MODULATED QMF FILTER BANKS WITH PERFECT RECONSTRUCTION

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Necessary and sufficient conditions for perfect reconstruction (PR) in a modulated filter bank are derived. It is shown that, for a bank of M filters of length L, PR can be obtained when $L = 2KM$, for any positive integer K, whereas previous results guaranteed PR only for $K = 1$.

[appeared in *Electronics Letters*, vol. 26, no. 13, pp. 906-907, June 1990]

Introduction

Quadrature mirror filter (QMF) banks are essential building blocks of a subband/transform coder. Generally, one wants the filter bank to have the perfect reconstruction (PR) property[1], which means that, in absence of coding, a delayed copy of the original signal is reconstructed exactly in the critically-sampled analysis-synthesis system of Fig. 1. Although there are useful filter banks[2] that are not exactly PR, if a PR filter bank with a fast implementation can be found, it is usually preferable. General conditions for PR have been derived by Vaidyanathan[1]. Modulated filter banks are a class of QMF banks where all the impulse responses $h_k(n)$ in Fig. 1 are obtained from modulated versions of a single low-pass filter prototype[3–5].

Princen and Bradley[4], and Vetterli and Le Gall[5] have shown that an M-channel PR modulated filter bank can be obtained with FIR analysis and synthesis filters as long as their impulse responses have a length $L = 2M$. The modulated lapped transform (MLT) is a particular modulated filter bank that has a very fast algorithm[3]. In this letter we will show that PR can be obtained not only when $L = 2M$, but also when $L = 2KM$, with K a positive integer. In applications such as transmultiplexing and speech coding, the possibility of using longer filters means less interference among the subband signals, and keeping the modulated structure means a possibility of computationally efficient implementations.

Perfect Reconstruction

The reconstructed signal in Fig. 1 is given by[6]

$$y(n) = \sum_{l=-\infty}^{\infty} x(l)\ \alpha(n,l) \tag{1}$$

where

$$\alpha(n,l) = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{M-1} f_k(n - mM)\ h_k(mM - l) \tag{2}$$

Assuming that the analysis and synthesis filters are FIR, of length L, the overall delay from $x(n)$ to $y(n)$ is $L-1$, and PR means that $y(n) = x(n-L+1)$. From Eqns. (1) and (2), PR is obtained if and only if

$$\alpha(n, l-L+1) = \delta(n-l) \tag{3}$$

where $\delta(n)$ is the unitary impulse.

A modulated filter bank is a particular form of the integer-band SSB filter bank[4–6], defined by

$$f_k(n) = h(n)\sqrt{\frac{2}{M}}\ \cos\left[\frac{\pi}{M}\left(k+\tfrac{1}{2}\right)\left(n+\frac{M+1}{2}\right)\right] \tag{4}$$

for $n = 0, 1, \ldots, L-1$, with $h_k(n) = f_k(L-1-n)$ and $h(n) = h(L-1-n)$. The window $h(n)$ is called the low-pass prototype, because the frequency responses of all filters are just shifted versions of $H(e^{j\omega})$. Substituting Eqn. (4) into (2), we get

$$\alpha(n, l-L+1) = \sum_{m=-\infty}^{\infty} h(n-mM)\ h(l-mM)\ \beta(n,l,m) \tag{5}$$

where

$$\beta(n,l,m) = \frac{2}{M}\sum_{k=0}^{M-1} \cos\left[\tfrac{\pi}{M}\left(k+\tfrac{1}{2}\right)\left(n-mM+\tfrac{M+1}{2}\right)\right] \cdot \cos\left[\tfrac{\pi}{M}\left(k+\tfrac{1}{2}\right)\left(l-mM+\tfrac{M+1}{2}\right)\right] \tag{6}$$

