Lapped Orthogonal Vector Quantization

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Abstract

The block processing inherent in the use of traditional vector quantization (VQ) schemes typically gives rise to perceptually distracting blocking artifacts. We demonstrate that such artifacts can, in general, be virtually eliminated via an efficient lapped VQ strategy. With lapped VQ schemes, blocks are obtained from the source in an overlapped manner, and reconstructed via superposition of overlapped codevectors. The new scheme, which we term lapped orthogonal vector quantization (LOVQ), requires no increase in bit rate and, in contrast to other proposed approaches, no significant increase in computational complexity or memory requirements. Attractively, the use of LOVQ also leads to a modest increase in coding gain over traditional VQ schemes of comparable complexity. In addition to the theory, results of experiments involving speech and image sources are also presented.

1 Introduction and Background

Vector quantization (VQ) plays an important role in a wide range of signal coding and data compression applications [1]. In a typical application, involving imagery or speech for example, the signal is partioned into contiguous blocks of equal size, each of which corresponds to a vector of signal samples. Each vector is then represented by one of a set of candidate codevectors that is closest to the vector with respect to some distortion or distance measure. This set is referred to as the codebook, and is available to decoder as well. As a result, for each block only the index of the codevector need be transmitted to allow suitable reconstruction of the block at the receiver.

VQ systems are generally memory-intensive, but the memory requirements are symmetric with respect to the encoder and decoder. The codebook size is $\mathcal{O}(2^{RN})$ where R is the prescribed bit rate and N is the block size. This behavior, coupled with the fact that the codevector lengths obviously grow linearly with N, means that the codebook memory requirements grow dramatically with block size.

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By contrast, the computational requirements of VQ systems are highly asymmetric. A full codebook search at the encoder has a computational complexity comparable to the memory requirements, viz., $\mathcal{O}(2^{RN})$ per signal sample. Decoding complexity is negligible however, since it requires a simple table lookup. This asymmetry is particularly well-suited to a variety of applications, such as database browsing. However, VQ systems and subsystems are also widely used in a wide spectrum of other applications, including videoconferencing and digital audio systems.

It is well known, in fact, that VQ is an asymptotically optimal compression strategy in the sense that given a sufficiently long block length and suitably designed codebook, the rate-distortion bound for the source can be approached arbitrarily closely. However, the memory and computational requirements strongly limit block lengths, and as a result the asymptotic limits are rarely approached in practice. The use of constrained or structured codebooks can reduce the computational and/or memory requirements, allowing larger block sizes to be used [2] [3] [1]. However, with such constraints, VQ is generally no longer an asymptotically optimal scheme.

An important class of coding systems that can be interpreted as a form of VQ with constrained codebooks is the traditional approach of using a linear block transform followed by scalar quantization [4]. The corresponding decoder then reconstructs the quantized coefficients and applies the inverse transform. As is well-known, the resulting system is equivalent to a VQ system in which the codebook corresponds to a rotated cartesian lattice of codevectors. In effect, it is this special structure that leads to a fast-searchable codebook. The memory requirements of such systems are are dramatically reduced, to $\mathcal{O}(N2^R)$. Moreover, if a fast-computable transform is used, the computational complexity at both the encoder and decoder is $\mathcal{O}(\log N)$ per sample. However, although reasonable performance can often be achieved via transform coding, its performance does not approach the rate-distortion bound with increasing block size.

The need to use finite block sizes in VQ systems not only limits how closely the rate-distortion bound can be approached, but also leads to unnatural and perceptually distracting blocking artifacts. In effect, mean-square coding distortion is not minimized because interblock dependencies are not exploited, and blocking artifacts arise because the distortion that is introduced by the coding process has statistics that are periodic with a period equal to the block size.

In this paper, we develop a highly efficient strategy for effectively eliminating blocking artifacts in VQ systems, and which, as a side benefit, also leads to a reduction in overall mean-square distortion. Specifically, in Section 2 we exploit an interpretation of lapped transform coding as a constrained lapped VQ system to develop and optimize a powerful generalization of the lapped transform paradigm as our main result. In Section 3 we then explore several attractive performance characteristics of the new strategy.

