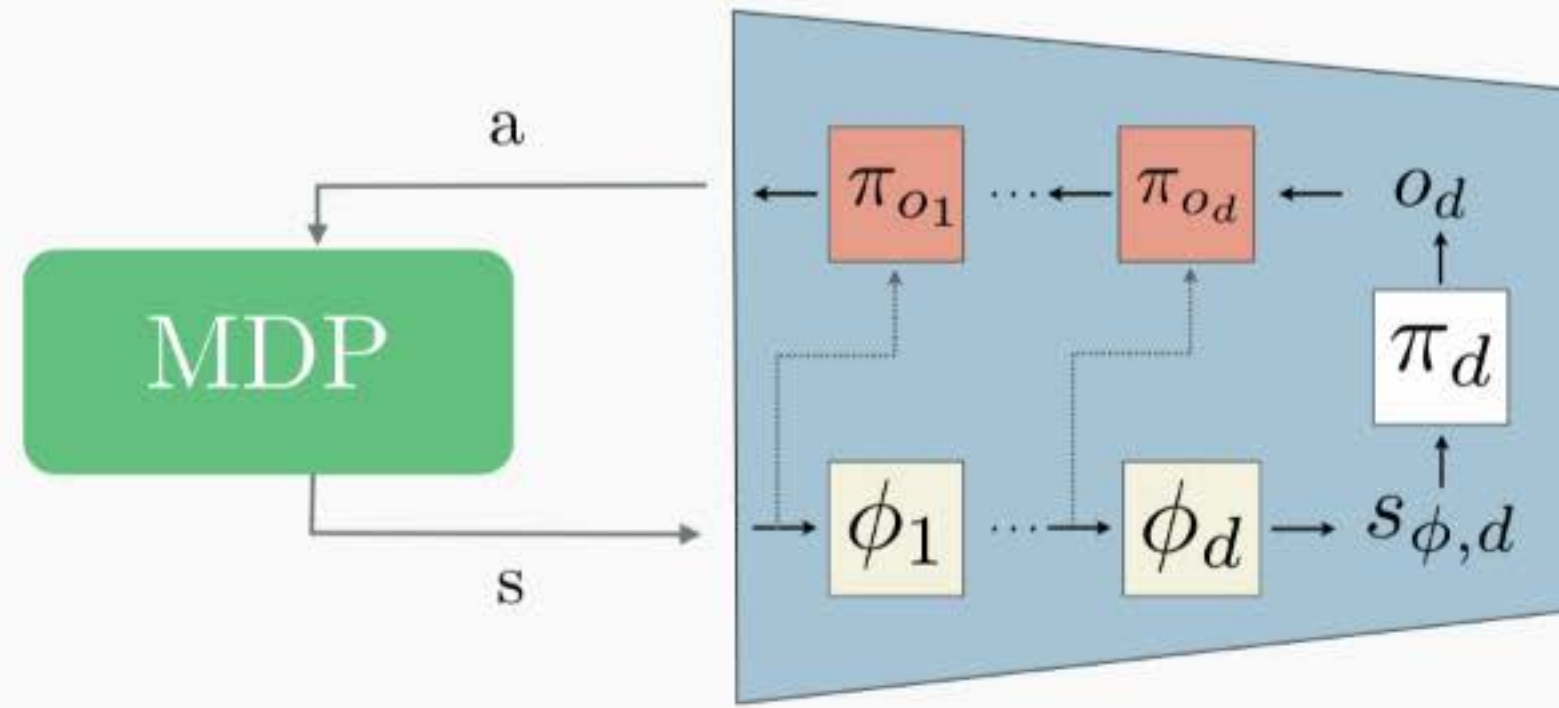
$$|V^*(s_0) - V^{\pi_{\phi, \mathcal{O}_\phi}^*}(s_0)| \leq f(\epsilon_{\mathcal{O}_\phi})$$