Using the law of cosines and the fact that

$$\frac{1}{M}\sum_{k=0}^{M-1}\cos\left[\frac{\pi}{M}\left(k+\tfrac{1}{2}\right)l\right] = \begin{cases} (-1)^r, & l = 2rM \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

we get

$$\beta(n,l,m) = \beta_1(n,l,m) + \beta_2(n,l,m) \tag{8}$$

where

$$\beta_1(n,l,m) = \begin{cases} (-1)^r, & l = n-2rM \\ 0, & \text{otherwise}\,, \end{cases} \tag{9}$$

and

$$\beta_2(n,l,m) = \begin{cases} (-1)^{v+1-m}, & l = (2v+1)\,M-n-1 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

Assuming that $L = 2KM$, for K a positive integer, using Eqns. (9) and (10) in (5), and applying the PR condition in Eqn. (3), we obtain, after some algebraic manipulations, the PR condition

$$\sum_{m=-\infty}^{\infty} h(n+mM)\,h(n+mM+2rM) = \delta(r) \tag{11}$$

Since $h(n)$ is FIR, only a few terms in the above equation will actually be nonzero.

Therefore, as long as the low-pass prototype $h(n)$ satisfies Eqn. (11), the modulated filter bank will have the PR property. Thus, perfect reconstruction modulated filter banks of any length can be designed. Even with the enforcement of the PR condition in Eqn. (11), there are enough degrees of freedom to allow $h(n)$ to have flexible combinations of transition width and stopband attenuation.

Examples

For the traditional case of $K = 1$, Eqn. (11) reduces to

$$h^2(n) + h^2(n + M) = 1 \tag{12}$$

which is precisely Princen-Bradley PR condition[4,5] for $L = 2M$. In this case, a good choice for the window $h(n)$ is that of the MLT, given by[3]

$$h(n) = \sin\left[\frac{\pi}{2\,M}\,(n + \tfrac{1}{2})\right] \tag{13}$$

For $K = 2$, i.e., $L = 4M$, Eqn. (11) becomes

$$h^2(n) + h^2(n + M) + h^2(n + 2M) + h^2(n + 3M) = 1 \tag{14}$$

and

$$h(n)\,h(n + 2M) + h(n + M)\,h(n + 3M) = 0 \tag{15}$$

A family of windows $h(n)$ that satisfy the above equations can be defined by

$$\begin{array}{rcl} h(M/2 - 1 - i) & = & -s_i\; s_{M-1-i} \\ h(M/2 + i) & = & s_i\; c_{M-1-i} \\ h(3M/2 - 1 - i) & = & c_i\; s_{M-1-i} \\ h(3M/2 + i) & = & c_i\; c_{M-1-i} \end{array} \tag{16}$$

where $c_i \equiv \cos(\theta_i)$ and $s_i \equiv \sin(\theta_i)$, for $i = 0, 1, \ldots, M/2 - 1$. PR is guaranteed for any set of angles θ_i, but for $h(n)$ to be a good low-pass filter a possible set of angles is

$$\theta_i = \left[\left(\frac{1 - p}{2M}\right)(2i + 1) + p\right]\frac{(2i + 1)\pi}{8M} \tag{17}$$

The free parameter p typically varies in the range $[0, 1]$, and controls the frequency responses of the filters in the analysis and synthesis banks.

In Fig. 2 we show the frequency response of the first subband, $M = 8$, $|H_0(e^{j\omega})| = |F_0(e^{j\omega})|$ for the MLT filter bank, and for the new filter bank with $K = 2$, for $p = 1/2$ and $p = 1$. We note that the new filter bank has a narrower transition width than the MLT, due to the longer filter impulse responses obtained with $K = 2$. With $p = 1/2$, the maximum stopband gain is about 9 dB lower than that of the MLT.