2 Mitigation of Blocking Artifacts: Lapped VQ

One class of techniques for mitigating artifacts in block processing systems such as VQ involves applying a temporally- or spatially-varying filter to the reconstructed signal at the decoder [5] [6] [7]. Such techniques can be combined with suitable prefiltering to substantially reduce blocking artifacts, though at the expense of an increase in the overall mean-square reconstruction error [8] [9] [10].

More efficient and effective systems have generally resulted through the use of lapped block processing strategies. For example, in unconstrained (full-search) VQ systems, blocking artifacts can be reduced by extending the reconstruction codevectors beyond the block boundaries at the decoder. A mean-square optimized overlapping reconstruction codebook can lead to a noticeable reduction of blocking artifacts and a reduction of the reconstruction error [11]. However, a disadvantage of this particular approach is the increase in decoding complexity and memory requirements due to the increased decoder codebook size.

In this section, we develop an efficient lapped VQ scheme in which blocks are acquired in a

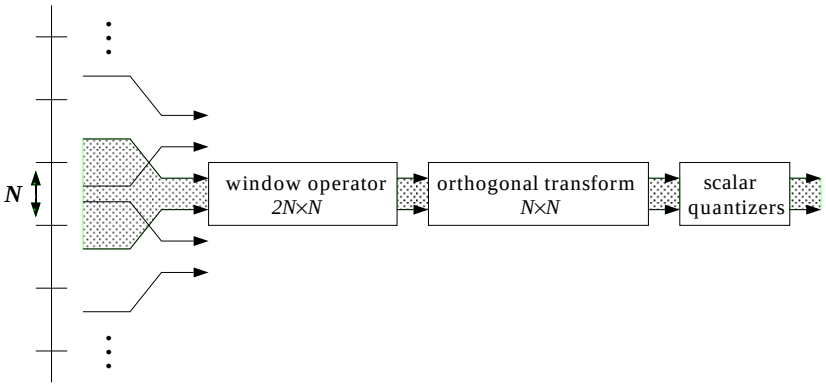


Figure 1: *Transform coding with lapped transforms. Typically, the quantizer block is formed from a set of N independent scalar quantizers.*

lapped manner at the encoder, and reconstructed in a lapped manner at the decoder. As we will demonstrate, this technique produces performance enhancements similar to those in [11], but without requiring any increase in the coder or decoder codebook sizes. This new scheme can be interpreted as a powerful generalization of lapped transform coding schemes. Accordingly, we begin our development with a summary of the relevant concepts and results on this topic.

2.1 Lapped Orthogonal Transforms

Lapped transform coding can be viewed as a lapped VQ strategy with a highly structured codebook, in much the same way as conventional transform coding can be viewed as a conventional VQ strategy with a highly structured codebook. Moreover, the use of lapped transforms with suitable orthogonality properties can achieve a significant reduction in blocking artifacts, and also simultaneously a reduction in mean-square reconstruction error over nonlapped transform coding.

Lapped transforms were developed based on the notion of representing the input signal as a combination of overlapping basis functions. Although other sizes are also used in practice, often the basis functions from adjacent blocks overlap by 50% on each side of the block, so that their length is twice the block size [9] [12]. With such a lapped transform, the transform matrix is no longer $N \times N$, but rather is $N \times 2N$, mapping a block of $2N$ input samples into a block of N transform coefficients, as shown in Fig. 1. Each length-$2N$ block in a lapped transform system cannot be exactly reconstructed from its N transform coefficients. However, when the transform basis functions satisfy the additional "orthogonality in the tails" constraint [12]—so that the collection of basis functions for all blocks constitute a complete orthonormal set—then, in the absence of quantization, perfect reconstruction of the signal can be achieved by superimposing the overlapped blocks at the decoder. These are referred to as lapped orthogonal transform (LOT) systems.

