

Dynamic Redeployment to Counter Congestion/ Starvation in Vehicle Sharing Systems

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Motivation: Bike Sharing Systems

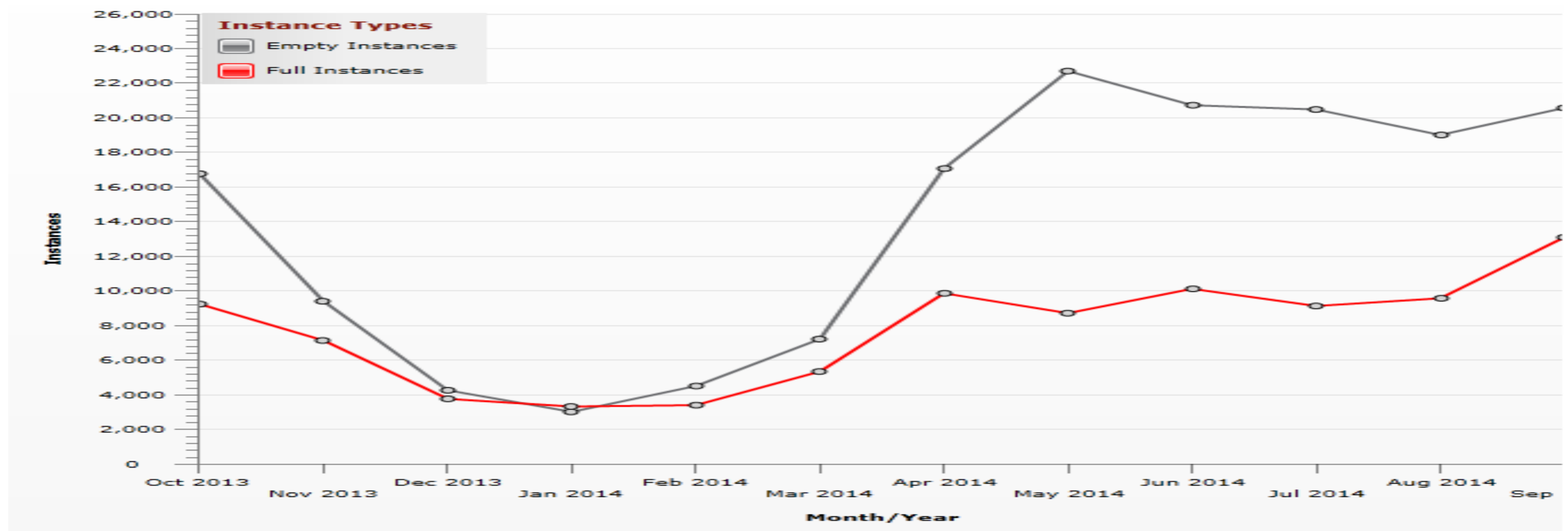
- Examples
 - Bike Sharing (Capital Bikeshare, Hubway, etc.): 747 active systems



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- Examples
 - Bike Sharing (Capital Bikeshare, Hubway, etc.): 747 active systems
- **Alternative transportation to reduce carbon emissions and traffic congestion**

Motivation: Bike Sharing Systems



- Problem: Lost demand because of insufficient vehicles at right places/times
 - Increased use of private transportation and hence carbon emissions
 - Reduced revenue

Related Work

- Static Redeployment (once at the end of day)
 - *Raviv and Kolka (2013), Raviv et al. (2013), Raidl et al. (2013)*
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- Dynamic Redeployment (matching of producer and consumer station)
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- **Our Approach:**
 - MILP to jointly consider dynamic routing and redeployment problem [**DRRP**]
 - Lagrangian dual decomposition to improve the scalability.
 - Abstraction mechanism by grouping the nearby base stations to reduce the decision problems.

Challenge

- Input: A **DRRP** is compactly defined using following tuple

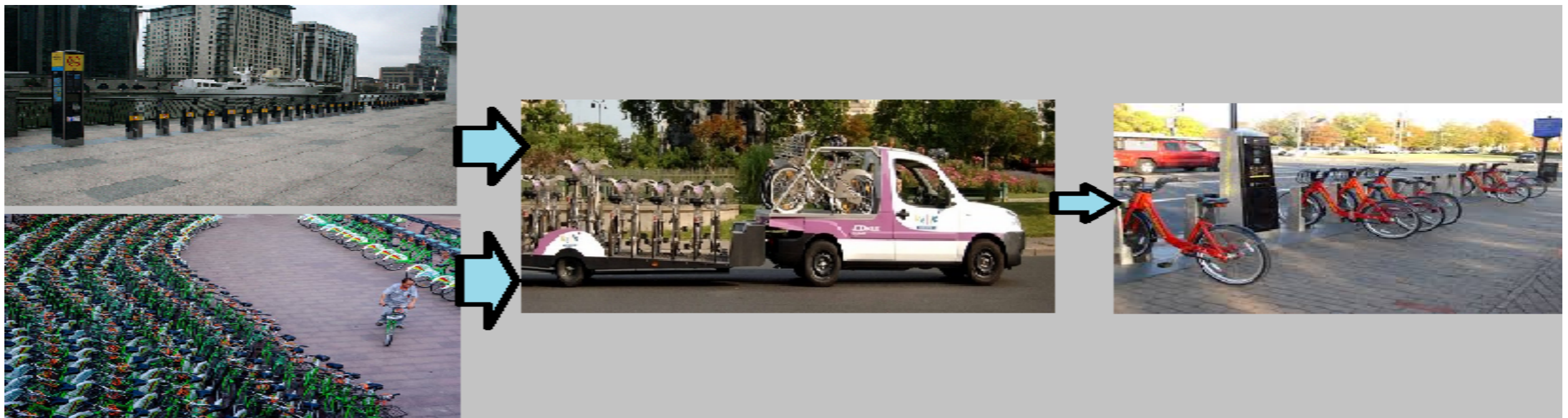
$$\langle \mathcal{S}, \mathcal{V}, \mathbf{C}^\#, \mathbf{C}^*, \mathbf{d}^{\#,0}, \mathbf{d}^{*,0}, \{\sigma_v^0\}, \mathbf{F}, \mathbf{R}, \mathbf{P} \rangle$$

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- Outputs:
 - Number of vehicles to be redeployed, \mathbf{y}
 - Routes for carriers, \mathbf{z} to make redeployments

