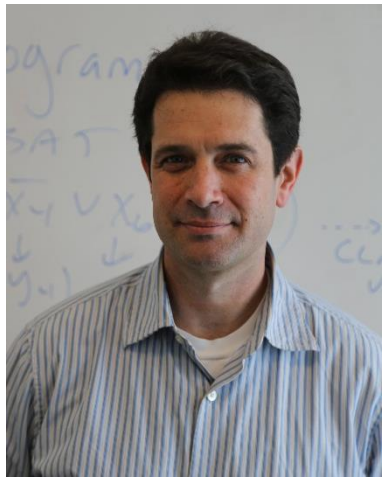
Efficiently Verifying Outsourced Cloud Computation: From Theory to Practice

Bryan Parno

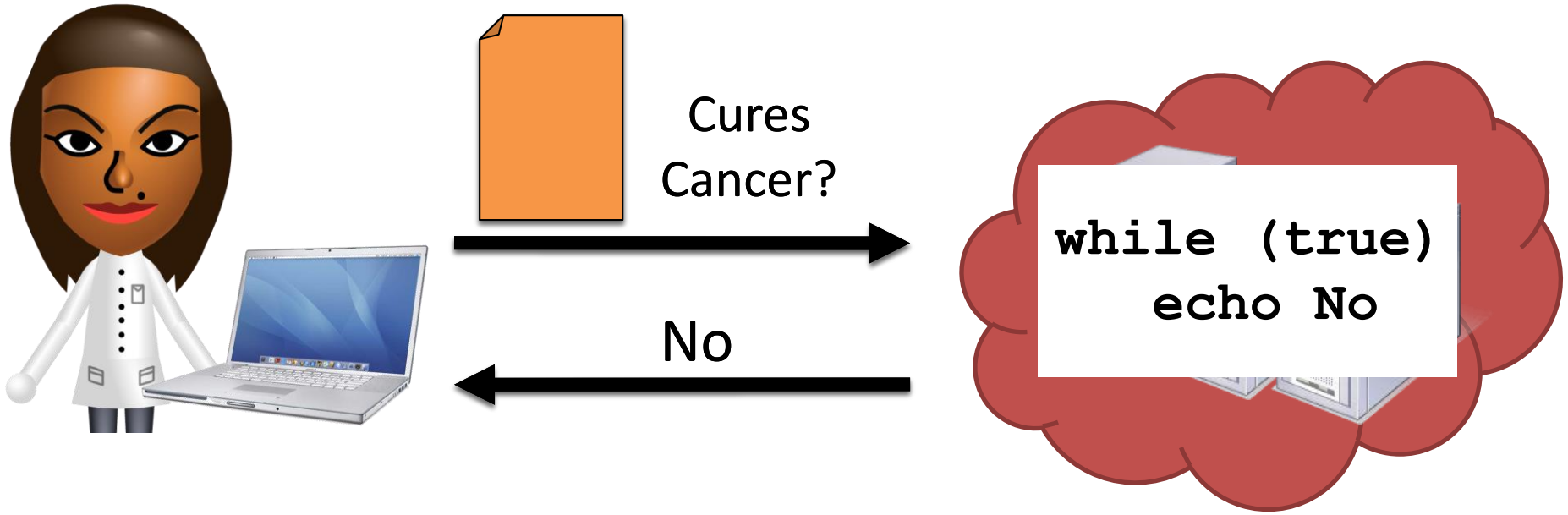
Microsoft Research



Michael Mitzenmacher

Mike Walfish

Outsourcing Computation

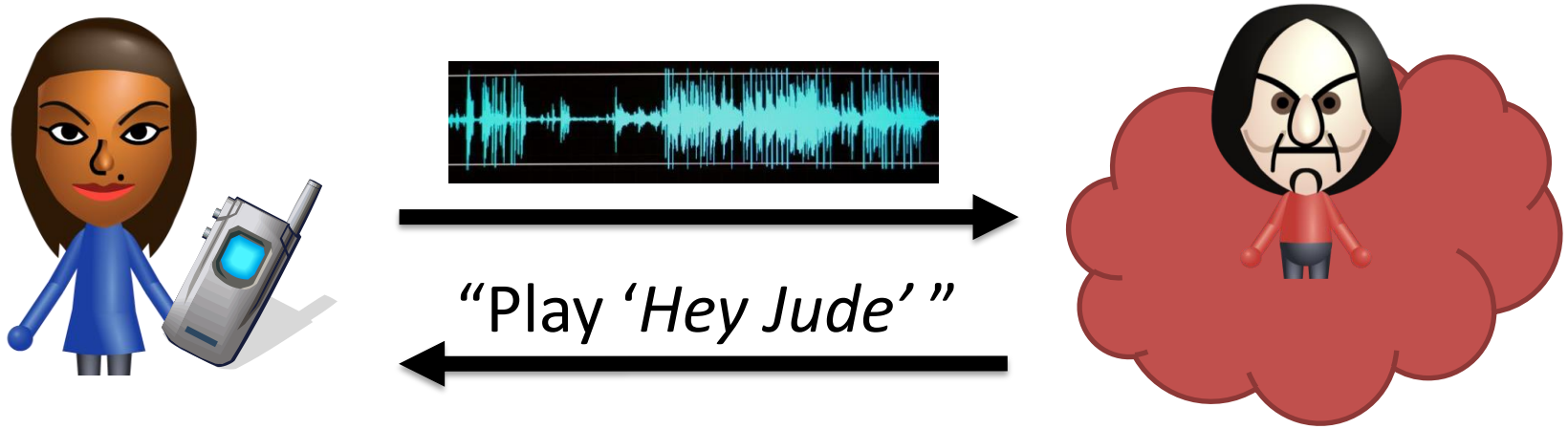


AWS Customer Agreement

WE… MAKE NO REPRESENTATIONS OF ANY KIND … THAT THE SERVICE OR THIRD PARTY CONTENT WILL BE UNINTERRUPTED, ERROR FREE OR FREE OF HARMFUL COMPONENTS, OR THAT ANY CONTENT … WILL BE SECURE OR NOT OTHERWISE LOST OR DAMAGED.

Why Verify Computation?