# Hierarchical Abstraction





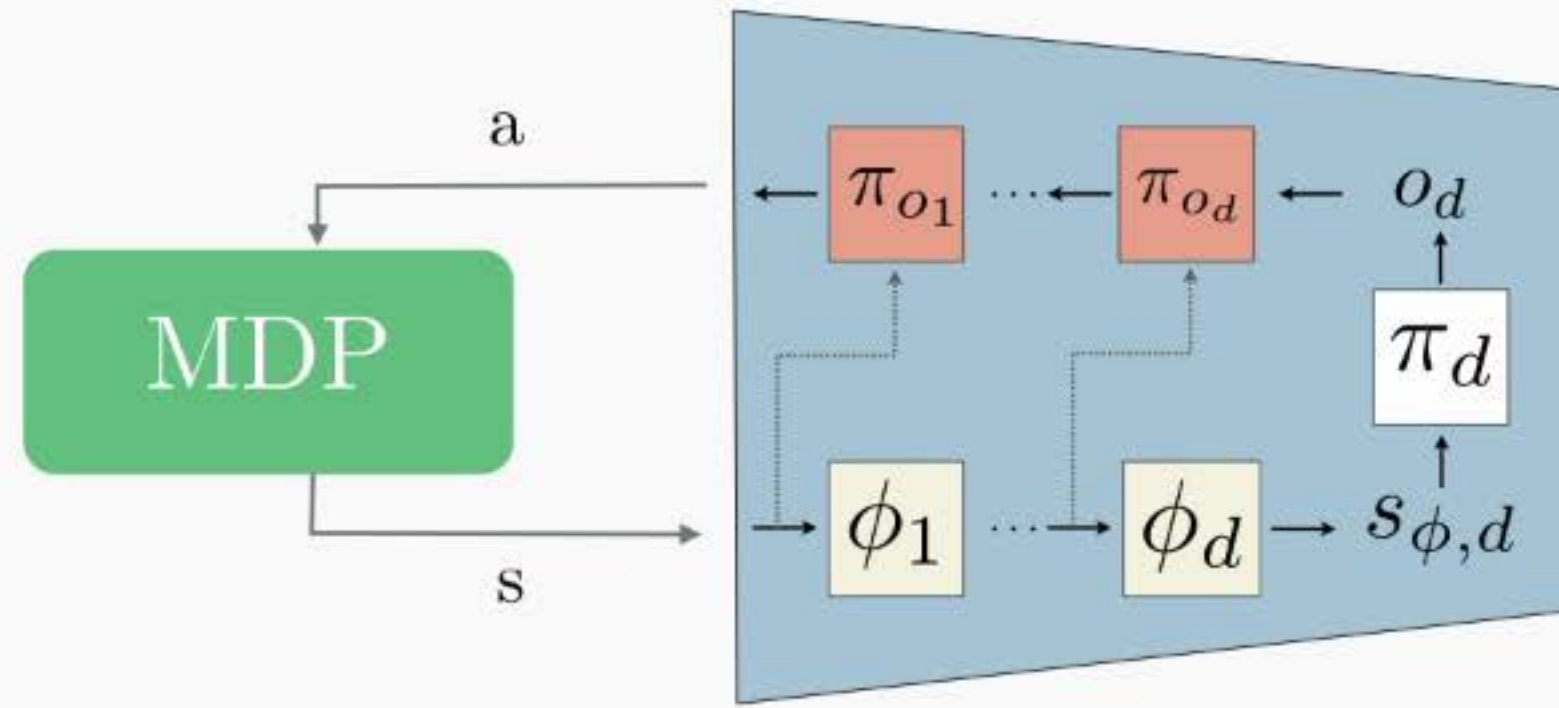
# Hierarchical Abstraction



**Theorem.** Well chosen  $\phi, \mathcal{O}_\phi$ , preserve value:

$$|V^*(s_0) - V^{\pi_d^*}(s_0)| = O(d)$$

# Hierarchical Abstraction



**Theorem.** Well chosen  $\phi, \mathcal{O}_\phi$ , preserve value:

$$|V^*(s_0) - V^{\pi_d^*}(s_0)| = O(d)$$

# Work Overview

## Abstraction in RL



*[ICML 2019a]*

*[AAAI 2019]*

*[AISTATS 2020]*

# Work Overview

## Abstraction in RL



- [ICML 2016]*    *[ICML 2018a]*
- [ICML 2019a]***    *[ICML 2019b]*
- [AAAI 2019]***    *[IJCAI 2019]*
- [Opinions in Behavioral Sci. 2019]*
- [AISTATS 2020]***

# Work Overview

## Abstraction in RL



[ICML 2016] [ICML 2018a]  
[**ICML 2019a**] [ICML 2019b]  
[**AAAI 2019**] [IJCAI 2019]  
[Opinions in Behavioral Sci. 2019]  
[**AISTATS 2020**]

## Human Cognition

[CCN 2019] [AAAI 2020]