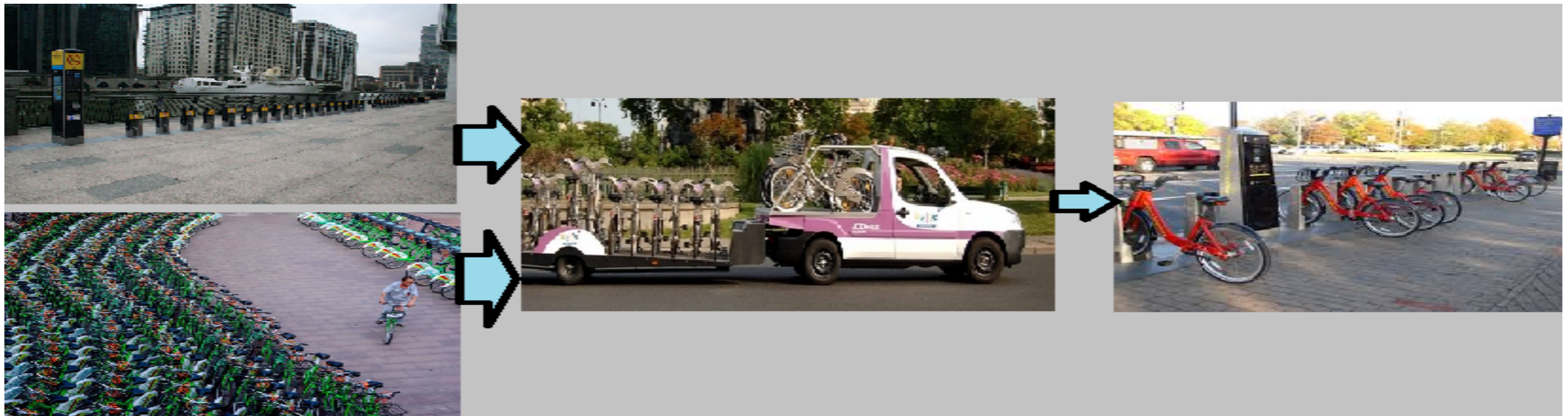


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- Outputs:
 - Number of vehicles to be redeployed, \mathbf{y}
 - Routes for carriers, \mathbf{z} to make redeployments
- **Objective: Maximize revenue (increasing satisfied demand + reducing carrier fuel costs)**



Approach: Linear Optimization

$$\min_{\mathbf{y}^+, \mathbf{y}^-, \mathbf{z}} - \sum_{t, k, s, s'} R_{s, s'}^{t, k} \cdot x_{s, s'}^{t, k} + \sum_{t, v, s, s'} P_{s, s'} \cdot z_{s, s', v}^t$$

Maximize revenue

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Maximize revenue

$$d_s^{\#, t} + \underbrace{\sum_{k, \hat{s}} x_{\hat{s}, s}^{t-k, k}}_{\text{Bikes inflow}} - \sum_{k, s'} x_{s, s'}^{t, k} + \underbrace{\sum_v (y_{s, v}^{-, t} - y_{s, v}^{+, t})}_{\text{Redeployed bikes}} = d_s^{\#, t+1}, \quad \forall t, s$$

Flow preservation of bikes at stations

Approach: Linear Optimization

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$$x_{s, s'}^{t, k} \leq d_s^{\#, t} \cdot \frac{F_{s, s'}^{t, k}}{\sum_{k, \hat{s}} F_{s, \hat{s}}^{t, k}}, \quad \forall t, k, s, s'$$

Actual flow \propto Observed Flow

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Flow preservation of vehicles in carriers

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Flow preservation of vehicles in carriers

$$\sum_{k \in S} z_{s, k, v}^t - \sum_{k \in S} z_{k, s, v}^{t-1} = \sigma_v^t(s), \quad \forall t, s, v$$

$$\sum_{j \in S, v \in \mathcal{V}} z_{s, j, v}^t \leq 1, \quad \forall t, s$$

Enforcing right movement of carriers between stations

Approach: Linear Optimization

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Enforcing right movement of carriers between stations

$$y_{s, v}^{+, t} + y_{s, v}^{-, t} \leq C_v^* \cdot \sum_{i \in S} z_{s, i, v}^t, \quad \forall t, s, v$$

Redeployment should respect the routing strategy

Key Idea 1: Lagrangian Dual Decomposition (LDD)

- Observation:
 - Minimal dependency between y (redeployment) and z (routing variables)

$$\begin{aligned}
 \min_{y,z} \quad & - \sum_{t,k,s,s'} R_{s,s'}^{t,k} \cdot x_{s,s'}^{t,k} + \sum_{t,v,s,s'} P_{s,s'} \cdot z_{s,s',v}^t \\
 \text{s.t.} \quad & d_s^{\#,t} + \sum_{k,\hat{s}} x_{\hat{s},s}^{t-k,k} - \sum_{k,s'} x_{s,s'}^{t,k} + \\
 & \sum_v (\check{y}_{s,v}^t - \hat{y}_{s,v}^t) = d_s^{\#,t+1}, \quad \forall t, s \\
 & x_{s,s'}^{t,k} \leq d_s^{\#,t} \cdot \frac{F_{s,s'}^{t,k}}{\sum_{k,\hat{s}} F_{s,\hat{s}}^{t,k}}, \quad \forall t, k, s, s' \\
 & d_v^{*,t} + \sum_{s \in \mathcal{S}} [(\hat{y}_{s,v}^t - \check{y}_{s,v}^t)] = d_v^{*,t+1}, \quad \forall t, v \\
 & \sum_{k \in \mathcal{S}} z_{s,k,v}^t - \sum_{k \in \mathcal{S}} z_{k,s,v}^{t-1} = \sigma_v^t(s), \quad \forall t, s, v \\
 & \sum_{j \in \mathcal{S}, v \in \mathcal{V}} z_{s,j,v}^t \leq 1, \quad \forall t, s \\
 & \hat{y}_{s,v}^t + \check{y}_{s,v}^t \leq C_v^* \cdot \sum_i z_{s,i,v}^t, \quad \forall t, s, v
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Redeployment

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Routing

$$\hat{y}_{s,v}^t + \check{y}_{s,v}^t \leq C_v^* \cdot \sum_i z_{s,i,v}^t, \quad \forall t, s, v$$