As Fig. 1 implies, the $2N \times N$ transform matrix $\mathbf{Q}$ of any LOT system can be factored into the product of a $2N \times N$ window operator matrix $\mathbf{W}$ and an $N \times N$ orthogonal transform matrix $\mathbf{U}$. Moreover, it is evident that this factorization is not unique. However, for some LOT systems, this factorization can be performed so that $\mathbf{W}$ is a sparse matrix and $\mathbf{U}$ can be implemented via a fast algorithm [12]. For example, for the class of LOT systems referred to as modulated lapped transform (MLT) systems, the resulting $\mathbf{U}$ can be efficiently implemented via an $\mathcal{O}(\log N)$ per sample algorithm, and $\mathbf{W}$ via an algorithm whose complexity per sample is *independent* of block size, so that the overall complexity is $\mathcal{O}(\log N)$ per sample.

2.2 Lapped Orthogonal Vector Quantization

Efficient lapped VQ systems result from generalizing the lapped transform coding systems discussed in the previous section. In lapped transform systems, the N transform coefficients generated for each block via the lapped transform are quantized via individual scalar quantizers. As a result, lapped transform coding corresponds to a lapped VQ strategy with a highly constrained codebook. In the remainder of this section, we focus on systems where the codebook is substantially less constrained. In particular, we replace the bank of N scalar quantizers in Fig. 1 with an unconstrained mean-square optimized vector quantizer whose codewords are length N. We refer to the resulting systems as lapped orthogonal vector quantization (LOVQ) systems.

When VQ is used in place of the bank of scalar quantizers in the LOT structure of Fig. 1, the implementation of the lapped transform component of the system can be substantially simplified. In particular, the $N \times N$ orthogonal transform matrix $\mathbf{U}$ can be eliminated with no impact on performance. This is because this matrix merely induces a (generalized) rotation of N-dimensional vector space, so its effect can be conveniently absorbed into the VQ subsystem design provided the VQ is unconstrained [1]. It is important to stress, however, that the window operator cannot be absorbed into the VQ since its dimension is $2N \times N$.

The resulting LOVQ encoder structure, which is equivalent to an LOT followed by VQ, is depicted in Fig. 2(a). The corresponding decoder structure, which is equivalent to a VQ decoder followed by the LOT inverse, is depicted in Fig. 2(b). Recall that the VQ decoder in Fig 2(b) is a simple table lookup operation: the appropriate length-N codevector is selected according to the received index. The window operator inverse, in turn, maps successive length-N codevectors into overlapping length-$2N$ codevectors which are superimposed to generate the reconstruction at the output.

Not surprisingly, the choice of window operator has a significant impact on the performance of the resulting system, both in terms of mitigating blocking artifacts and reducing mean-square coding distortion. Furthermore, the structure of this operator affects the additional computational complexity inherent in the use of LOVQ over conventional VQ systems. From these perspectives, a particularly attractive choice for the window operator is that corresponding to the MLT. As we discussed, this window operator has a computational complexity per sample that is independent of the with block size. Its implementation via the orthogonal butterflies is depicted in Fig. 3, where the butterfly transmittances are given by

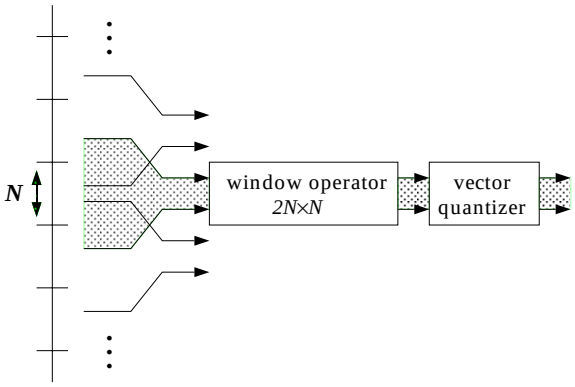
$$h[n] = \sin\left[\left(n + \frac{1}{2}\right)\frac{\pi}{2N}\right]. \tag{1}$$

This choice for the window operator leads to the overlapping length-$2N$ decoder codevectors having some intuitively appealing characteristics. To see this, note that each length-$2N$ codevector generated at the output of the LOVQ decoder in Fig. 2(b) is a linear combination of the N basis functions of the window operator, i.e., the columns of the window operator matrix. For the specific choice of the MLT window operator these basis functions all taper smoothly to zero at both ends, a result one might expect of a reconstruction of the lapped blocks via superposition that avoids blocking artifacts.