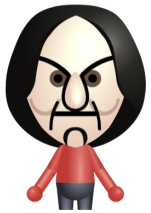
Mobile Offloading



Non-Malicious Providers

- Verifiable results:
  - Encourage cloud adoption
  - Command a premium
  - Help shed liability

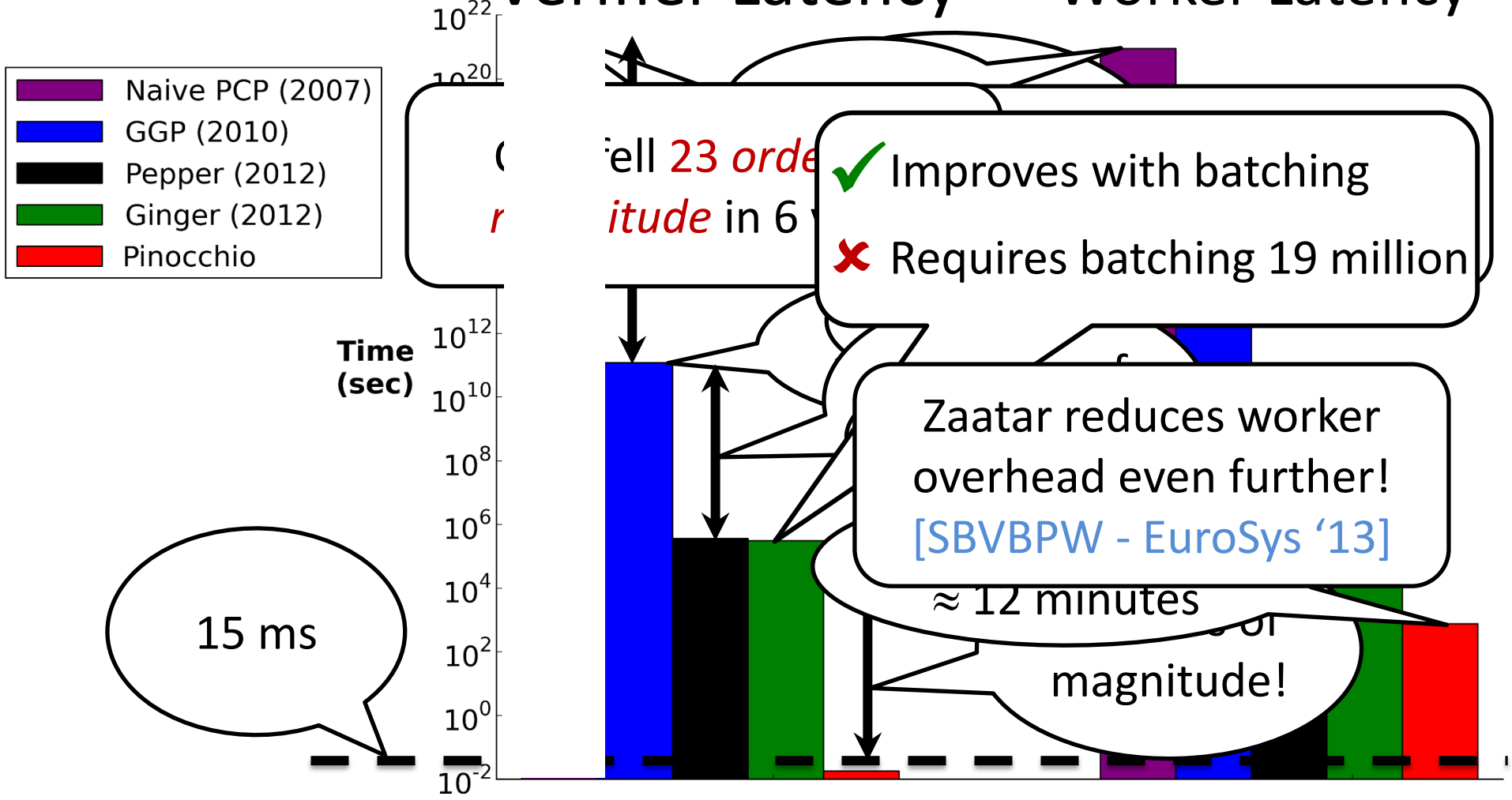
Relative Performance



100x100 matrix mult.

Verifier Latency

Worker Latency



Qualitative Comparison

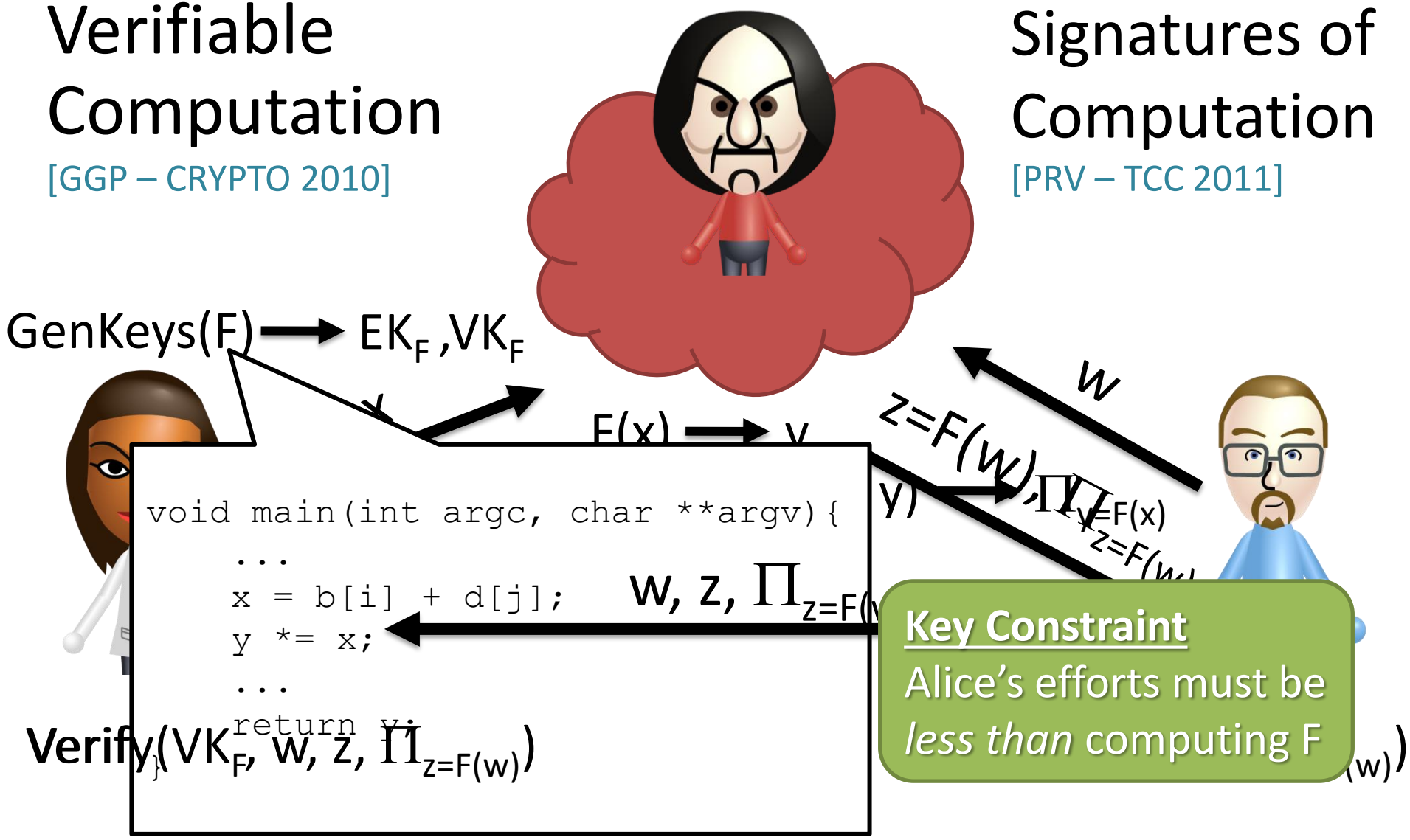
	FHE [Gennaro et al. '10] [Chung et al. '10]
Privacy	✓
Practical Performance	✗
Public Verification	✗
General Purpose	✓
Security Features	