Key Idea 1: Lagrangian Dual Decomposition (LDD)

- Observation:
 - Minimal dependency between y (redemption) and z (routing variables)
- Lagrangian Dual decomposition on joint constraints
- Update price variable in the master function.

$$\min_{y,z} - \sum_{t,k,s,s'} R_{s,s'}^{t,k} \cdot x_{s,s'}^{t,k} + \sum_{t,v,s,s'} P_{s,s'} \cdot z_{s,s',v}^t$$

$$\text{s.t. } d_s^{\#,t} + \sum_{k,\hat{s}} x_{\hat{s},s}^{t-k,k} - \sum_{k,s'} x_{s,s'}^{t,k} + \sum (\tilde{y}_{s,v}^t - \hat{y}_{s,v}^t) = d_s^{\#,t+1}, \quad \forall t, s$$

Redemption

$$x_{s,s'}^{t,k} \leq d_s^{\#,t} \cdot \frac{F_{s,s'}^{t,k}}{\sum_{k,\hat{s}} F_{s,\hat{s}}^{t,k}}, \quad \forall t, k, s, s'$$

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Routing

Redemption + Routing

Key Idea 1: Lagrangian Dual Decomposition (LDD)

- Observation:
 - Minimal dependency between y (redemption) and z (routing variables)
- Lagrangian Dual decomposition on joint constraints
 - Update price variable in the master function.
 - **Primal extraction based on routing feasibility**
 - **Strong upper and lower bounds**

$$\min_{y,z} - \sum_{t,k,s,s'} R_{s,s'}^{t,k} \cdot x_{s,s'}^{t,k} + \sum_{t,v,s,s'} P_{s,s'} \cdot z_{s,s',v}^t$$

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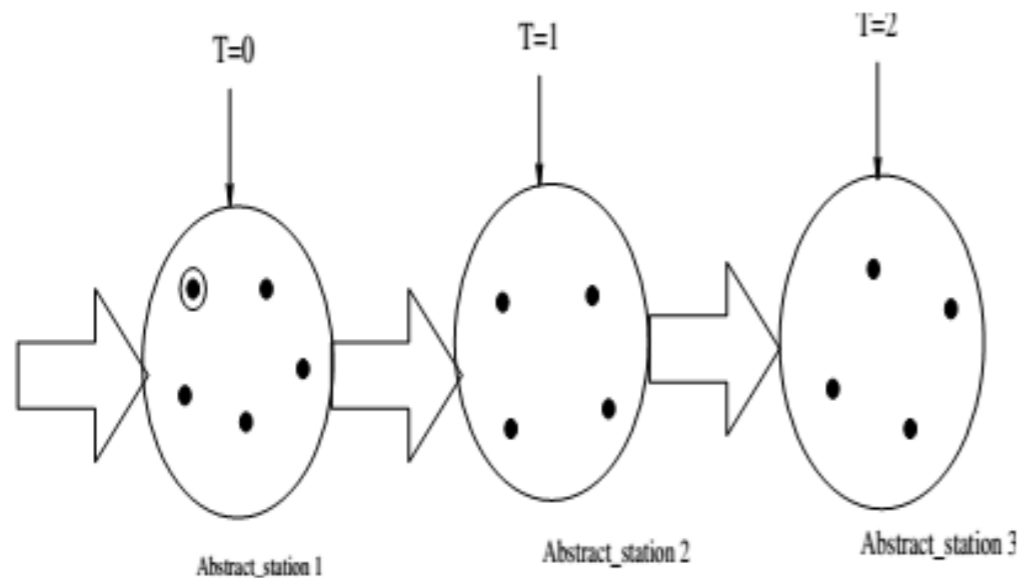
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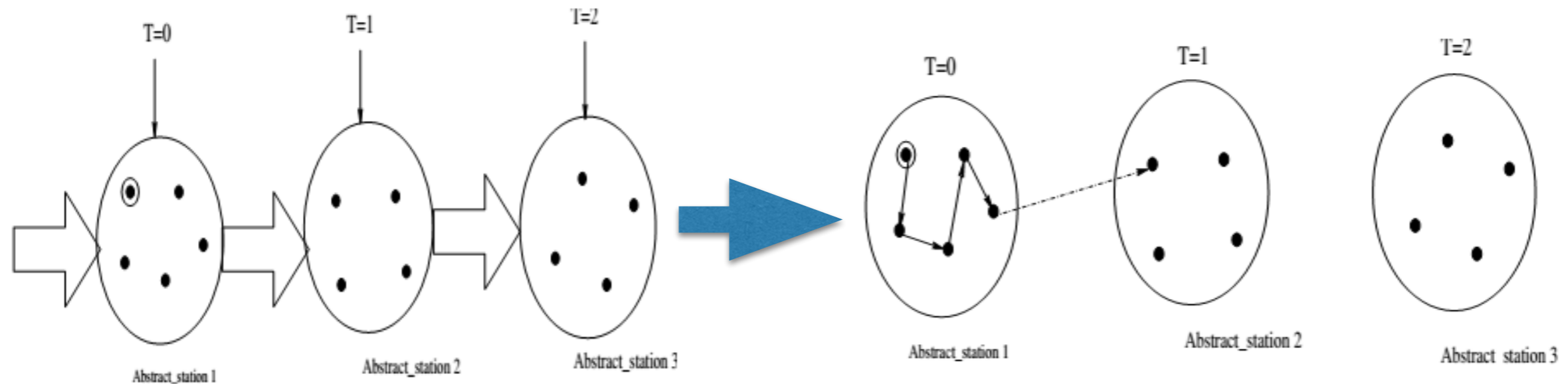
Redemption + Routing

