POST-SAMPLING ALIASING CONTROL FOR NATURAL IMAGES

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ABSTRACT

Sampling and reconstruction are usually analyzed under the framework of linear signal processing. Powerful tools like the Fourier transform and optimum linear filter design techniques, allow for a very precise analysis of the process. In particular, an optimum linear filter of any length can be derived under most situations. Many of these tools are not available for non-linear systems, and it is usually difficult to find an optimum non-linear system under any criteria. In this paper we analyze the possibility of using non-linear filtering in the interpolation of subsampled images. We show that a very simple (5x5) non-linear reconstruction filter outperforms (for the images analyzed) linear filters of up to 256x256, including optimum (separable) Wiener filters of any size.

1. INTRODUCTION

In digital signal processing, it is often necessary to alter the sampling rate of a discrete signal. We usually refer to *decimation* (or sub-sampling) as the operation of selecting a subset of the original samples of the signal; i.e., reducing the sampling rate by an integer factor. We refer to *interpolation* as the operation of increasing the sampling rate by an integer factor by estimating the value of intermediate samples. Previous work in this area [1, 2, 3] was based on the Shannon Sampling theorem, which states that the signal should be band-limited (by filtering) before (sub)sampling, and the interpolation should consist of up-sampling followed by filtering. In the most common case, both filters should be low-pass, with cut-off at the Nyquist frequency.

Since optimum linear filters have been derived for most practical situations, recent work has concentrated either on subjective effects of aliasing[4], or on non-linear techniques[5, 6, 7, 8].

In developing non-linear sampling/interpolation systems, the lack of some key tools used in the analysis of linear systems (e.g., Fourier transforms, Shannon theorem) has limited the success of the early work on non-linear filters[7, 8]. However, recent results on Critical Morphological Sampling[5, 6] provide a morphological equivalent of the Shannon sampling theorem, and can be useful in developing better pre- and post-filters based on non-linear techniques.

Critical Morphological Sampling is similar to traditional techniques in the sense that it also requires pre-filtering before subsampling. In some applications, this pre-filtering maybe undesirable, or even impossible, in which case the signal is simply subsampled, without any pre-filtering, or a very simple filter is used. An example of increasing importance is video processing, where the high data rate and memory restrictions often limit the filtering to very short windows.

In this paper we show how it is possible to reduce the effects of aliasing by using non-linear **reconstruction** techniques. We analyze a specific reconstruction technique which uses a 5x5 reconstruction filter. We compare the results of the technique with those obtained by using FIR reconstruction filters with and without pre-filtering. The proposed technique outperforms all linear reconstruction techniques when not using a pre-filter. Even when a pre-filter is used, the traditional (linear) technique requires a much higher computation effort to provide equivalent performance.

Section 2 formally defines the problem, and presents some optimum linear solutions. Section 3 presents the proposed non-linear interpolation filter, Section 4 gives the results of simulations on some test images, and Section 5 presents some insight into what is "wrong" with linear interpolators. Section 6 presents some conclusions and further research directions.

2. THE PROBLEM AND LINEAR SOLUTIONS

The problem is that of interpolating an image that has been downsampled without an anti-aliasing filter (see Figure 1). Note that we consider only the case of downsampling by 2:1. We want to compare the performance of several filters under m.s.e. and m.a.e. criteria.

If no information about the signal is available, an ideal low-pass filter is generally used as a prototype, and an FIR filter with linear phase is designed to approximate the prototype under some optimality criterion[9]. For example, square window filter design minimizes the mean-squared difference between the filter frequency response and that of the prototype, and Parks-McClellan (equiripple) design

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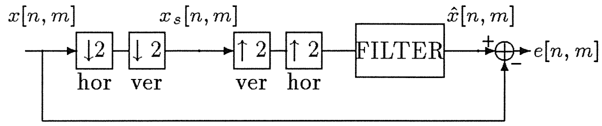