Verifiable Computation

[GGP – CRYPTO 2010]

Signatures of Computation

[PRV – TCC 2011]



Pinocchio: Nearly Practical Verifiable Computation

Bryan Parno
Jon Howell
Microsoft Research

Craig Gentry
Mariana Raykova
IBM Research

Contributions

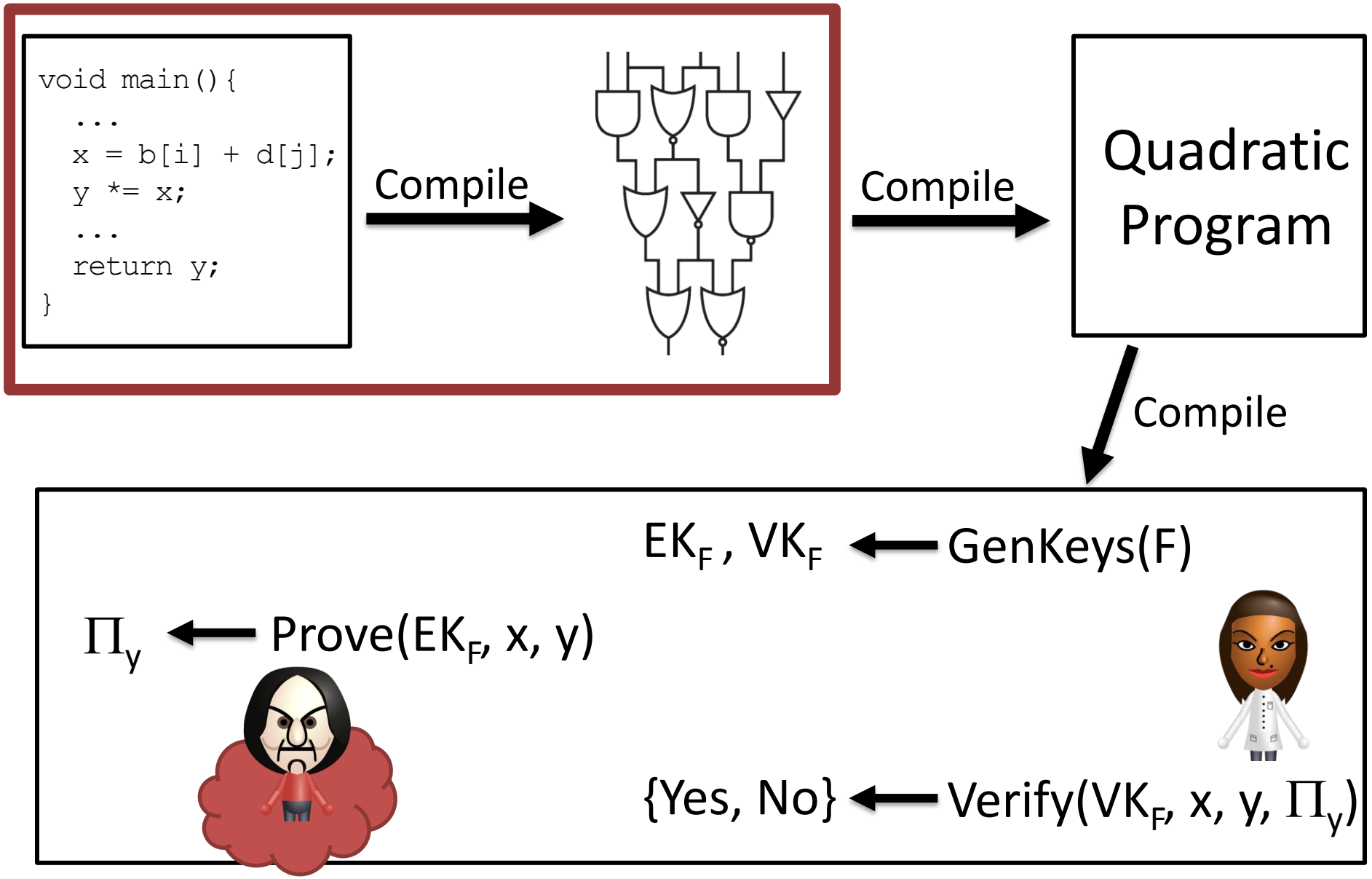
- New cryptographic protocol for general-purpose public verifiable computation
  - Also supports succinct zero-knowledge arguments

- Quadratic programs, a new, highly efficient encoding of general computations

	Good asymptotics	Concrete Performance
• Key setup:	O(N)	
• Worker:	O(N log N)	60x faster
• Verification:	O(IO)	10^7x faster
• Signature:	O(λ) bytes	288 bytes