Key Idea 2: Abstraction



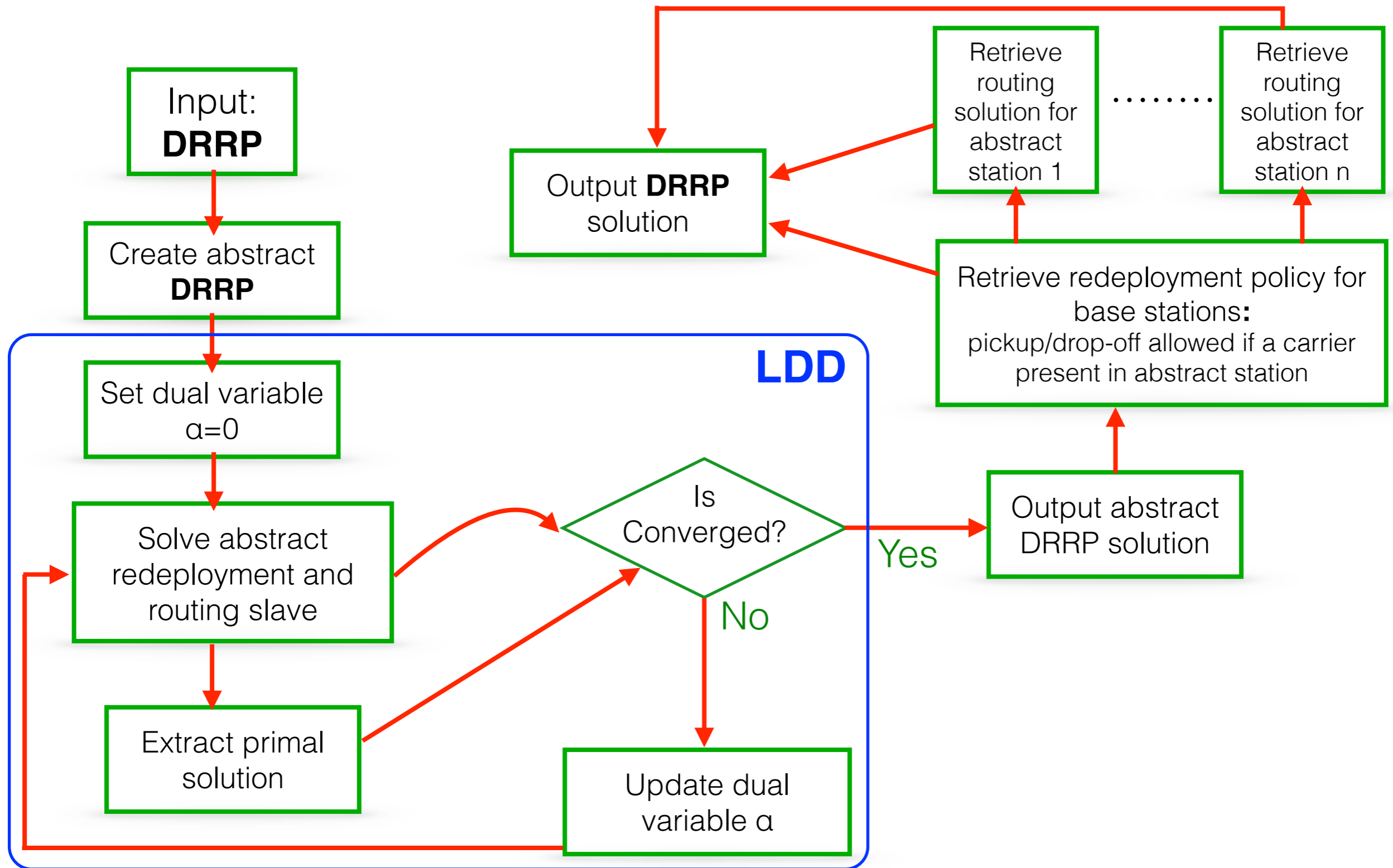
- Grouping of stations
 - Group base stations into abstract stations
 - Solve abstract problem using LDD

Key Idea 2: Abstraction



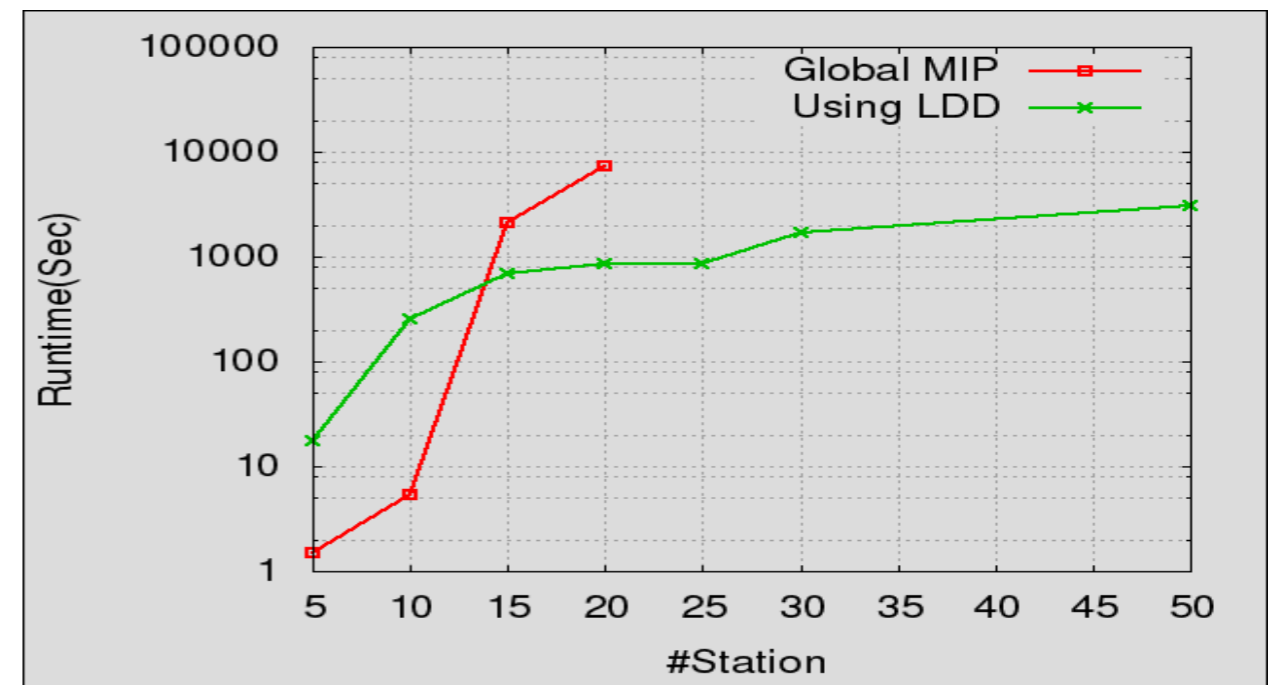
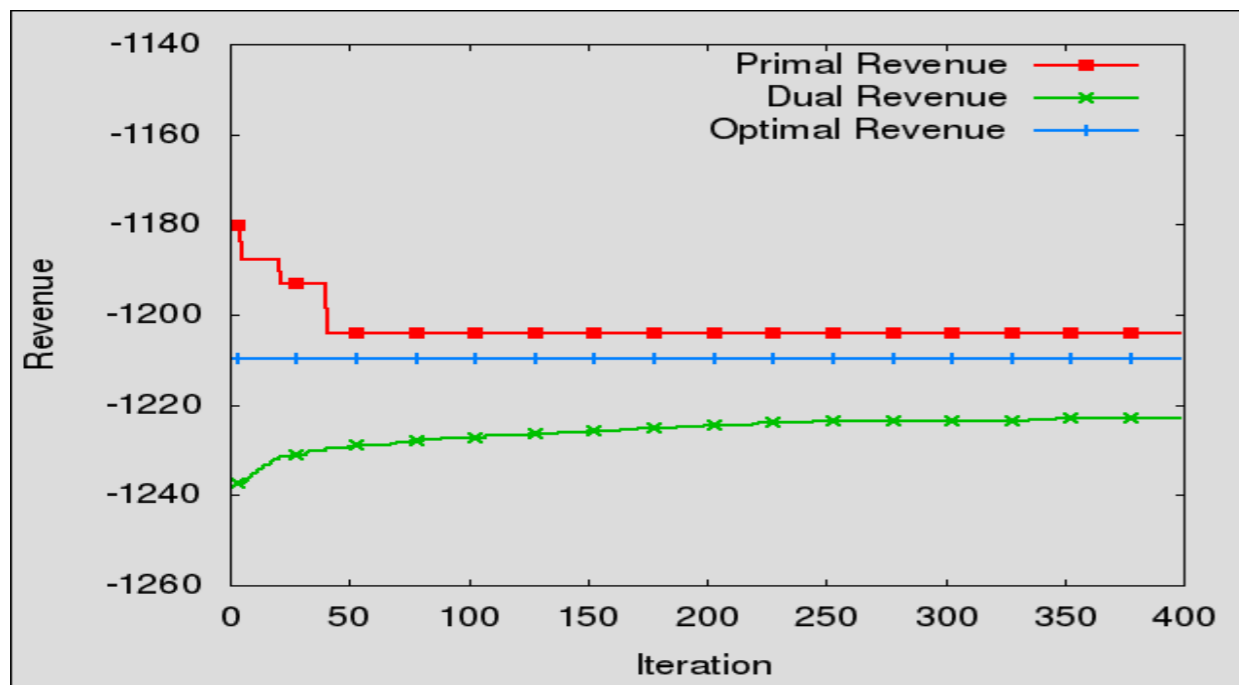
- Grouping of stations
 - Group base stations into abstract stations
 - Solve abstract problem using LDD
- Retrieve redeployment and routing strategy from solution to the abstract problem
 - Involves solving an optimization problem

