# Reconstruction of Complex Dynamical Networks

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## Background

*A group of blind men touch an elephant to learn what it is like. They conclude that the elephant is like a wall, snake, spear, tree, fan or rope, depending upon where they touch. They have a heated debate and the conflict is never resolved.*

## Aims

- What is the topology of the network?
- What are the directions and strengths of coupling links between nodes?
- What kind of linear/nonlinear functions govern the dynamics of each node?
- What are the parameters of these functions?

## Model Transformation

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{n} w_{ij} g_{ij}(x_j(t), x_i(t)) + \eta_i(t)$$

$$\dot{x}_{ik}(t) = \sum_{l=1}^{N_{f_i}} w_{f_i}^{(l)}(k) f^{(l)}(x_i(t)) + \sum_{j=1}^{n}\sum_{r=1}^{N_{g_{ij}}} w_{g_{ij}}^{(r)}(k,:) g^{(r)}(x_j(t), x_i(t)) + \eta_{ik}(t)$$

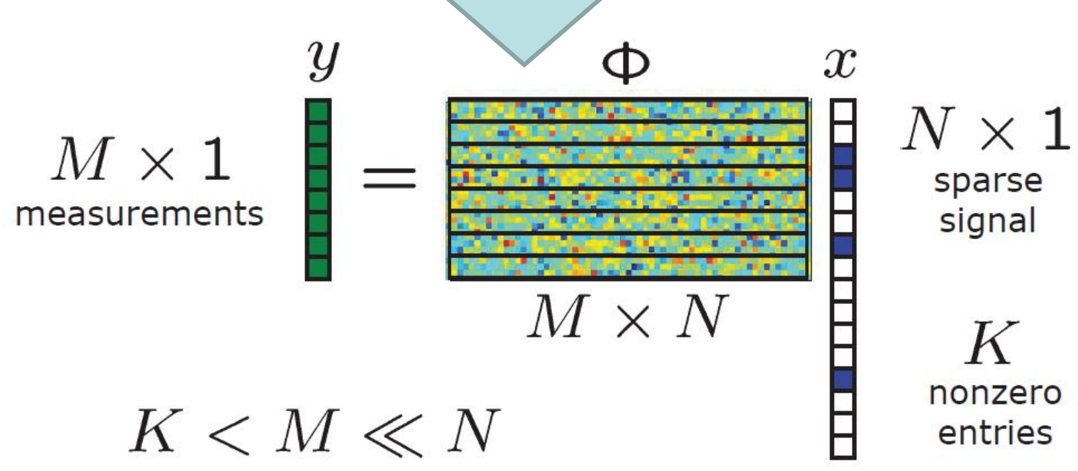
$$e_{ik}(t_{T+1}) = F(x_i(t_T)) W_{f_{ik}} + G_i(x(t_T)) W_{g_{ik}} + \xi_{ik}(t_T)$$

$$\mathbf{y}_{ik} \triangleq [e_{ik}(t_1), \ldots, e_{ik}(t_M)]^T \in \mathbb{R}^{M\times 1}$$

$$\Phi_{ik} \triangleq \begin{bmatrix} F(x_i(t_0)) & G_i(x(t_0)) \\ F(x_i(t_1)) & G_i(x(t_1)) \\ \vdots & \vdots \\ F(x_i(t_{M-1})) & G_i(x(t_{M-1})) \end{bmatrix} \in \mathbb{R}^{M\times N_i}$$