Figure 1: General block diagram of the system

minimizes the maximum approximation error. If information about the spectrum of the (original) signal is available, then better filters can be derived. In [2], Oetken, et. al. derive the FIR filter that minimizes the m.s.e. of the estimate. Their results could be used to derive optimum separable linear filters for our problem. The ideas could also be extended to the design of non-separable filters, but this would require dealing with a 4-D autocorrelation matrix. Instead, we note that these filters, based on the knowledge of the original signal spectrum, are in fact just Wiener filters, where the aliasing component is considered noise. In other words, the ideal 1-D filter can be expressed as:

$$W(\omega) = \frac{\Phi_x(\omega)}{\Phi_x(\omega) + \Phi_x(2\pi - \omega)}, \tag{1}$$

where $\Phi_x(\omega)$ is the power spectrum of the *original* (non-aliased) signal. This can be easily extended to the 2-D non-separable case by including all the 3 components of the aliasing:

$$W(\omega_1, \omega_2) = \frac{\Phi_x(\omega_1, \omega_2)}{\Phi_x(\omega_1, \omega_2) + \Phi_a(\omega_1, \omega_2)} \tag{2}$$

where

$$\Phi_a(\omega_1, \omega_2) = \Phi_x(2\pi - \omega_1, \omega_2) + \Phi_x(\omega_1, 2\pi - \omega_2) + \Phi_x(2\pi - \omega_1, 2\pi - \omega_2). \tag{3}$$

Since we want these filters only for comparison purposes, we computed only the 256x256 (separable and non-separable) Wiener filters, since these can be designed directly in the frequency domain. These filters can be considered as upper bounds for the performance of a linear filter of smaller length.

3. THE NON-LINEAR STRATEGY

Non-linear reconstruction techniques have already found important applications in areas where traditional techniques cannot be applied, as is the case for example with binary images[10]. In such cases, non-linear filtering is required, but even in a more general (gray-level) situation, non-linear reconstruction filters can be designed to explore the inherently non-band-limited nature of sharp-edges present in most images.

The non-linear reconstruction filter we analyze in this paper is a modified rank-order filter (an L-filter)[11]. It consists of averaging the result of the samples at rank .50 and .51 when using the weights in the mask of Figure 2-a. A non-linear filter cannot generally be decomposed into polyphase sub-filters. Nevertheless, since we only apply this filter to the up-sampled signal, a polyphase decomposition is possible, and will greatly reduce computations. Figures 2-b through 2-e show the 4 sub-masks corresponding to this

(a)

0	.1	0	.1	0
.1	.25	.3	.25	.1
0	.3	1	.3	0
.1	.25	.3	.25	.1
0	.1	0	.1	0

(b)

0	0	0
0	1	0
0	0	0

(c)

.25	.25
.25	.25

(d)

.1	.3	.1
.1	.3	.1

(e)

.1	.1
.3	.3
.1	.1

Figure 2: (a)Coefficients for the weighted median filter used for reconstruction; (b-e) polyphase sub-filters.

Filter	size	m.s.e.	m.a.e.
without pre-filtering			
Non-linear	5^2	26.75	7.05
Low-pass	11^2	26.17	7.33
Low-pass	256^2	25.55	8.62
Wiener (separable)	256^2	26.46	7.47
Wiener (non-sep.)	256^2	26.62	8.67
with pre-filtering			
Low-pass	2×7^2	24.53	16.99
Low-pass	2×11^2	27.21	8.81
Low-pass	2×256^2	28.03	7.36

Table 1: m.s.e. and m.a.e. performance of some filters

decomposition. Notice that the first sub-filter (Figure 2-b) is just the identity (as one would expect). The second sub-filter (Figure 2-c) has all 4 weights equal, and therefore is just the average between the rank 2 and rank 3 samples in the 2x2 mask, and can be implemented using 4 comparisons, 1 sum, and 1 shift (division by 2). The last two sub-filters (Figures 2-b and 2-c) are equivalent to averaging the two center rank samples in a weighted rank order filter, with weights (1,1,3,3,1,1), and can be implemented (in the worst case) with 9 comparisons, 1 sum, and 1 shift. Therefore, this filter can be implemented using less than 6 comparisons, 1 sum and 1 shift (division by 2) per sample (no multiplications). This is approximately the computational effort for a typical separable 3x3 FIR filter (3 multiplies and 4 sums per sample).