LDD+Abstraction



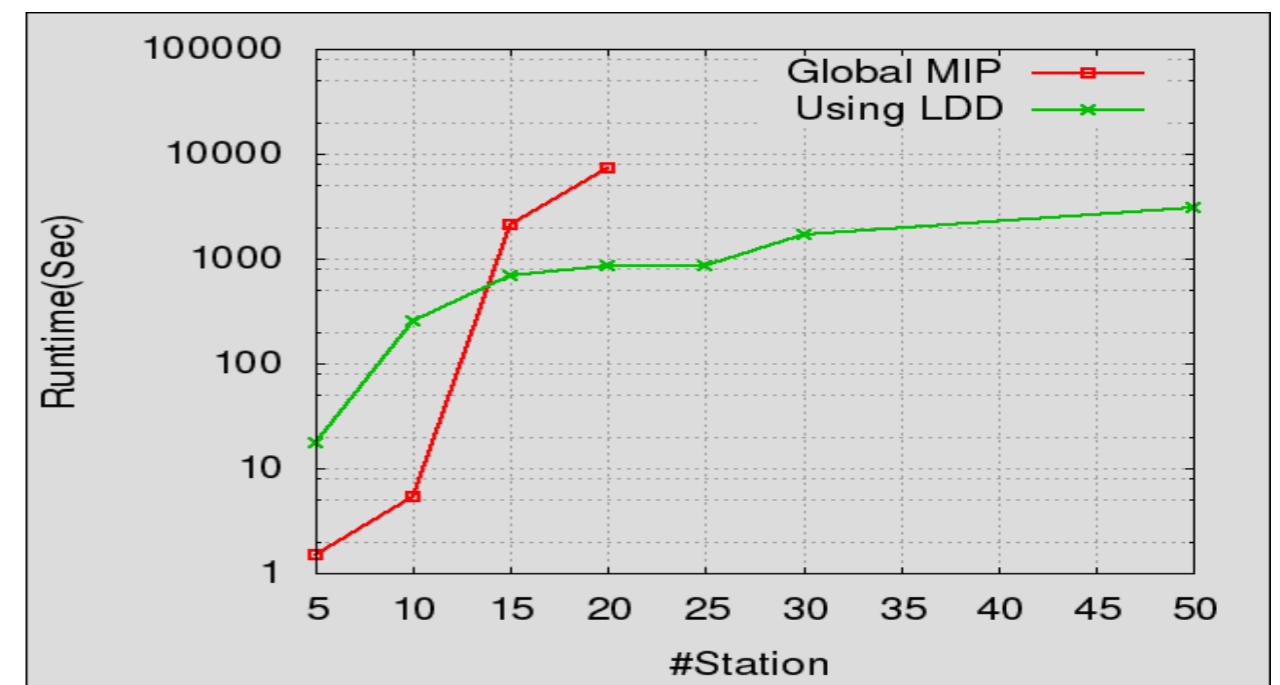
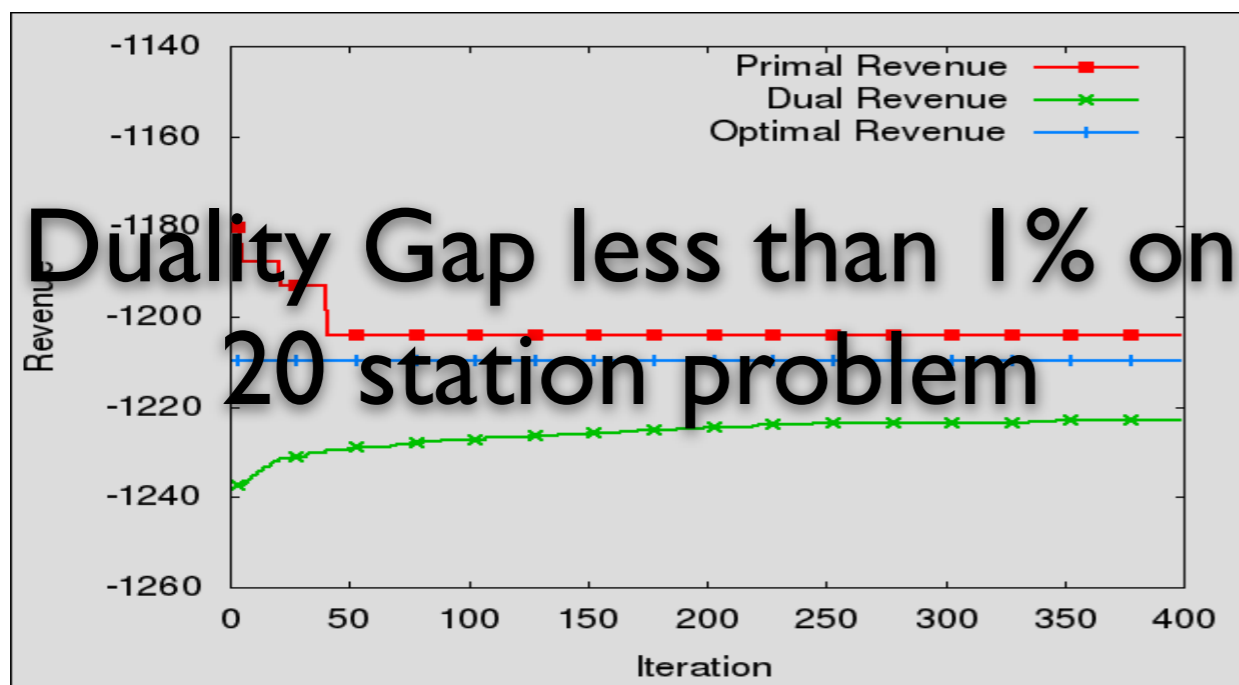
Experimental Results