# Work Overview

## Abstraction in RL



*[ICML 2016]*    *[ICML 2018a]*  
***[ICML 2019a]***    *[ICML 2019b]*  
***[AAAI 2019]***    *[IJCAI 2019]*  
*[Opinions in Behavioral Sci. 2019]*  
***[AISTATS 2020]***

## Multitask Learning

*[ICAPS 2015]*    *[ICML 2018b]*

## Human Cognition

*[CCN 2019]*    *[AAAI 2020]*

# Work Overview

## Abstraction in RL



*[ICML 2016]*    *[ICML 2018a]*  
***[ICML 2019a]***    *[ICML 2019b]*  
***[AAAI 2019]***    *[IJCAI 2019]*  
*[Opinions in Behavioral Sci. 2019]*  
***[AISTATS 2020]***

## Human Cognition

*[CCN 2019]*    *[AAAI 2020]*

## Multitask Learning

*[ICAPS 2015]*    *[ICML 2018b]*

## Computational Sustainability



*[EnvirolInfo 2017]*  
*[RLDM 2017]*  
*[IAAI 2018]*

# Work Overview

## Abstraction in RL



*[ICML 2016]*    *[ICML 2018a]*  
***[ICML 2019a]***    *[ICML 2019b]*  
***[AAAI 2019]***    *[IJCAI 2019]*  
*[Opinions in Behavioral Sci. 2019]*  
***[AISTATS 2020]***

## Human Cognition

*[CCN 2019]*    *[AAAI 2020]*

## Multitask Learning

*[ICAPS 2015]*    *[ICML 2018b]*

## Computational Sustainability



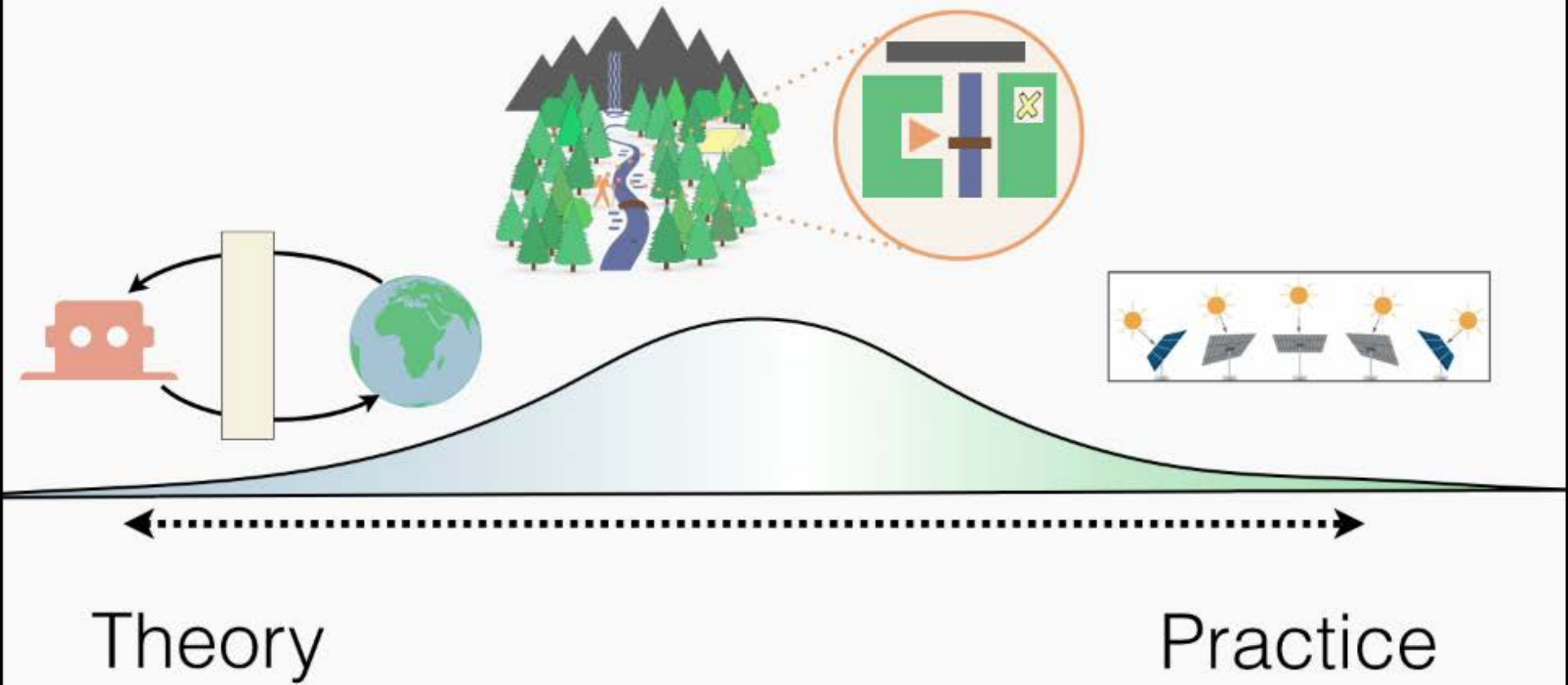
*[EnvirolInfo 2017]*  
*[RLDM 2017]*  
*[IAAI 2018]*