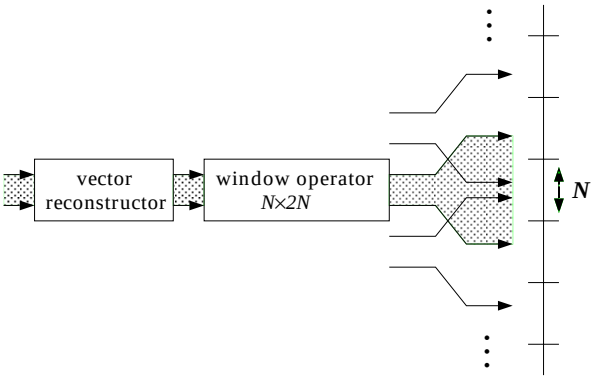
Note that with this fast-computable window, the LOVQ system complexity in terms of both computation and memory requirements is dominated by its VQ subsystem, and thus is comparable to that for traditional VQ systems. This makes LOVQ an attractive alternative to the lapped VQ scheme described in [11], which requires a decoder codebook whose vectors are of length $2N$.

2.3 Optimization of Coding Gain in LOVQ Systems

Within the class of LOVQ systems, it is natural to seek that yielding both minimal block artifacts and minimal overal coding distortion. Fortunately, these objectives are nonconflicting. In this section we describe a framework for optimizing LOVQ systems.



(a) LOVQ encoding



(b) LOVQ decoding

Figure 2: *Implementation of lapped orthogonal vector quantization.*

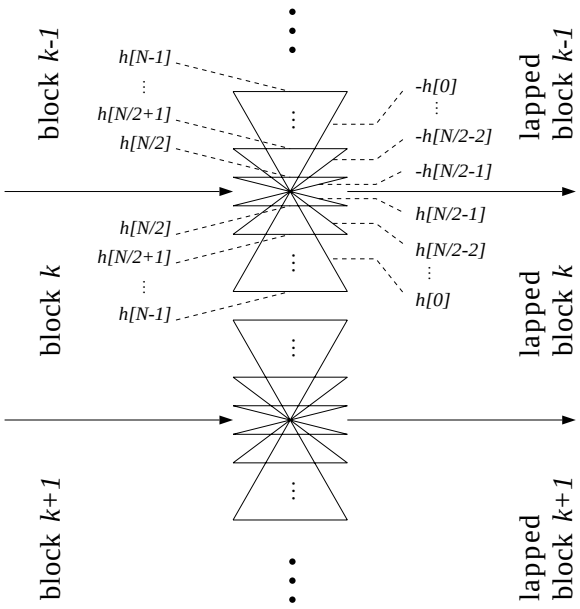


Figure 3: *LOVQ window operator at the encoder. The inverse window operator to be used at the decoder is the transposition of this signal flow graph.*

To begin, let us focus on the length-N vectors of coefficients generated by the window operator. When the original source $x[n]$ is, for example, stationary, the sequence of overlapping length-$2N$ vectors $\mathbf{x}$ at the input to the window operator is a stationary vector source with Toeplitz covariance matrix $\mathbf{R_X}$. In turn, the length-N vectors of transform coefficients $\mathbf{y}$ at the input to the vector quantizer is also a stationary vector source with covariance matrix

$$\mathbf{R_Y} = \mathbf{W}\mathbf{R_X}\mathbf{W}^{\mathrm{T}}.$$

For ergodic sources, the mean-square distortion-rate function for blocks of size N is bounded according to [13]