- One synthetic data set and two real data sets:
 - Capital Bikeshare (305 stations, 6 carriers)
 - Hubway (95 stations, 4 carriers)
- Strategy of redeployment and routing for the entire day (30 minute decisions)
- Obtain strategy from part of the datasets and execute on another part



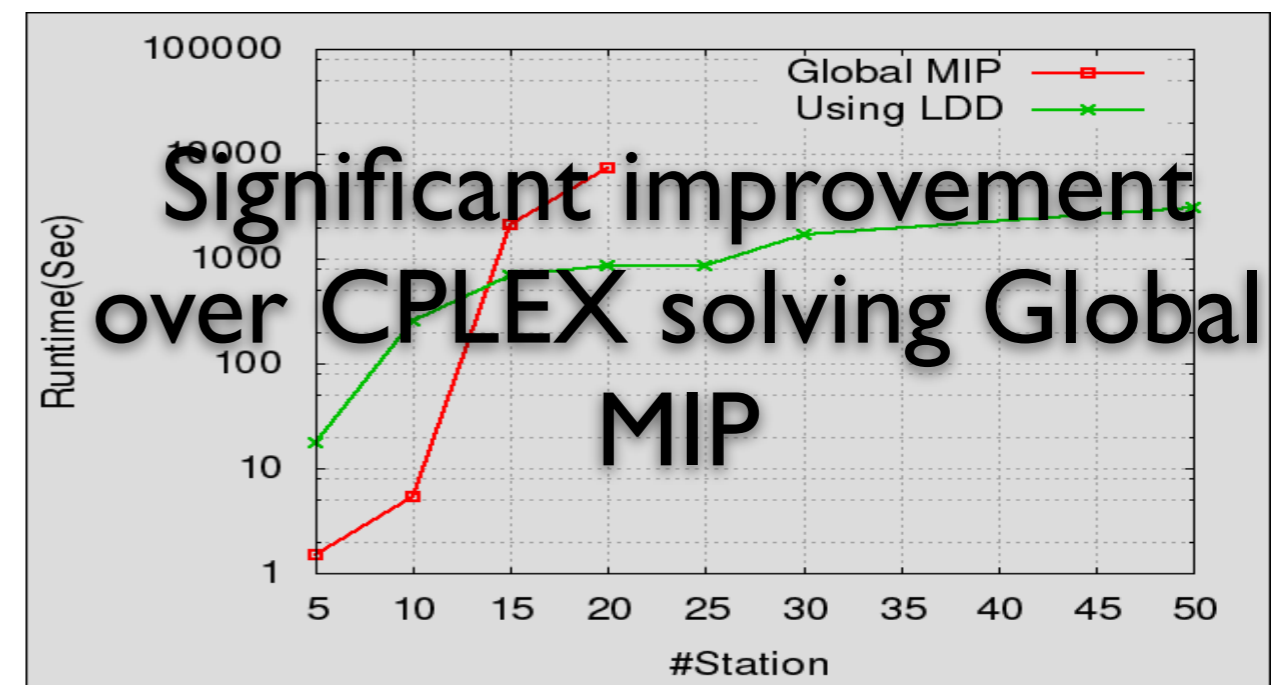
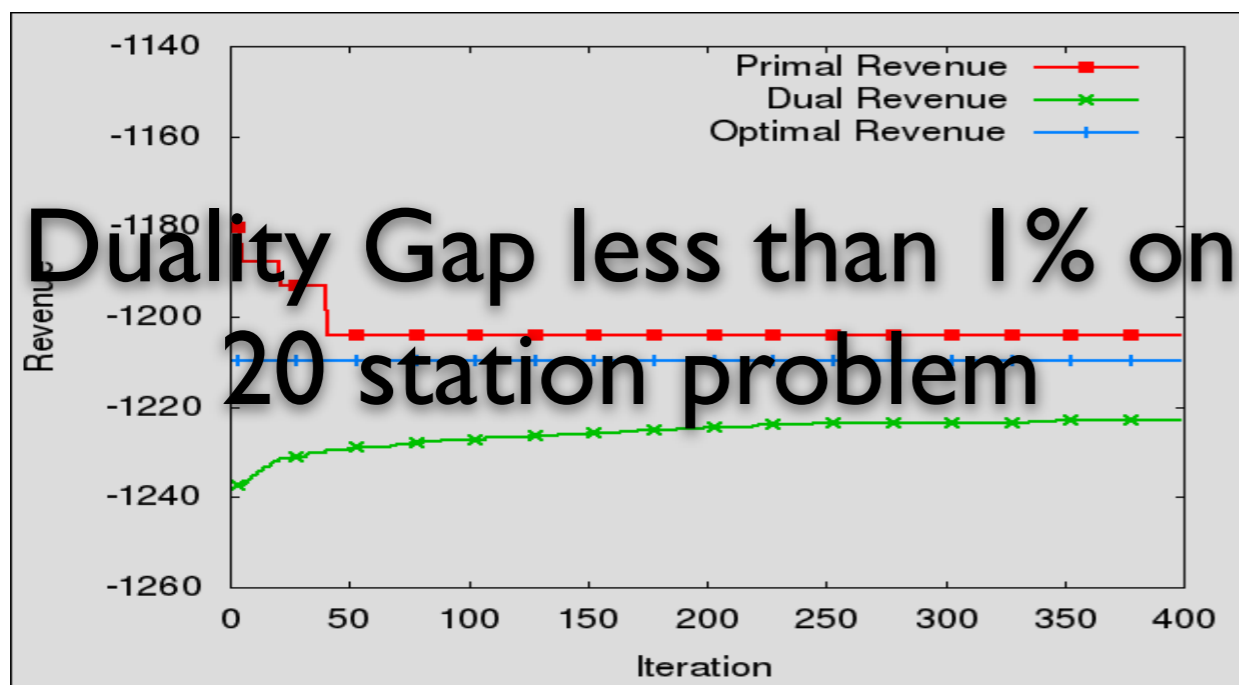
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Experimental Results on Real Datasets