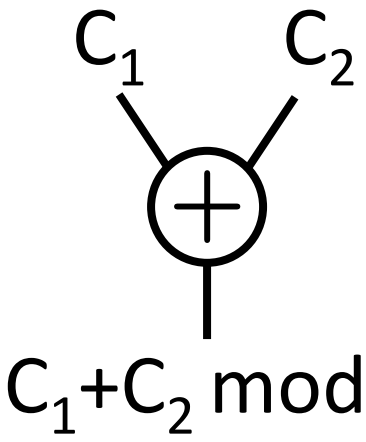
- First VC system where verification beats native C

Pinocchio's Verification Pipeline

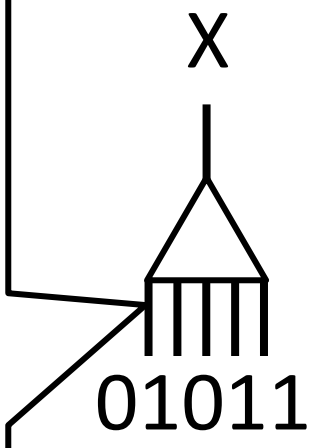


Compiling C to Circuits

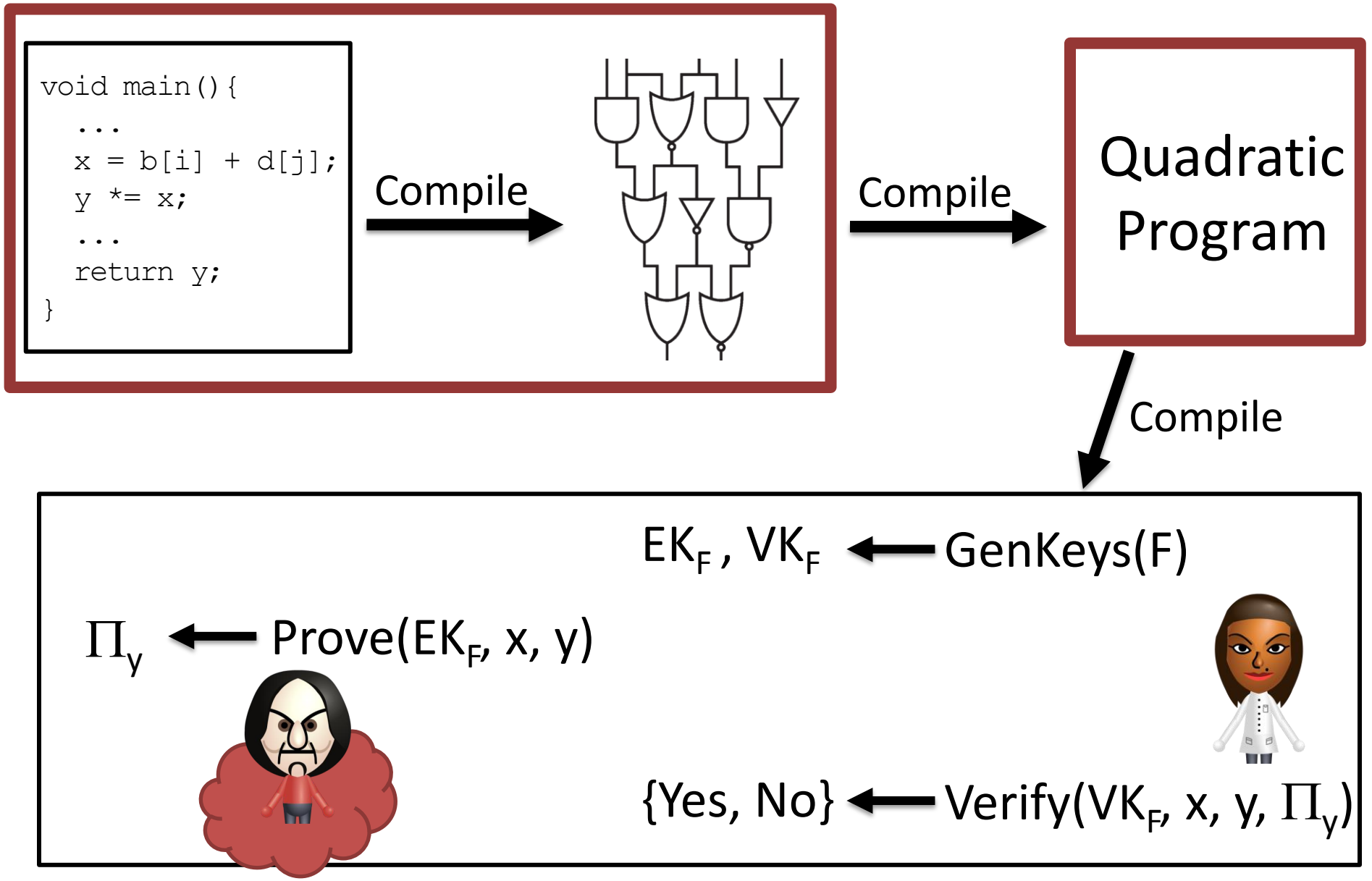
- Compiler understands a subset of C
  - Global, function, block-scoped variables
  - Arithmetic and bitwise operators
  - Functions, conditionals, bounded loops
  - Static initializers
  - Arrays, structs, pointers
  - Preprocessor syntax
- Outputs an *arithmetic* circuit with wire values $C_i \in \mathbb{F}_p$



Boolean	Arithmetic
And	A*B
Or	(A+B) – A*B
Not	1-A
Xor	(A+B) – 2 A*B



Pinocchio's Verification Pipeline



Quadratic Programs

[GGPR – EuroCrypt 2013]

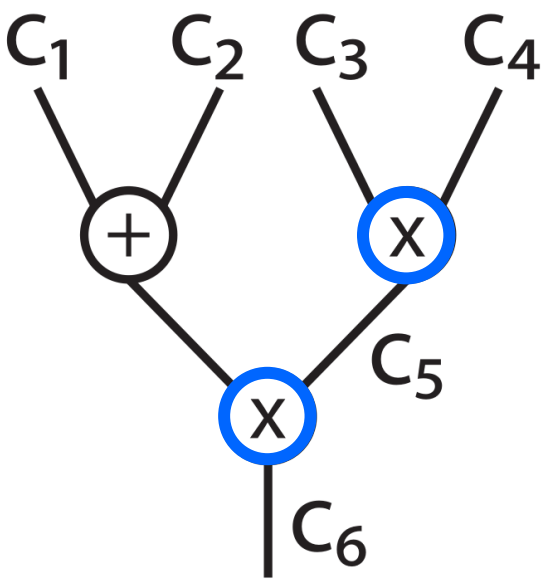
- An efficient encoding of computation
  - Lends itself well to cryptographic protocols