$$D_N(R) \leq \sigma_x^2 \, \gamma_N^2 \, 2^{-2R}, \tag{2}$$

where σ_x^2 is the variance of the source, and where γ_N^2 is the spectral flatness measure for the source, i.e.,

$$\gamma_N^2 = \frac{\left[\prod_{k=0}^{N-1} \lambda_k\right]^{1/N}}{\frac{1}{N}\sum_{k=0}^{N-1} \lambda_k} = \frac{N\left[\det \mathbf{R_Y}\right]^{1/N}}{\mathrm{tr}\,\mathbf{R_Y}} = \frac{N\left[\det(\mathbf{W}\mathbf{R_X}\mathbf{W}^{\mathrm{T}})\right]^{1/N}}{\mathrm{tr}(\mathbf{W}\mathbf{R_X}\mathbf{W}^{\mathrm{T}})} \tag{3}$$

with λ_k denoting the kth eigenvalue of $\mathbf{R_Y}$.

The bound (2) suggests that optimum VQ performance is obtained when the spectral flatness measure γ_N^2 is minimized. Thus, the desired optimization is to minimize (3) over all possible window operators $\mathbf{W}$ subject to the constraint that the operators correspond to orthogonal transformations. This constraint can be expressed in the form

$$\mathbf{W}\mathbf{W}^{\mathrm{T}} \quad = \quad \mathbf{W}_1\mathbf{W}_1^{\mathrm{T}} + \mathbf{W}_2\mathbf{W}_2^{\mathrm{T}} = \mathbf{I} \tag{4a}$$

$$\mathbf{W} \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{W}^{\mathrm{T}} = \mathbf{W}_1 \mathbf{W}_2^{\mathrm{T}} = \mathbf{0} \tag{4b}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the zero matrix, both of size $N \times N$, and where

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{bmatrix}.$$

In addition, it is sometimes convenient to further constrain the window operator to have a fast implementation of the form described by Fig. 3. In this case, the orthogonality conditions (4) are equivalent to the condition that the window sequence $h[n]$ satisfy [12]

$$h^2[n] + h^2[N-1-n] = 1.$$

for $n = 0, 1, \ldots, N/2 - 1$.

Interestingly, for first-order autoregressive sources $x[n]$, for which the autocorrelation function is

$$R_x[k] = \sigma^2 \rho^{|k|},$$

the MLT window operator is asymptotically near-optimal, i.e., as $\rho \to 1$ except for very small block sizes. For $N = 2$, the optimal window sequence differs from that of the MLT, but can be readily computed, yielding

$$h[0] = \sin(\pi/6) \qquad h[1] = \cos(\pi/6). \tag{5}$$

3 LOVQ Performance Characteristics

Experiments involving speech and image data were conducted to verify the anticipated reduction in blocking artifacts. In the set of experiments involving vector quantization of speech, LOVQ based on the MLT window is compared with conventional VQ, where the VQ block size is $N = 12$ and the rate is $R = 0.5$ bits/sample. Some typical codevectors from the respective codebooks are depicted in Fig. 4. The codebook was designed from training data using an Linde-Buzo-Gray algorithm [1] initialized with codevectors randomly selected from the training data. A typical segment of the decoded speech waveform for each of the two systems is depicted in Fig. 5. While traditional VQ led to visibly and audibly significant blocking artifacts, these were effectively eliminated when LOVQ was used, as Fig. 5 reflects.

In the set of experiments involving vector quantization of imagery, LOVQ based on the MLT-window was compared to traditional VQ with 4×4 blocks ($N = 16$) at rate $R = 0.5$ bits/sample. Fig. 6 illustrates the performance of the respective systems on a test image of size 128×128 pixels, and 8 bits/pixel resolution. As Fig. 6 reflects, while using traditional VQ the reconstruction has prominent blocking artifacts, using LOVQ blocking effects are again effectively eliminated.