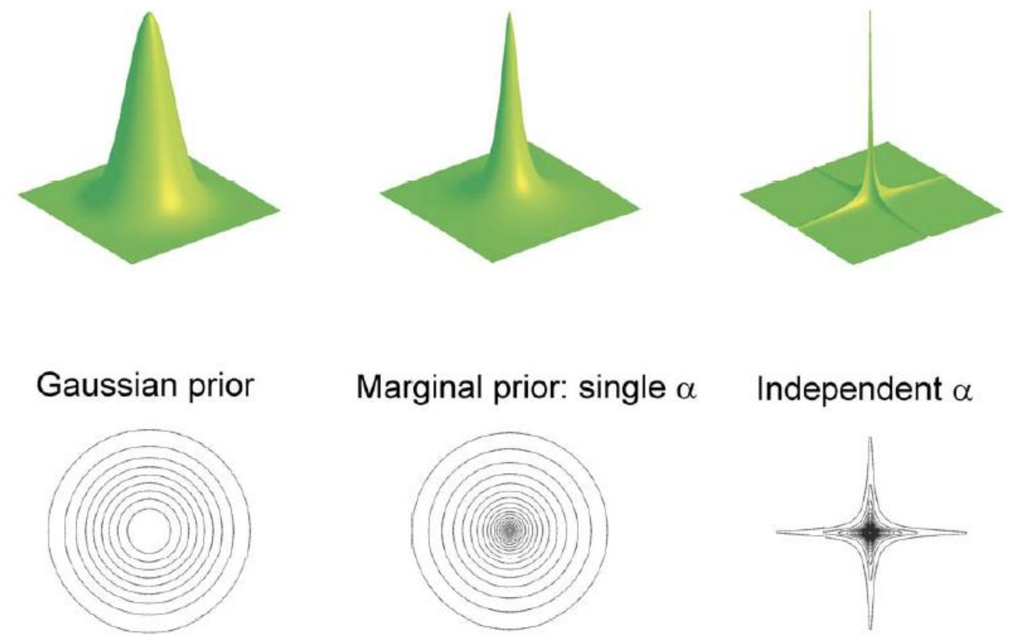
$$\mathbf{x}_{ik} \triangleq [W_{f_{ik}}, W_{g_{ik}}] \in \mathbb{R}^{N_i \times 1}$$

$$\Xi_{ik} \triangleq [\xi_{ik}(t_1), \ldots, \xi_{ik}(t_{M-1})]^T \in \mathbb{R}^{M\times 1}$$



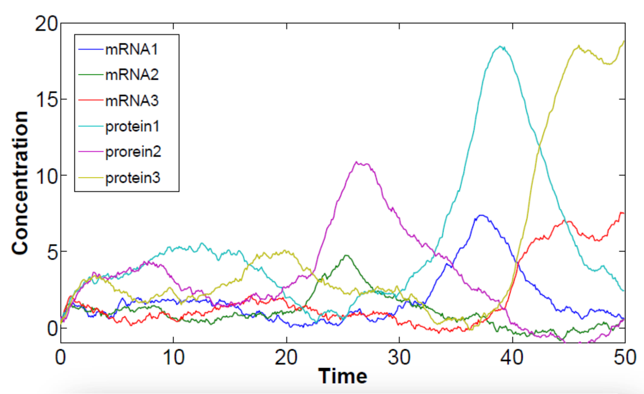

## Method

$$p(\mathbf{w}_i|\alpha_i) = \prod_{j=1}^{N} \mathcal{N}\left(\mathbf{w}_i(j)|0, \alpha_{ij}^{-1}\right)$$