The filter have been designed in order to preserve the sharpness of edges. It can be shown that it preserves every edge that can be identified in the 5x5 window, as well as any flat or slanted regions spanning the whole window.

4. RESULTS

In order to compare the performance of the proposed strategy with the traditional linear filtering strategy, we applied the scheme of Figure 1 using several filters. We computed mean absolute error (m.a.e.) and mean square error (m.s.e.) using 6 common images (Lena, ape, camera man, bridge, boy, and building). Table 1 shows the average results on these images. *Non-linear* refers to the proposed technique, which uses the 5x5 reconstruction filter described in Sec-

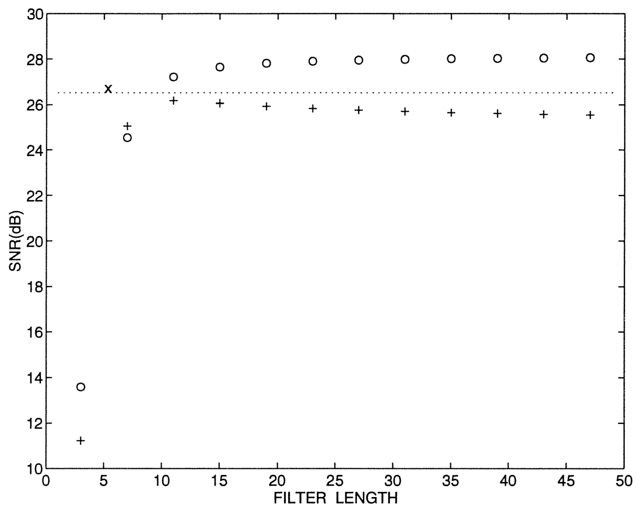

Figure 3: performance of the linear strategies for several filter lengths.

tion 3. *Low-pass* refers to the 11x11 and 256x256 (separable) low-pass filters. A low-pass is probably the filter most commonly used as prototype. The *Wiener* filter results refer to the 256x256 (separable and non-separable) Wiener filters, optimized for the specific 6 images used. It can be considered as the absolute upper bound for the performance of (separable and non-separable) linear filters. The last three rows of the table, refer to cases where an anti-aliasing filter *was* included (Low-pass filters, designed using a Hamming window).

Notice that the non-linear technique outperforms all strategies under the m.a.e. criteria (even including a pre-filter). For m.s.e., the results depend on the computational complexity allowed for the linear case (filter length used for the FIR filter), but is clearly favorable to the non-linear technique.

- If a pre-filter is not used, the non-linear 5x5 reconstruction filter outperforms FIR filters of any length, under both m.a.e. and m.s.e. criteria.
- If pre-filtering is allowed in the linear strategy, the proposed scheme still performs better under m.a.e.
- If pre-filtering is allowed, and under the m.s.e. criteria, two FIR filters of the order of approximately 9x9 will be required to match the performance of the non-linear strategy. This is many times the computational complexity of the non-linear strategy (remember that a pre-filter is about 4 times more complex than a post-filter of same size).

Figure 3 compares the performance of the filters for several filter lengths. The circles refer to FIR pre- and post-sampling. The crosses refer to FIR post-sampling filter only. The "x" corresponds to the proposed non-linear strategy (which does not include pre-filtering), and the horizontal line represents the upper bound for linear filter (non-separable 256x256 Wiener filter). Note that the performance of the low-pass FIR post-filters reach a peak around 11x11. This can be attributed to the fact that, at that length, the filter is a good approximation for the Wiener filter for these images, while the sharper cut-off filters will allow more of the mid-frequency aliasing to pass through.

The different nature of distortion for the different techniques can be perceived in Figure 4. The images correspond to a 100x100 segment of the 256x256 interpolated images. The PSNR figures refer to the whole image. Notice that the non-linear technique produce the sharpest edges. Among the linear low-pass filters, the 256x256 produces sharper edges than the 11x11, but it lets more aliasing pass through. Note also the similarity between the output of the 11x11 low-pass and the 1-D Wiener filter.