## Philosophy of AI

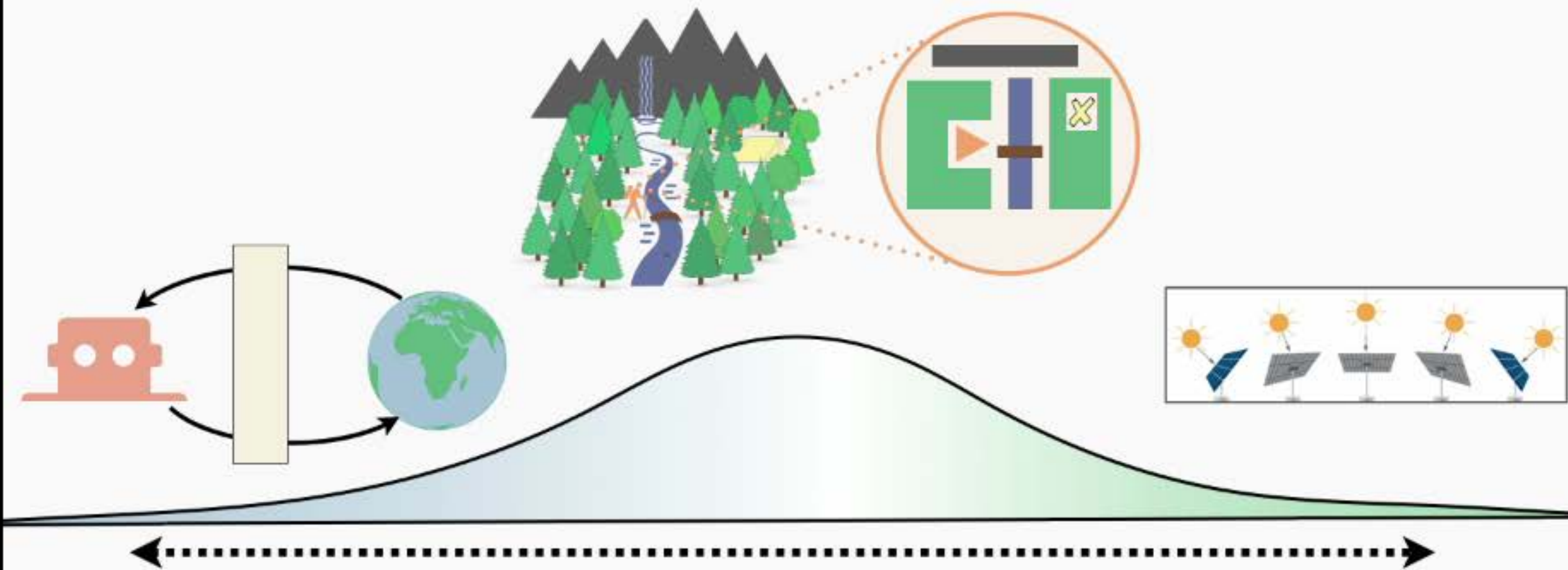
*[AIES Workshop 2016]*  
*[IACAP 2019]*



# Agenda

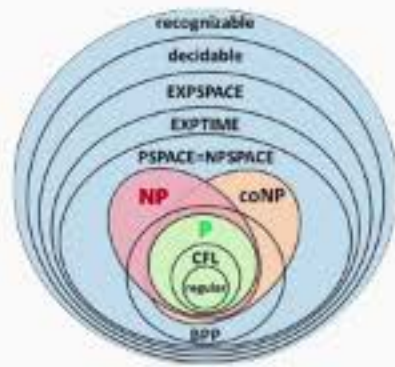


# Agenda



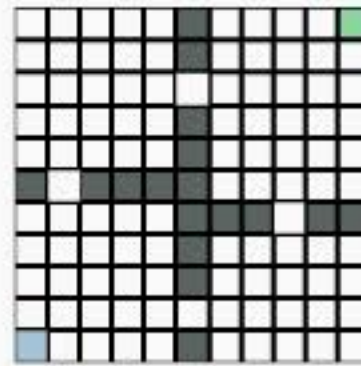
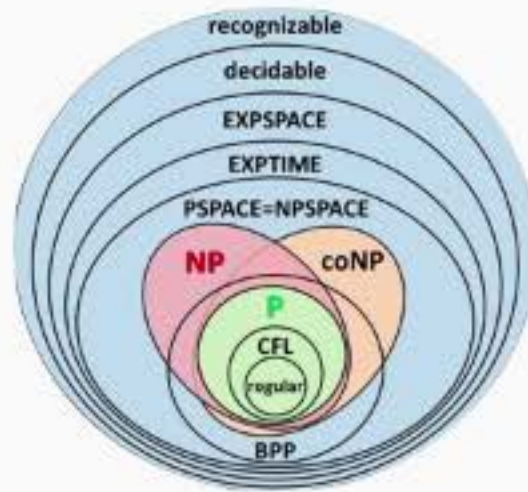
**Guiding Question:** *How can we ground computational problem solving to observation and action, rather than symbols?