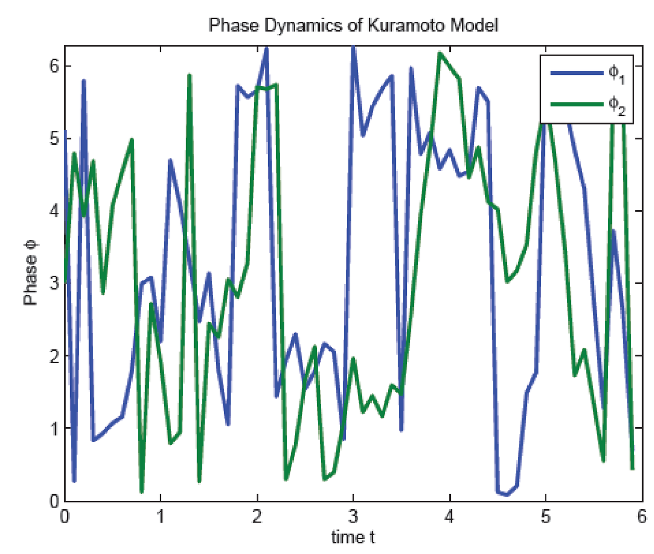

## Results

### Synthetic Genetic Repressilator

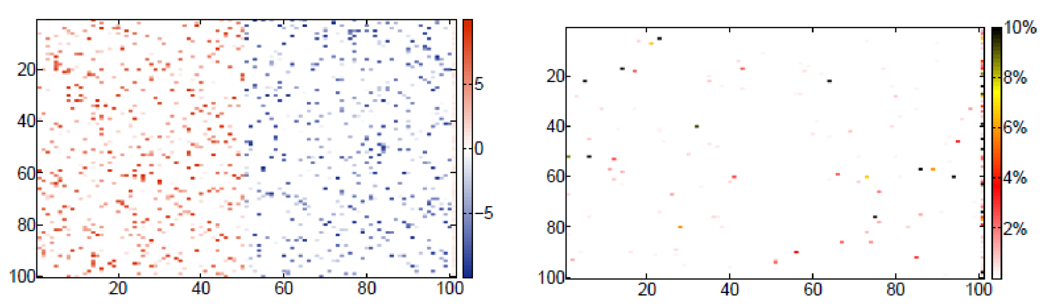



$$\frac{dx_1}{dt} = -\gamma_1 x_1 + \frac{\alpha_1}{1+x_6^{n_1}} + \theta_1,$$
$$\frac{dx_2}{dt} = -\gamma_2 x_2 + \frac{\alpha_2}{1+x_4^{n_2}} + \theta_2,$$
$$\frac{dx_3}{dt} = -\gamma_3 x_3 + \frac{\alpha_3}{1+x_5^{n_3}} + \theta_3,$$
$$\frac{dx_4}{dt} = -\gamma_4 x_4 + \beta_1 x_1,$$
$$\frac{dx_5}{dt} = -\gamma_5 x_5 + \beta_2 x_2,$$
$$\frac{dx_6}{dt} = -\gamma_6 x_6 + \beta_3 x_3.$$

### Kuramoto Oscillator



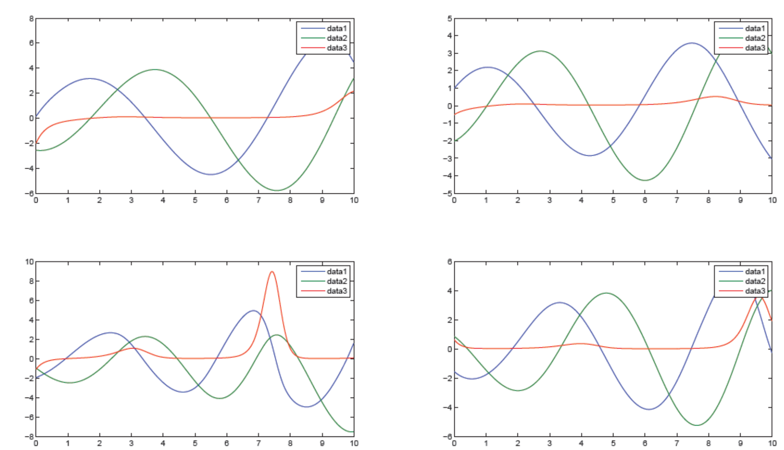

$$\underbrace{\dot{\phi}_i(t)}_{\text{Phase}} = \underbrace{\omega_i}_{\text{Natural Frequency}} + \underbrace{\sum_{j=1}^{n} w_{ij} g_{ij}(\phi_j(t) - \phi_i(t))}_{\text{Coupling Function}} + \underbrace{\eta_i(t)}_{\text{Noise}}$$