5. SOME INTERPRETATIONS

Under the morphological approach, images are considered as a combination of sets, instead of a linear combination of sinusoids, as in traditional linear systems analysis. From this point of view, "small" sets (e.g., an isolated impulse) should be removed (filtered out) from the signal before subsampling[5, 6], in order to avoid generating larger sets in the reconstructed signal (what can be considered "shape aliasing"). If it is known that this filtering has not been performed (as it was the case here), the consequences can be "controlled" by trying to identify some of these impulses in the subsampled signal, and not dilating those pixels. That is exactly what the non-linear reconstruction filter used in this paper does.

We can apply a similar analysis to linear interpolation. In this case, it is easy to see that the superposition requirement, together with the (desired) DC preservation, will *imply* that an isolated impulse be smeared into a shape whose sum of amplitudes be at least 4 times the amplitude of the original impulse. In other words, linear interpolators *need* this "shape aliasing" to preserve the DC component of the images.

6. CONCLUSIONS

In this paper we show that it is possible to mitigate the effects of aliasing **after** subsampling. Using the ideas introduced in this paper one can remove anti-aliasing filtering from the process, use a simple reconstruction filter, and yet obtain performance equivalent to much more complex FIR filter strategies.

This should find immediate application in several real-time video applications, where computational complexity is usually an issue, and where subsampling is often used as a way of reducing the amount of data, converting between different resolutions, or producing multiresolution pyramids.

It should be pointed out that the filter we presented in this paper will not necessarily perform well for other applications, or on radically different images. Non-linear techniques still lack powerful design tools. Recent developments, such as the Critical Morphological Sampling Theorem[5, 6] and the Slope transform[12] may be the basis for adequate design techniques in the near future.

Figure 4: Examples of the various reconstruction techniques: (a) original (100x100); (b) 5x5 non-linear filter (30.13 dB); (c) 256x256 low-pass (29.02 dB); (d) pixel replication; (e) 256x256 separable Wiener filter (30.02 dB); (f) 11x11 low-pass (29.96 dB).

7. REFERENCES

[1] R. W. Schafer and L. R. Rabiner, "A digital signal processing approach to interpolation," *Proc. IEEE*, vol. 61, pp. 692–702, June 1973.

[2] G. Oetken, T. W. Parks, and H. W. Schüssler, "New results in the design of digital interpolators," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 23, pp. 301–309, June 1975.

[3] H. S. Malvar and D. H. Staelin, "Optimal FIR pre- and postfilters for decimation and interpolation of random signals," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 67–74, Jan. 1988.

[4] M. Green, "The perceptual basis of aliasing and antialiasing," in *Human Vision, Visual Processing and Digital Display III*, vol. SPIE 1666, pp. 84–93, 1993.

[5] D. A. F. Florêncio and R. W. Schafer, "Critical morphological sampling." to be submitted to IEEE Trans. on Signal Processing.

[6] D. A. F. Florêncio and R. W. Schafer, "Homotopy and critical morphological sampling," in *SPIE's Symposium on Visual Comm. and Image Proc.*, Sept. 1994.

[7] I. Defée and Y. Neuvo, "Antialiasing median-type filters for image decimation and processing," in *Proc. European Signal Processing Conf.*, pp. 805–808, 1990.

[8] P. He, "Digital interpolation of stocastic signals from the viewpoint of estimation theory," in *Proc. European Signal Processing Conf.*, pp. 129–132, 1990.

[9] R. E. Crochiere and L. R. Rabiner, "Interpolation and decimation of digital signals — a tutorial review," *Proc. IEEE*, vol. 69, pp. 300–331, Mar. 1981.

[10] C. Tung, "Resolution enhancement technology in Hewlett-Packard laserjet printers," in *Proceedings of SPIE, vol. 1912*, pp. 440–448, 1993.

[11] A. C. Bovik, T. S. Huang, and J. D. C. Munson, "A generalization of median filtering using linear combinations of order statistics," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 31, pp. 1342–1350, Dec. 1983.

[12] P. Maragos, "Morphological systems theory: slope transforms, max–min differential equations, envelope filters and sampling," in *Mathematical morphology and its applications to image processing* (J. Serra and P. Soille, eds.), pp. 149–160, Kluwer Academic Publishers, 1994.