*

# Future Agenda



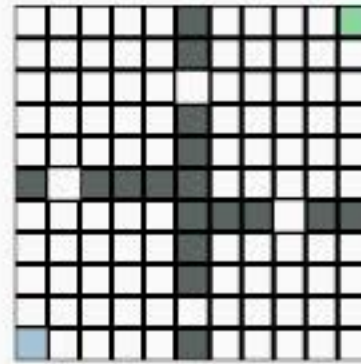
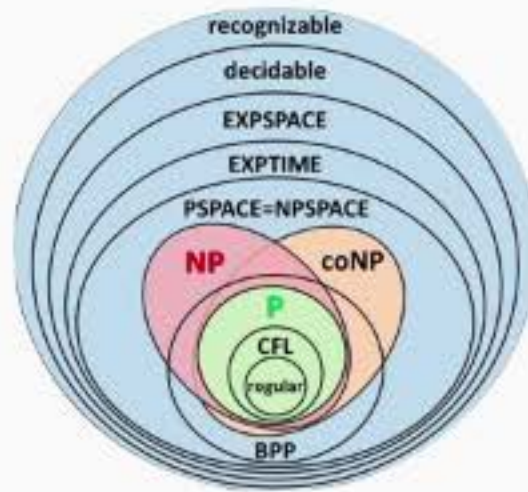
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# Future Agenda



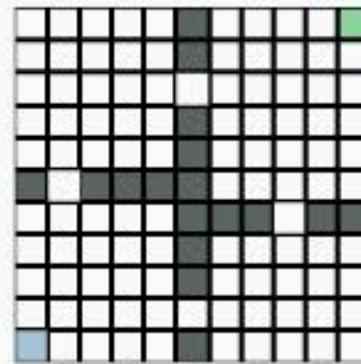
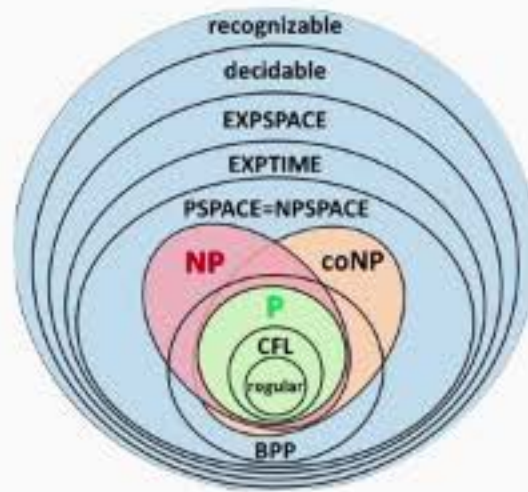
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# Future Agenda