(a) True weight matrix (b) NMSE matrix

## More results

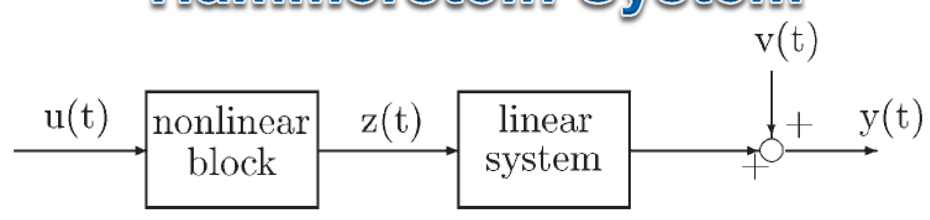
### Rössler Oscillator



$$\dot{p}_i(t) = -\omega_i q_i(t) - s_i(t) + \sum_{j=1}^{n} w_{g_{ij}^p} g_{ij}^p(p_j(t) - p_i(t)) + \eta_i^p(t)$$
$$\dot{q}_i(t) = \omega_i p_i(t) + a_i q_i(t) + \sum_{j=1}^{n} w_{g_{ij}^q} g_{ij}^q(q_j(t) - q_i(t)) + \eta_i^q(t)$$
$$\dot{s}_i(t) = b_i + (p_i(t) - c_i) s_i(t) + \sum_{j=1}^{n} w_{g_{ij}^s} g_{ij}^s(s_j(t) - s_i(t)) + \eta_i^s(t)$$

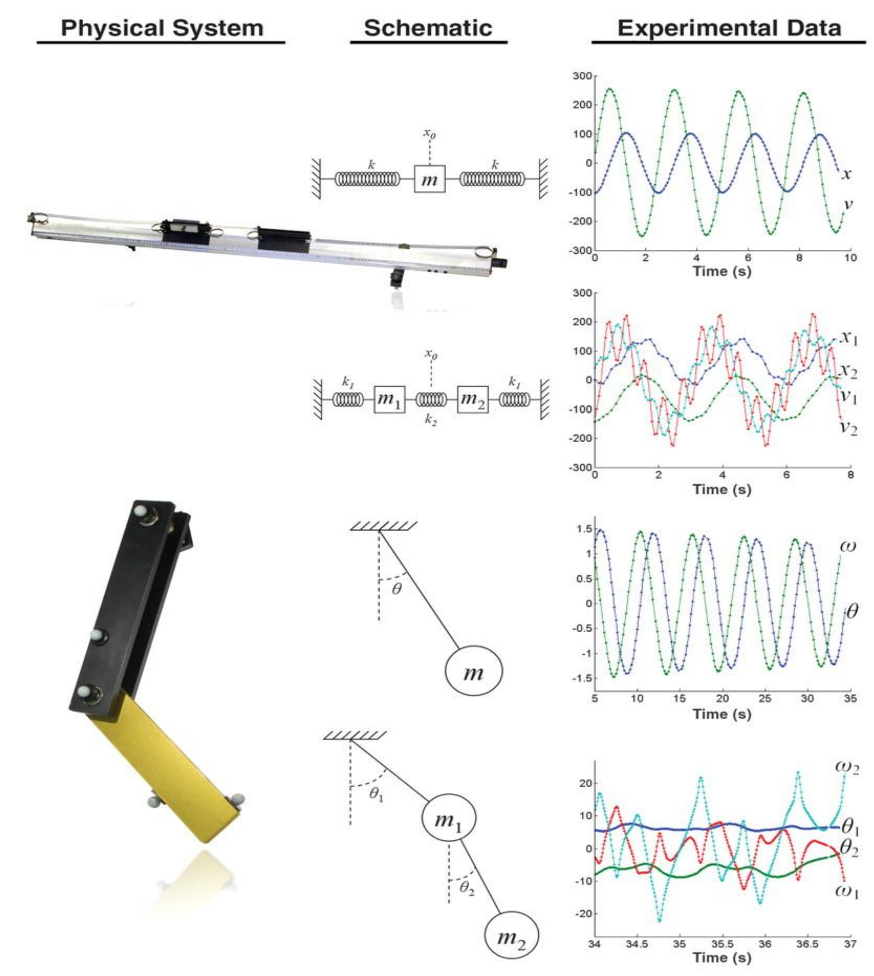
Local States — Local Function — Coupling Function — Noise

### Hammerstein System




$$y(t) = \sum_{j=1}^{n} b_j z(t-j) + v(t)$$
$$z(t) = \sum_{j=1}^{m} c_j g_j[u(t)].$$

### Natural Laws




*Data from "Distilling Free-Form Natural Laws from Experimental Data", Science 324, 81 (2009)*

## Conclusions

- The method requires time series data (noisy) and a series of candidate linear or nonlinear functions.
- Select a minimum number of candidate functions to fit the time series data.
- The network topology, functions with the corresponding parameters can be reconstructed.

### References

1. Pan W et al. *Control and Decision conference 2012, submitted.*

2. Pan W et al. *Physical Review Letters, submitted*.

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