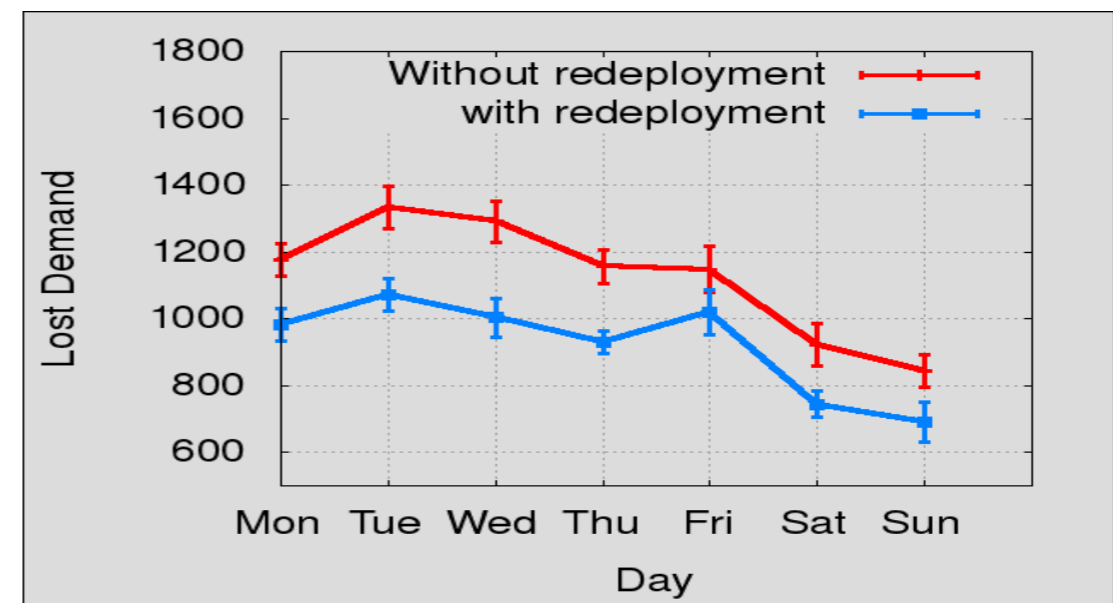
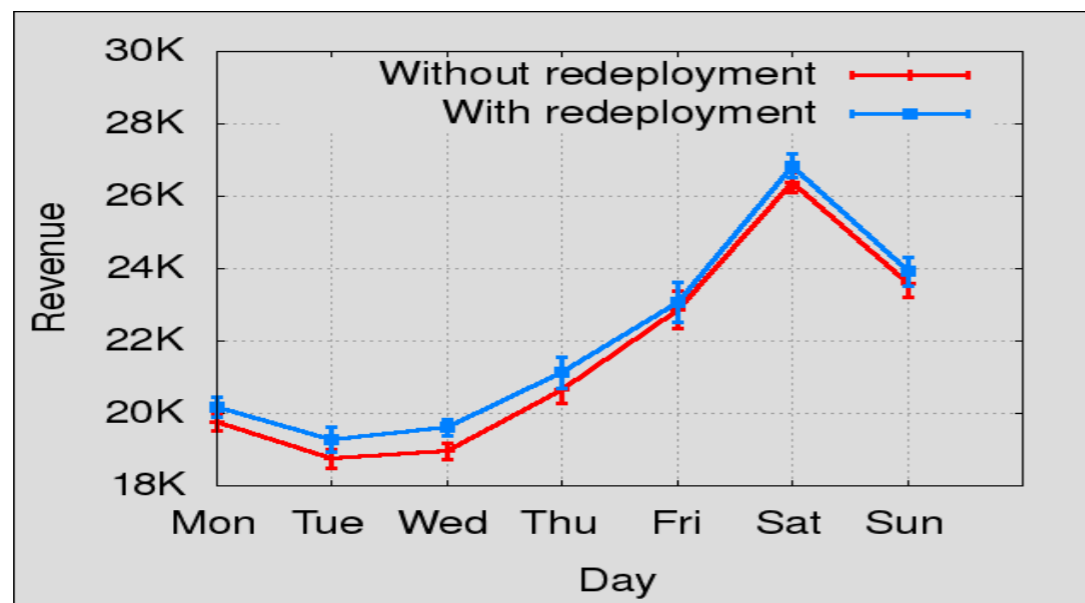
- Comparison with current practice (abstraction + LDD)
 - Demand follows poisson with mean observed flow
 - CapitalBikeshare Data:
 - Revenue increased by 3%
 - Lost demand reduced by up to 33.76%