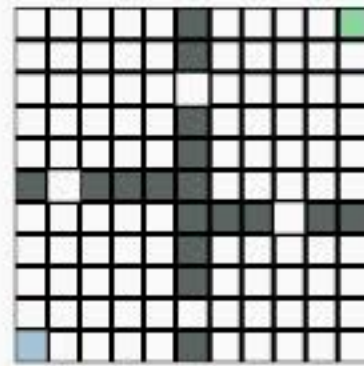
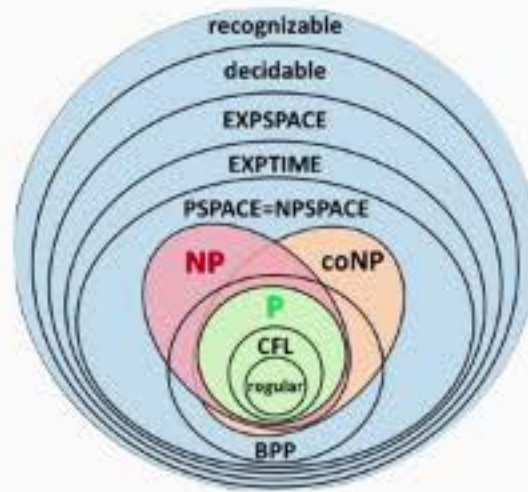
**Guiding Question:** *How can we ground computational problem solving to observation and action, rather than symbols?*

# Future Agenda



**Guiding Question:** *How can we ground computational problem solving to observation and action, rather than symbols?*

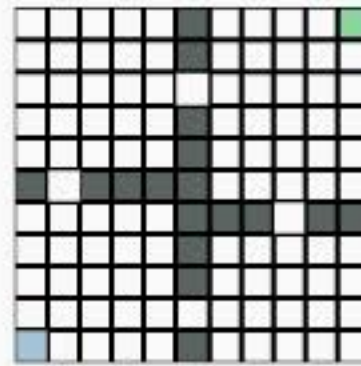
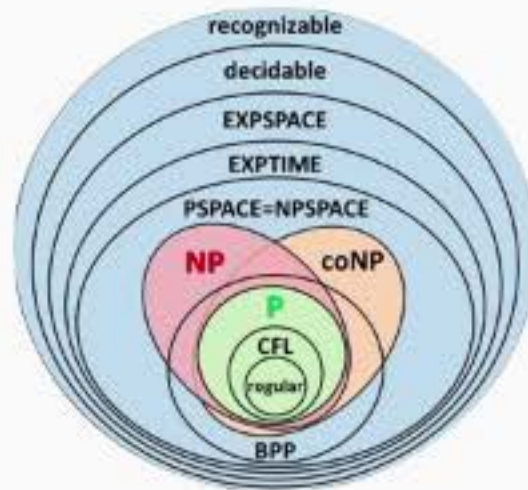
# Future Agenda



RL difficulty

**Guiding Question:** *How can we ground computational problem solving to observation and action, rather than symbols?*

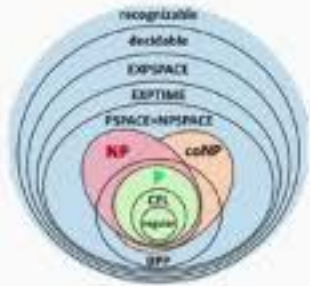
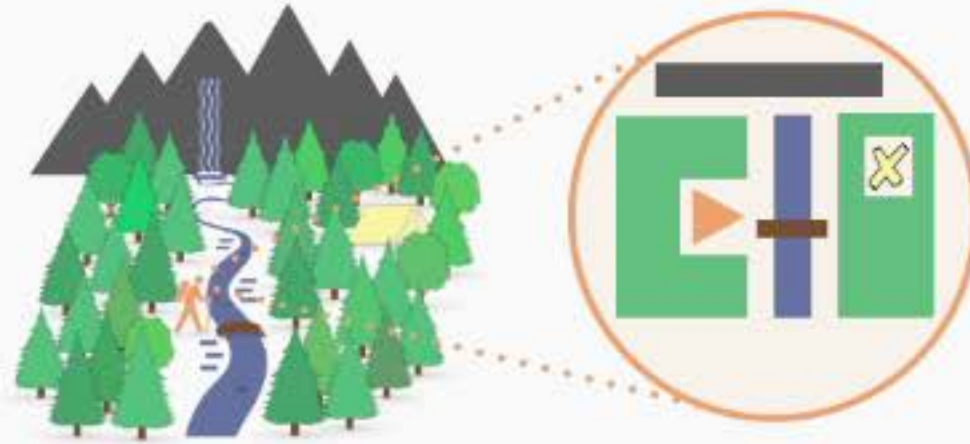
# Future Agenda