Conclusion

We have demonstrated that an M-band perfect-reconstruction QMF filter bank can be designed with basis on a modulated (SSB) filter bank, as long as the window $h(n)$ has length $L = 2KM$, for any positive integer K, and satisfies the PR condition in Eqn. (11). It was previously believed that perfect reconstruction could only be obtained when $L = 2M$. As an example, we have shown that a filter bank designed with $K = 2$ will lead to better band-pass responses than the modulated lapped transform[3] (which is a modulated QMF filter bank with $K = 1$). The main advantage of the modulated filter bank structure is that it has the potential of computationally efficient implementations. We are currently developing a fast algorithm for modulated filter banks with $K = 2$; it will be reported in the near future.

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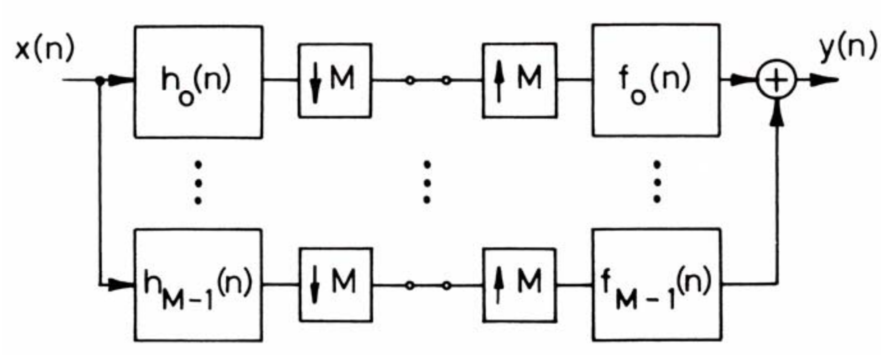

Fig. 1 Analysis-synthesis filter bank.

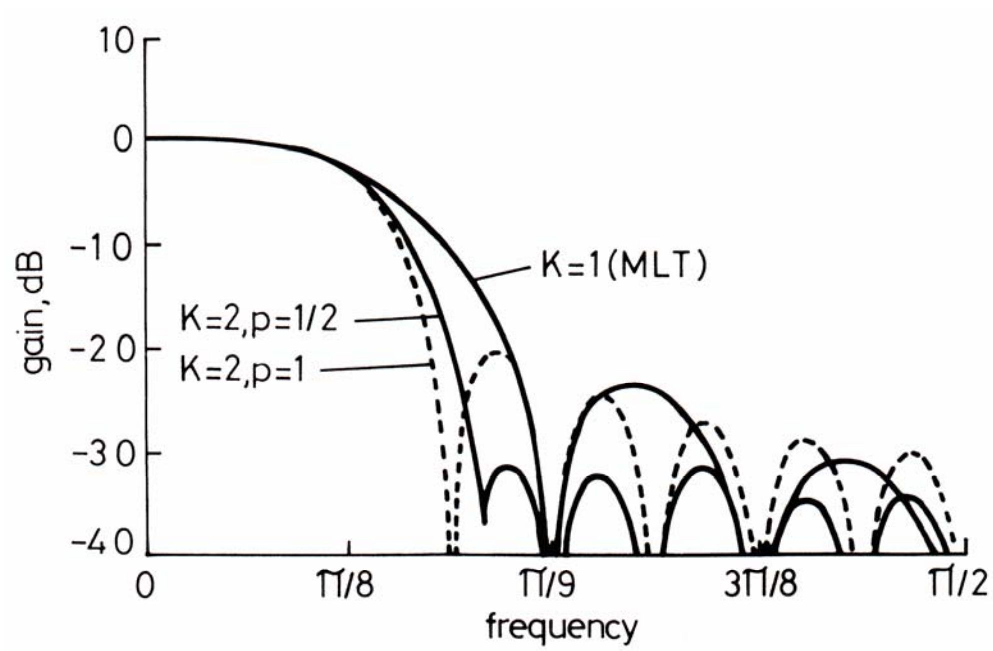

Fig. 2 Frequency responses for the first subband filter h_0, for the modulated lapped transform ($K = 1$), and the new filter banks ($K = 2$).