- Thm: Let C be an arithmetic circuit that computes F.
  There is a Quadratic Arithmetic Program (QAP)
  of size O(|C|) that computes F

  $\Rightarrow$ Can verify any poly-time (or even NP) function

- Related theorem for Boolean circuits and
  Quadratic Span Programs (QSPs)

Quadratic Arithmetic Program Intuition



$C_3 * C_4 == C_5$

$(C_1 + C_2) * C_5 == C_6$

⋮

Construct polynomials $D(z)$ and $P(z)$ that encode gate equations and wire values $\{C_i\}$

$(c_1, \ldots, c_m)$ is a valid set of wire values iff:

$$D(z) \text{ divides } P(z)$$

$$\equiv$$

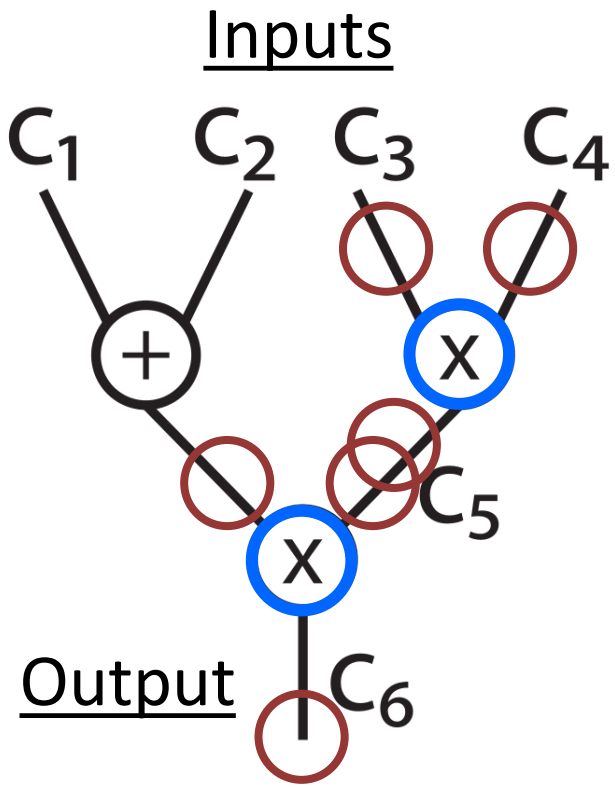
$$\exists\, H(z):\ H(z) \cdot D(z) == P(z)$$

$$\equiv$$

$$\forall\, r_i : D(r_i) == 0 \ \Rightarrow\ P(r_i) == 0$$

Crypto protocol checks divisibility at a random point, and hence cheaply checks correctness

Converting Arithmetic Circuit to QAPs



- Pick arbitrary root for each (x) : r_5 , r_6 from $\mathbb{F}$
- Define: $D(z) = (z - r_5)(z - r_6)$
- Define $P(z)$ via three *sets* of polynomials:
  $\{v_1(z), \ldots, v_m(z)\}$ $\{w_1(z), \ldots, w_m(z)\}$ $\{y_1(z), \ldots, y_m(z)\}$

	$z=r_5$	$z=r_6$
$v_1(z)$	0	1
$v_2(z)$	0	1
$v_3(z)$	1	0
$v_4(z)$	0	0
$v_5(z)$	0	0
$v_6(z)$	0	0

Left Inputs

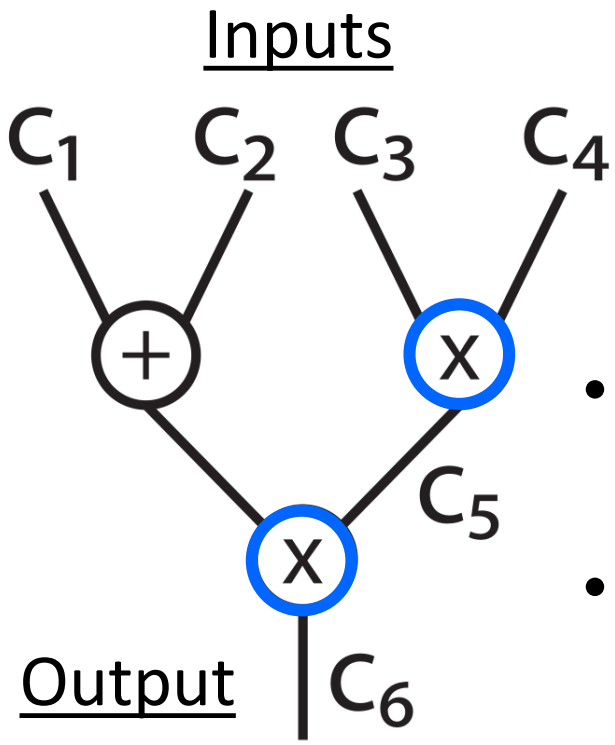
	r_5	r_6
$w_1(z)$	0	0
$w_2(z)$	0	0
$w_3(z)$	0	0
$w_4(z)$	1	0
$w_5(z)$	0	1
$w_6(z)$	0	0

Right Inputs

	r_5	r_6
$y_1(z)$	0	0
$y_2(z)$	0	0
$y_3(z)$	0	0
$y_4(z)$	0	0
$y_5(z)$	1	0
$y_6(z)$	0	1

Outputs

Why It Works



	x=r_5	x=r_6
$v_1(z)$	0	1
$v_2(z)$	0	1
$v_3(z)$	1	0
$v_4(z)$	0	0
$v_5(z)$	0	0
$v_6(z)$	0	0

	r_5	r_6
$w_1(z)$	0	0
$w_2(z)$	0	0
$w_3(z)$	0	0
$w_4(z)$	1	0
$w_5(z)$	0	1
$w_6(z)$	0	0

	r_5	r_6
$y_1(z)$	0	0
$y_2(z)$	0	0
$y_3(z)$	0	0
$y_4(z)$	0	0
$y_5(z)$	1	0
$y_6(z)$	0	1