**Guiding Question:** *How can we ground computational problem solving to observation and action, rather than symbols?*



# Future Agenda



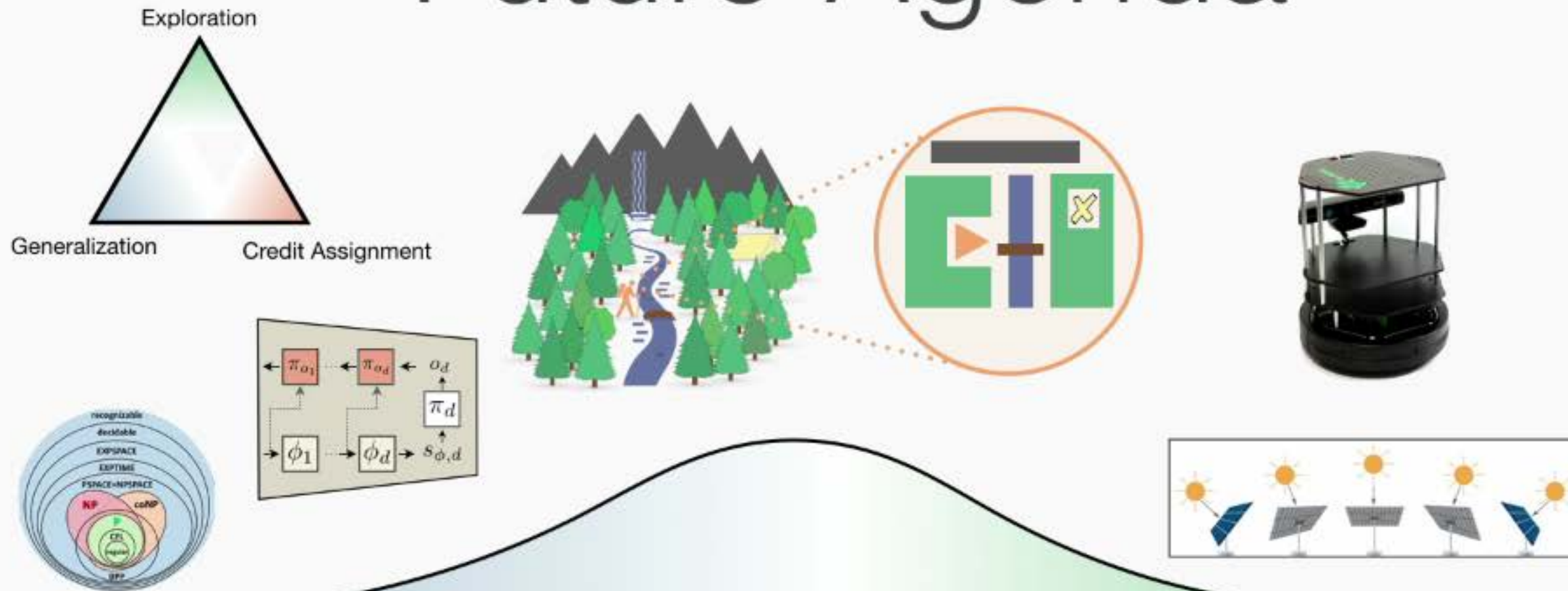
**Guiding Question:** *How can we ground computational problem solving to observation and action, rather than symbols?*