In both the above examples, the reduction of blocking artifacts was accompanied, appealingly, by a modest reduction in overall mean-square distortion as well. This byproduct is, in fact, predicted by the theory described in Section 2.3. In particular, Fig. 7 illustrates the coding gain that can be achieved using LOVQ with a fast window operator over conventional VQ with the same code-vector size N, as measured by the rate-distortion bound. In this figure, the source is a first-order autoregressive source $x[n]$ with correlation coefficient ρ. For the case $N = 2$, the window operator that was used corresponded to the window sequence (5); for $N > 2$, the MLT window operator was used. As Fig. 7 illustrates, greater coding gains are achieved for more strongly correlated sources and smaller block sizes. This is to be expected since there are more statistical dependencies that can be exploited by lapping in these cases.

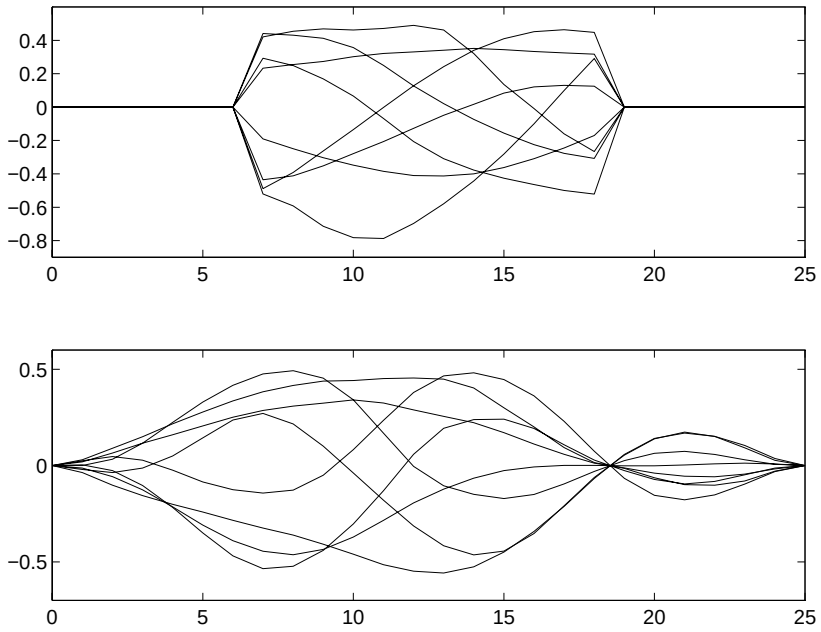


Figure 4: *Representative vectors from the codebook of VQ systems for speech where the VQ block size is $N = 12$ and the rate is $R = 0.5$ bits/sample. Top: vectors from the codebook of a traditional VQ system. Bottom: vectors from an LOVQ codebook. Note that the LOVQ codevectors based on the MLT window decay smoothly to zero at each end in order to reduce blocking artifacts.*

4 Conclusion

In this paper, LOVQ was developed as an efficient lapped VQ strategy that leads to dramatically reduced blocking artifacts when compared with traditional VQ systems. Moreover, as an attractive byproduct, with LOVQ this reduction is also accompanied by a modest reduction in overall mean-square distortion. Most importantly, these performance enhancements are achieved with negligible increase in system complexity or memory requirements. In particular, the overhead in complexity amounts to a total of only 1.5 additional multiplies and adds (MADs) per input sample.