	Whole day (5am-12am)		Peak period (5am-12pm)	
	Revenue gain	Lost demand reduction	Revenue gain	Lost demand reduction
Mean	3.47 %	22.72 %	7.74 %	30.58 %
Mon	2.33 %	22.46 %	4.48 %	25.55 %
Tue	3.07 %	28.56 %	7.86 %	37.10 %
Wed	3.30 %	31.16 %	8.95 %	44.88 %
Thu	2.86 %	33.76 %	6.04 %	35.97 %
Fri	2.51 %	27.37 %	4.50 %	28.15 %
Sat	3.87 %	23.61 %	4.33 %	24.30 %
Sun	3.01 %	26.00 %	4.04 %	36.51 %

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- Comparison with current practice (abstraction + LDD)
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 - Robust to small changes in mean demand**

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Experimental Results on Real Datasets (2)

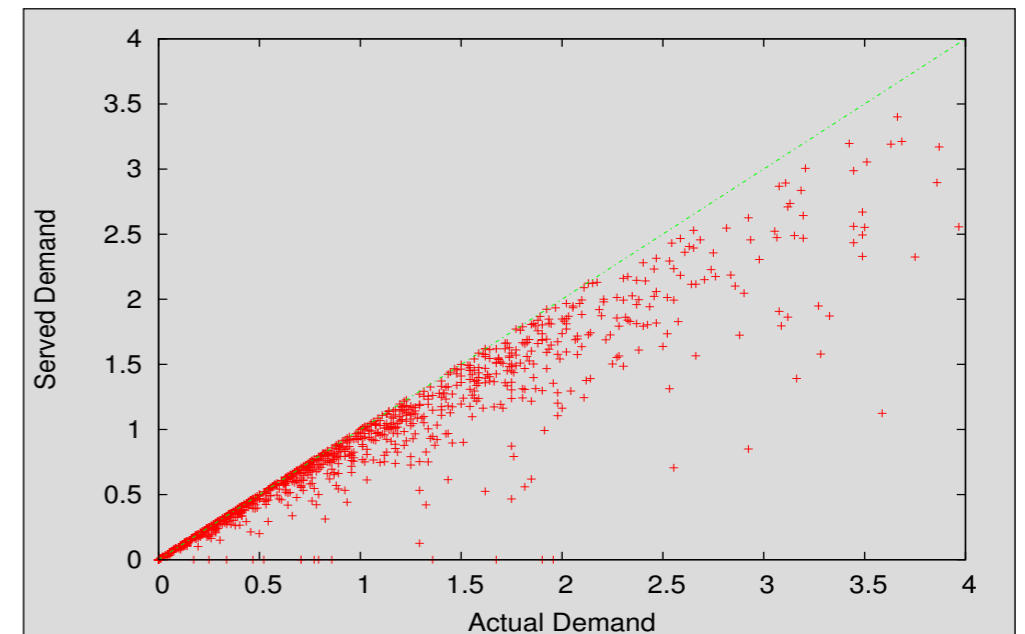
- Hubway Data:
 - Revenue increased by 5%
 - Lost demand reduced by 60% on average

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Revenue Gain (%)	3.94	5.93	4.45	5.90	6.27	2.20	3.15
Lost Demand Reduction(%)	42.6	60.7	58.5	54.7	77.2	69.8	74.0

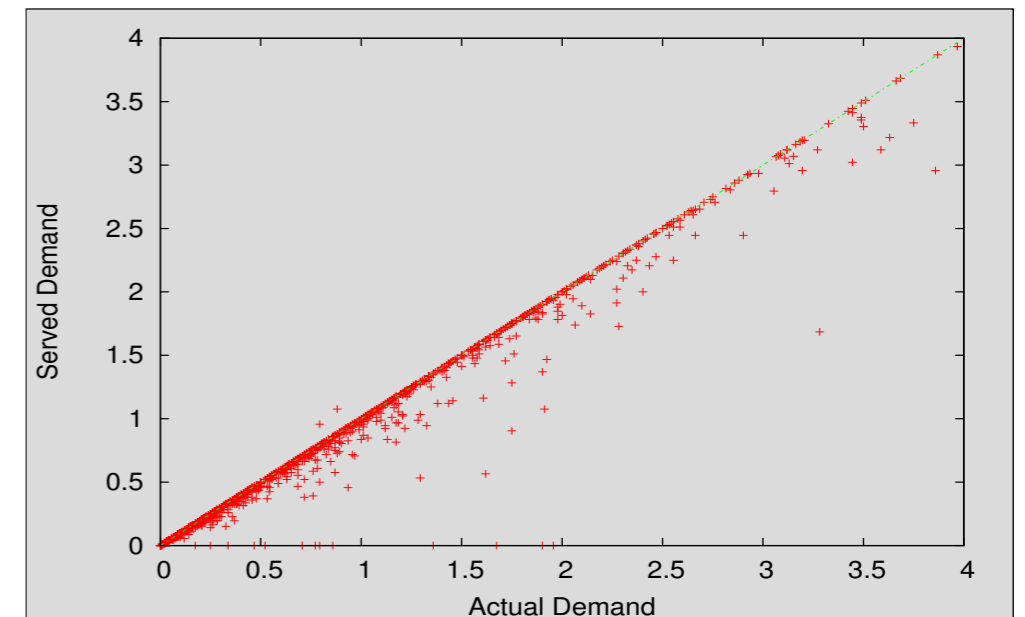
Experimental Results on Real Datasets (2)

- Hubway Data:
 - Revenue increased by 5%
 - Lost demand reduced by 60% on average
- Better matching of demand and supply
 - Ideally all the points should lie on the identity line

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Revenue Gain (%)	3.94	5.93	4.45	5.90	6.27	2.20	3.15
Lost Demand Reduction(%)	42.6	60.7	58.5	54.7	77.2	69.8	74.0



Matching without redeployment



Matching using our redeployment

Summary

- Dynamic redeployment of bikes
- Important large-scale problem with relevance to many cities
- Two techniques (Decomposition, Abstraction) to improve scalability and provide near-optimal solutions
- Reduces lost demand by over 20% on both datasets
- Robust to small changes in demand

Questions???



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Thank you!



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