# Future Agenda



**Guiding Question:** How can we ground computational problem solving to observation and action, rather than symbols?

# Future Agenda



**Guiding Question:** How can we ground computational problem solving to observation and action, rather than symbols?

# Thanks to Advisors!

*Masters*



Joshua  
Schechter



Stefanie  
Tellex

*PhD*



**Michael L.  
Littman**

*Undergrad*



David  
Liben-Nowell



Ana  
Moltchanova



George  
Konidaris



Peter  
Stone



Will  
Dabney



Fernando  
Diaz



Owain  
Evans

*Committee*

*Internships*

# Thanks to Collaborators!



Alekh  
Agarwal



Dilip  
Arumugam



Kavosh  
Asadi



Gabriel  
Barth-Maron



Stephen  
Brawner



Marie  
desJardins



Tom  
Griffiths



Yue  
Guo



D. Ellis  
Hershkowitz



Mark  
Ho



Yuu  
Jinnai



Khimya  
Khetarpal



Akshay  
Krishnamurthy



Lucas  
Lehnert



James  
MacGlashan



Doina  
Precup



Daniel  
Reichman



Emily  
Reif



John  
Salvatier



Robert  
Schapire



Andreas  
Stuhlmüller



Nathan  
Umbanhower



Edward  
Williams

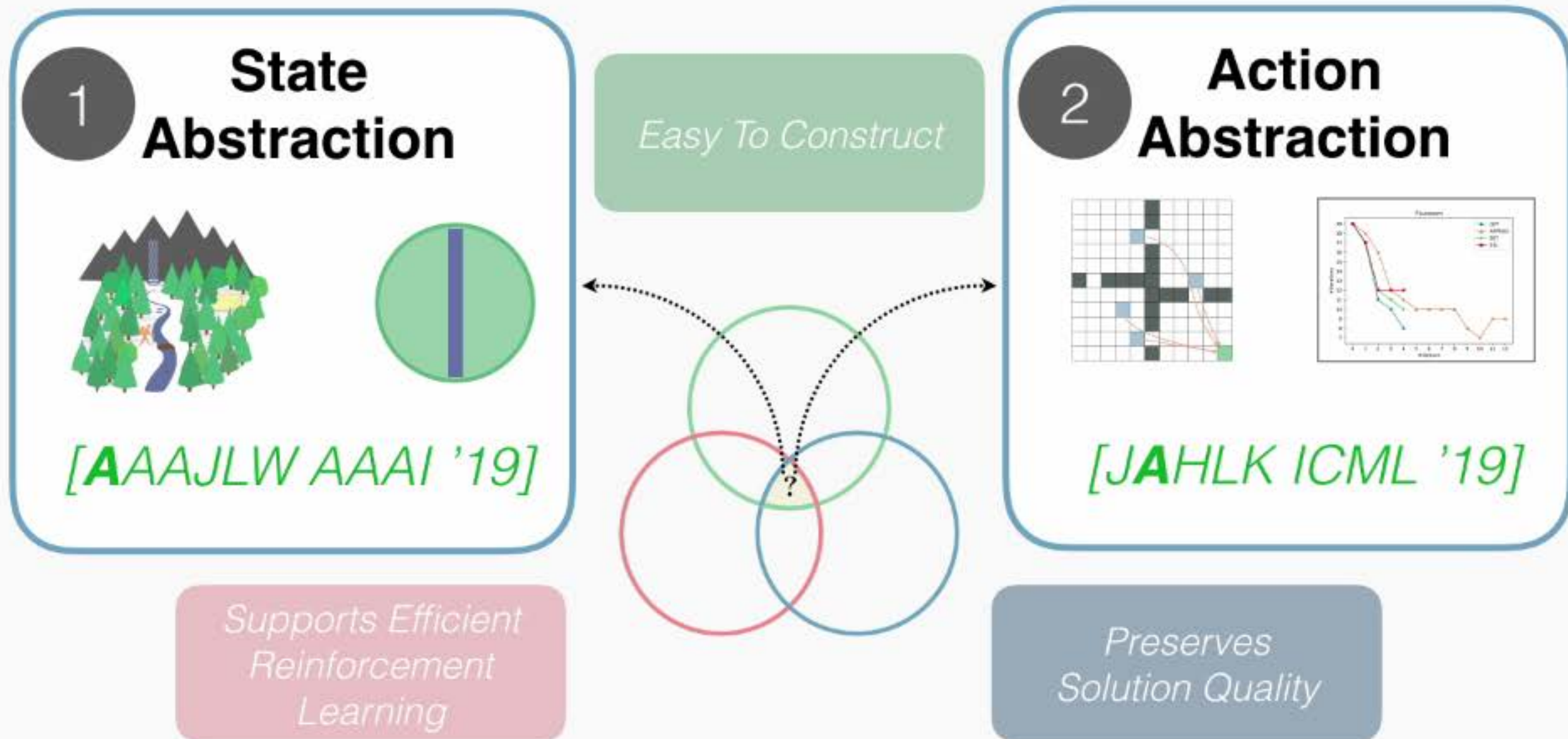


John  
Winder



Lawson  
Wong

# Summary



**Contact:** [david\\_abel@brown.edu](mailto:david_abel@brown.edu)

**Code:** [github.com/david\\_abel/simple\\_rl](https://github.com/david_abel/simple_rl)