- Define:

$$P(z) = (\sum c_i v_i(z))(\sum c_i w_i(z)) - (\sum c_i y_i(z))$$

- $D(z)$ divides $P(z)$ means:

$$\forall\, r_i : D(r_i) == 0 \;\Rightarrow\; P(r_i) == 0$$

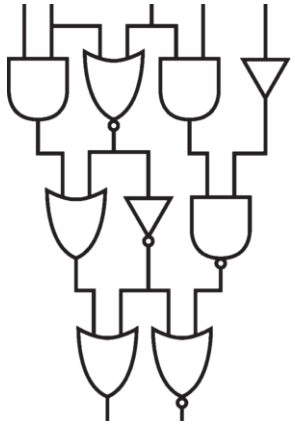
$$D(r_5) = 0 \quad P(r_5) = (c_3)(c_4) - (c_5)$$

$$D(r_6) = 0 \quad P(r_6) = (c_1 + c_2)(c_5) - (c_6)$$

Pinocchio's Verification Pipeline

```
void main(){
  ...
  x = b[i] + d[j];
  y *= x;
  ...
  return y;
}
```

Compile



Compile

Quadratic Program

$D(z)$

$v_1(z)...v_m(z)$

$w_1(z)...w_m(z)$

$y_1(z)...y_m(z)$

Compile

EK_F , VK_F ⟵ GenKeys(F)

Π_y ⟵ Prove(EK_F, x, y)

{Yes, No} ⟵ Verify(VK_F, x, y, Π_y)

Cryptographic Protocol (simplified)

GenKeys(F) $\longrightarrow$ EK_F , VK_F

Generate the QAP for F

Pick random s

Compute $EK_F = \{g^{v1(s)}, \ldots, g^{vm(s)}, g^{w1(s)}, \ldots, g^{wm(s)}, g^{y1(s)}, \ldots, g^{ym(s)}, g^{s\wedge i}\}$

Compute $VK_F = \{g^{D(s)}\}$

Prove(EK_F, x, y) $\longrightarrow$ Π_y

Evaluate circuit. Get wire values $c_1, \ldots, c_m$

Compute: $g^{v(s)} = \Pi\, (g^{v\_i(s)})^{c\_i}$,

$g^{w(s)} = \Pi\, (g^{w\_i(s)})^{c\_i}$

$g^{y(s)} = \Pi\, (g^{y\_i(s)})^{c\_i}$

Find H(z) s.t. H(z)*D(z) = V(z)*W(z)-Y(z)

Compute $g^{H(s)} = \Pi\, (g^{s\wedge i})^{h\_i}$

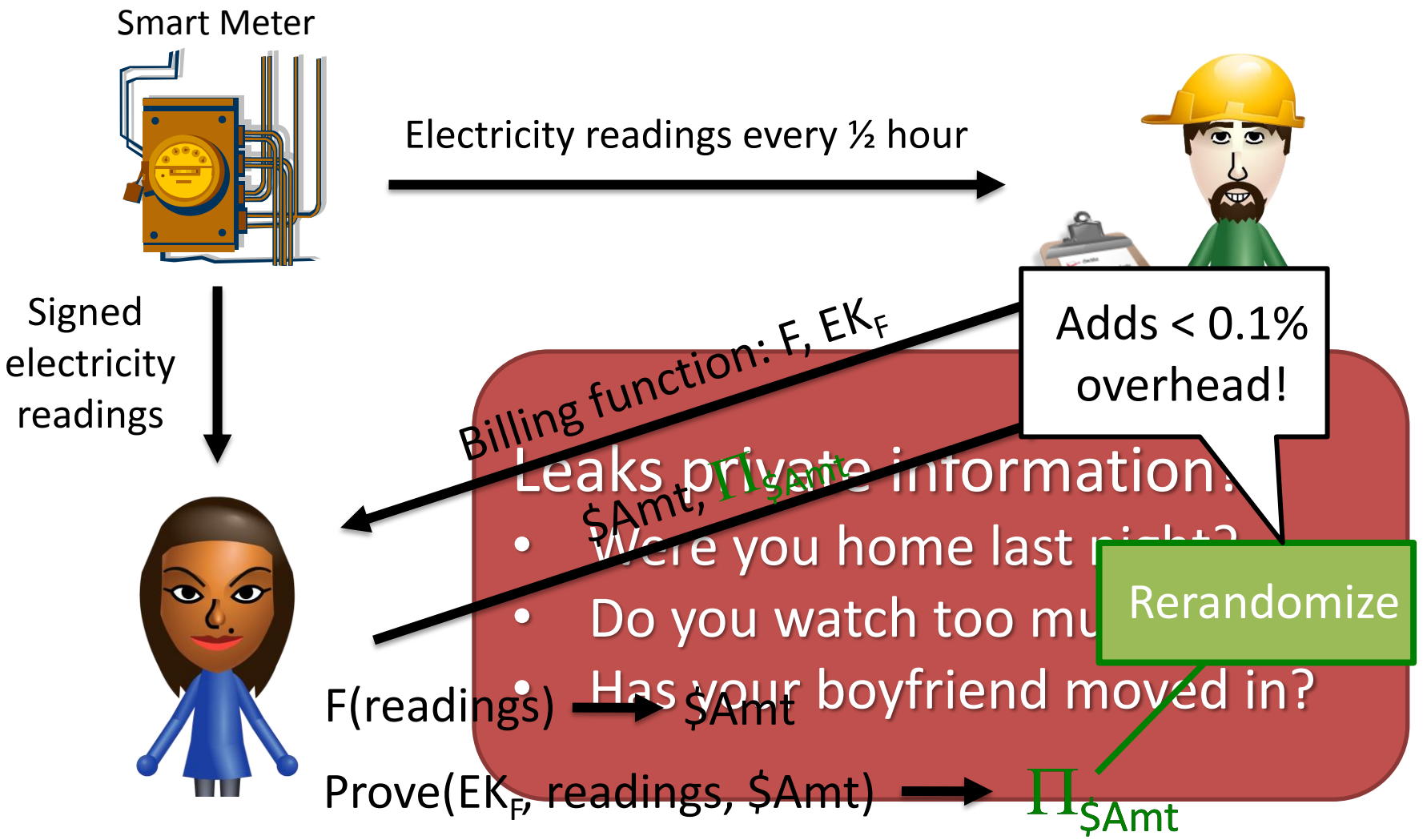
Proof is $(g^{v(s)}, g^{w(s)}, g^{y(s)}, g^{H(s)})$