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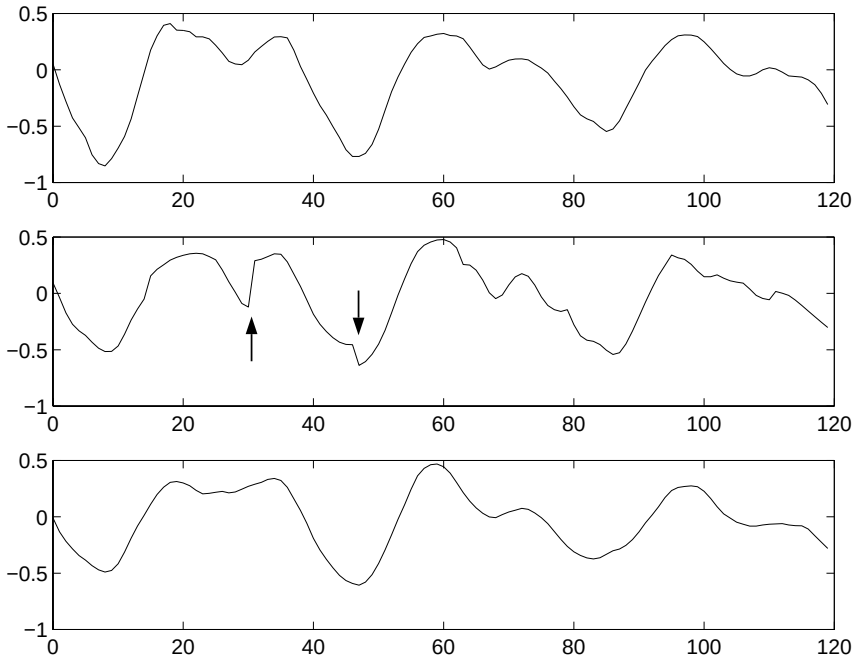

Figure 5: *Waveforms in example of vector quantization of speech when the VQ block size is* $N = 12$ *and the rate is* $R = 0.5$ *bits/sample. Top: original speech signal, sampled at 22.05 kHz. Middle: reconstruction when traditional VQ is used; the arrows indicate some of the prominent blocking artifacts. Bottom: reconstruction when LOVQ is used, which is effectively devoid of blocking artifacts.*

Figure 6: *Example involving vector quantization of imagery. The block size is* 4×4 *(*$N = 16$*) and the rate is* $R = 0.5$ *bits/sample. Left: original image,* 128×128 *pixels, 8 bits/pixel resolution. Middle: reconstruction when traditional VQ is used. Right: reconstruction when LOVQ based on the MLT-window is used; note that blocking effects are mitigated without loss of resolution.*

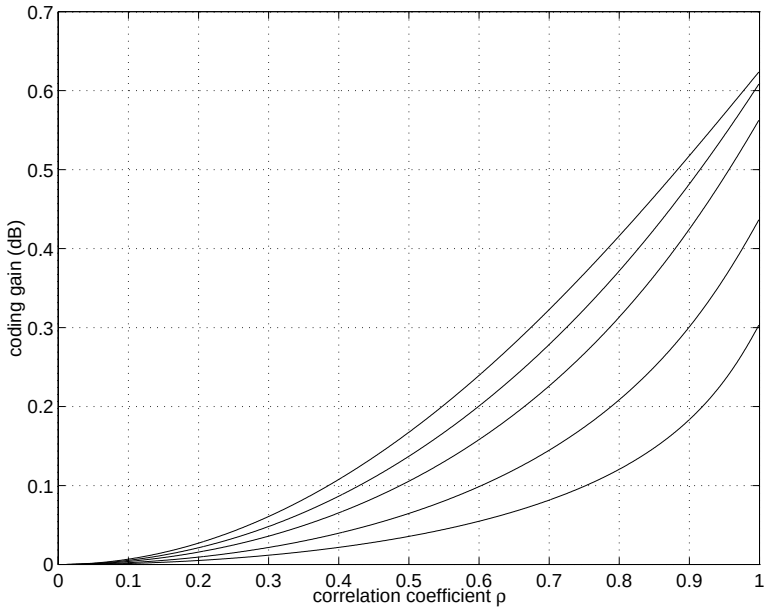

Figure 7: *Achievable coding gain of LOVQ over conventional VQ as a function of the correlation coefficient of a first-order autoregressive source. The successively lower curves correspond to codevector sizes $N = 2, 4, 8, 16, 32$. This gain, a byproduct of the LOVQ scheme, is achieved without any significant increase in system complexity.*

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