Verify(VK_F, x, y, Π_y) $\longrightarrow$ {Yes, No}

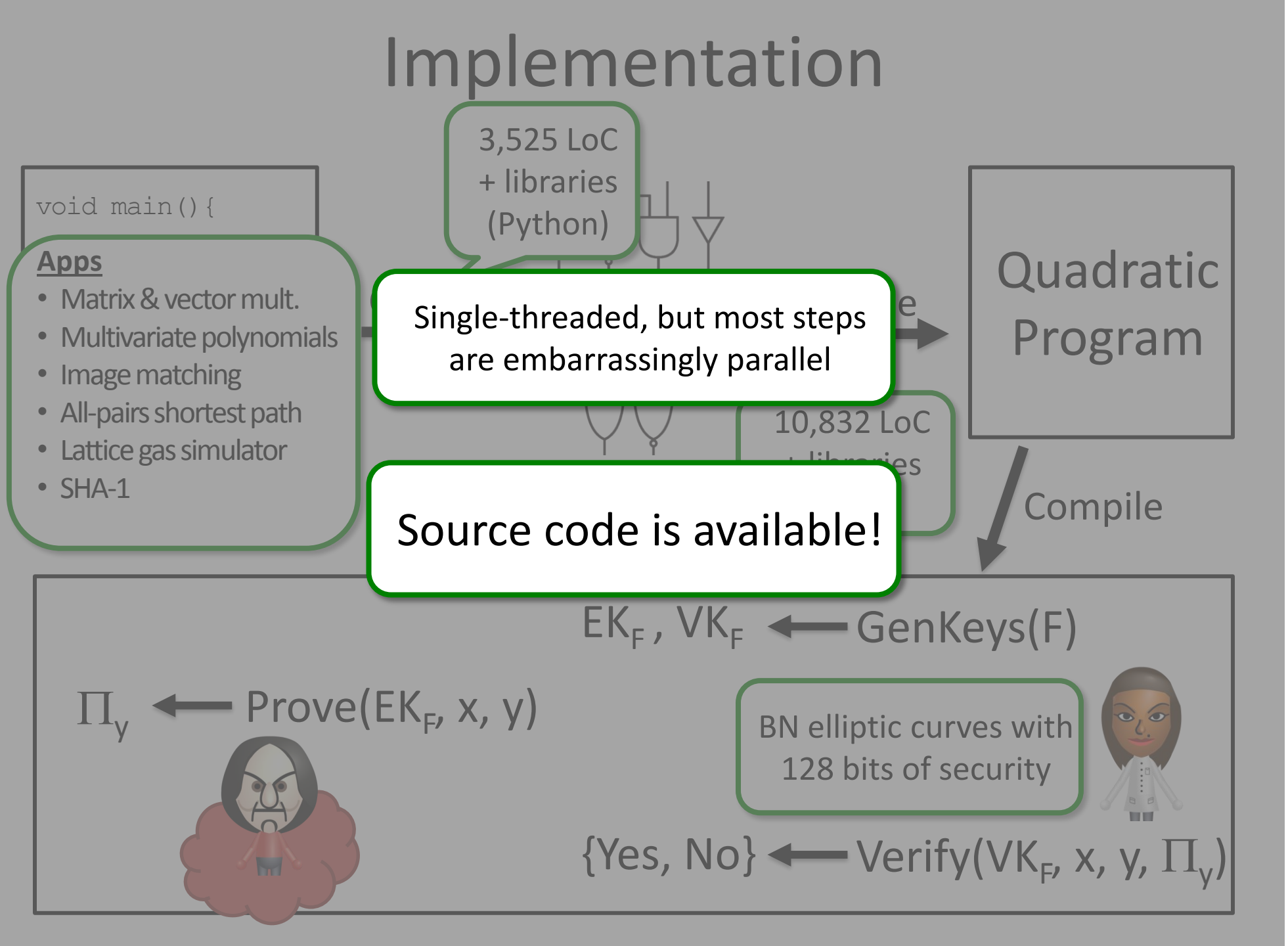
Check: $e(g^{v(s)}, g^{w(s)})/e(g^{y(s)}, g) =?= e(g^{h(s)}, g^{D(s)})$

e(·, ·) is a pairing:
$e(g^a, g^b) == e(g, g)^{ab}$

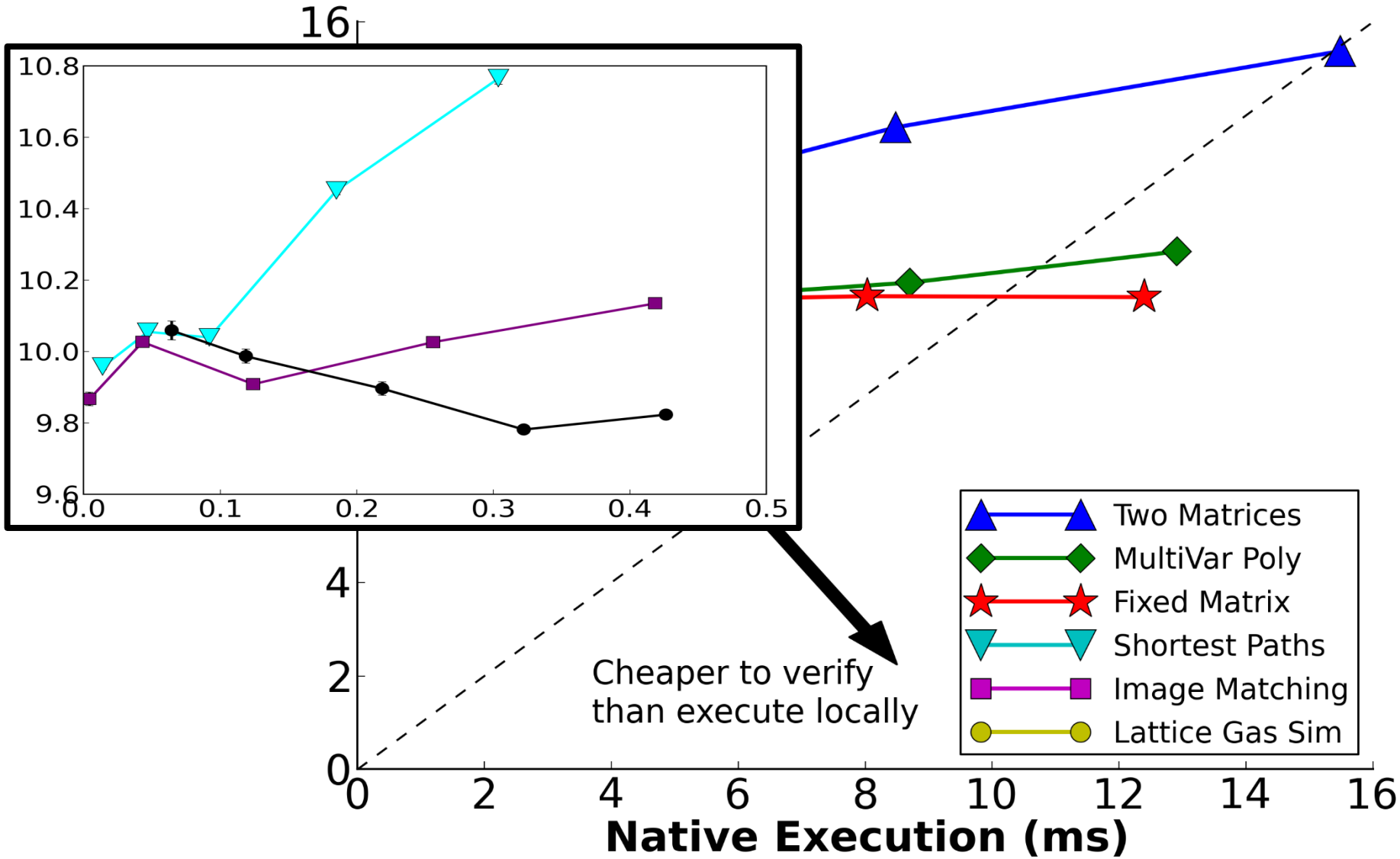
Making the Proof Zero Knowledge



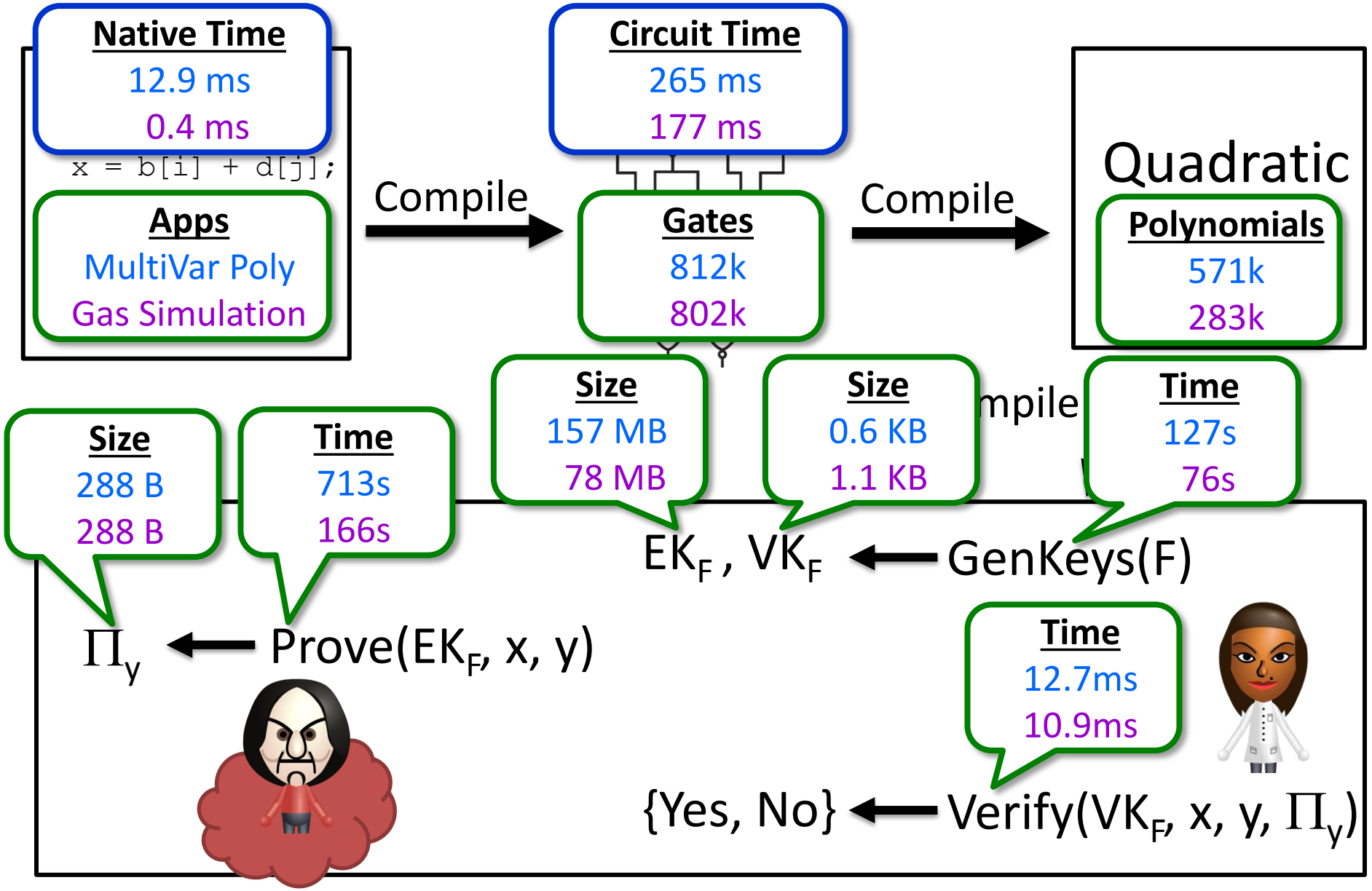
Implementation



Verification Time vs. Native Execution



Detailed Metrics



Future Work

- New cryptographic techniques to move beyond the circuit model
- Efficient computations over signed data
- Additional applications of quadratic programs, e.g., general attribute-based encryption

Summary

- Quadratic programs are a new tool for efficient public verifiable computation (and zero-knowledge proofs)
- Pinocchio compiles C programs into verifiable computation protocols
- Performance improved by orders of magnitude – verification now beats native execution (for some apps)

Thank you!

parno@microsoft.com