Recursive Program Synthesis

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Program Synthesis

Program Synthesis

Synthesis: Dreams $\Longrightarrow$ Programs

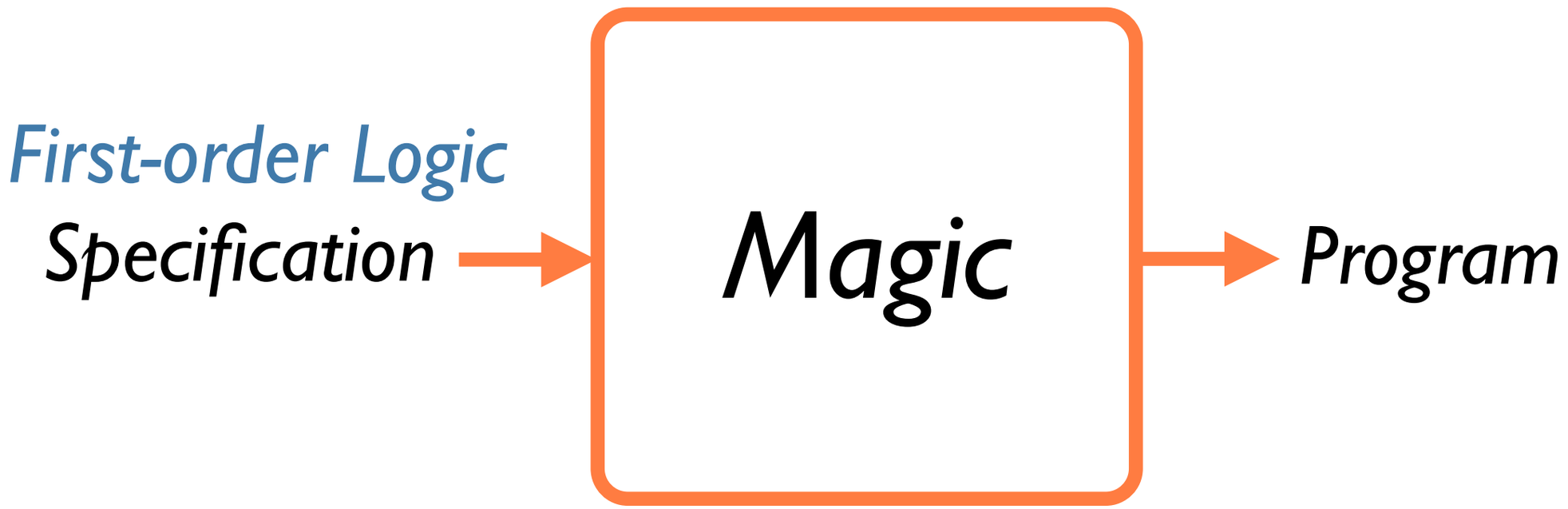
ZOHAR MANNA AND RICHARD WALDINGER

What are your Dreams?

Synthesis: Dreams $\Longrightarrow$ Programs

ZOHAR MANNA AND RICHARD WALDINGER

What are your Dreams?



Synthesis: Dreams $\Longrightarrow$ Programs

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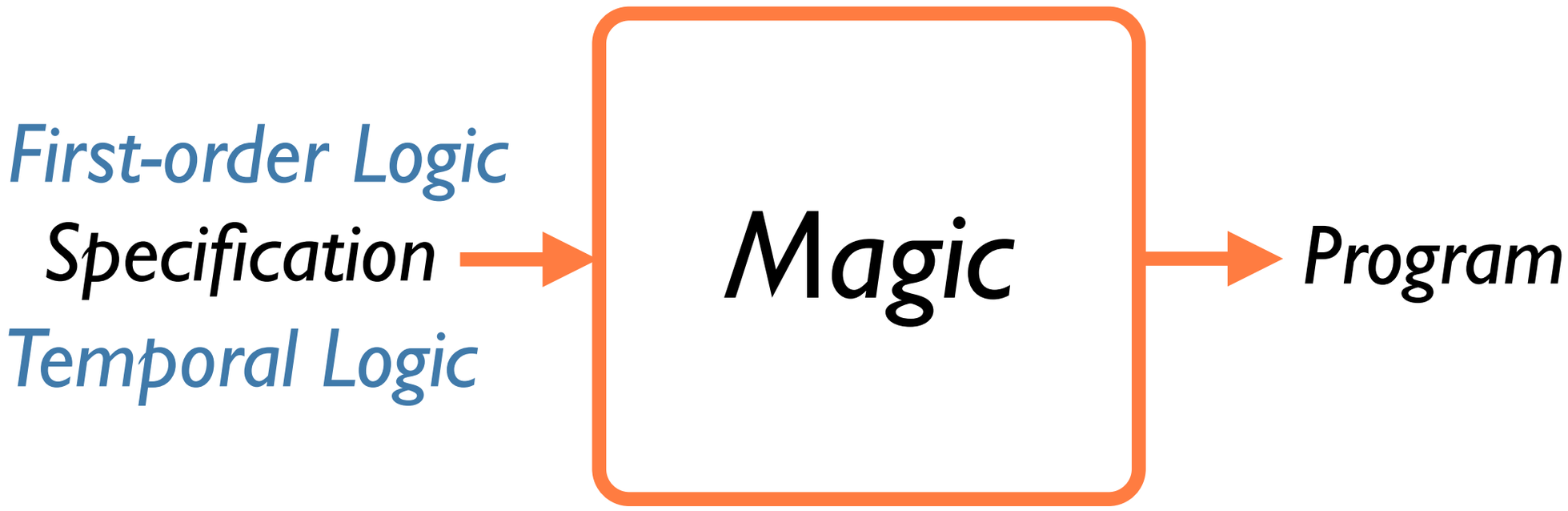

Synthesis: Dreams $\Longrightarrow$ Programs

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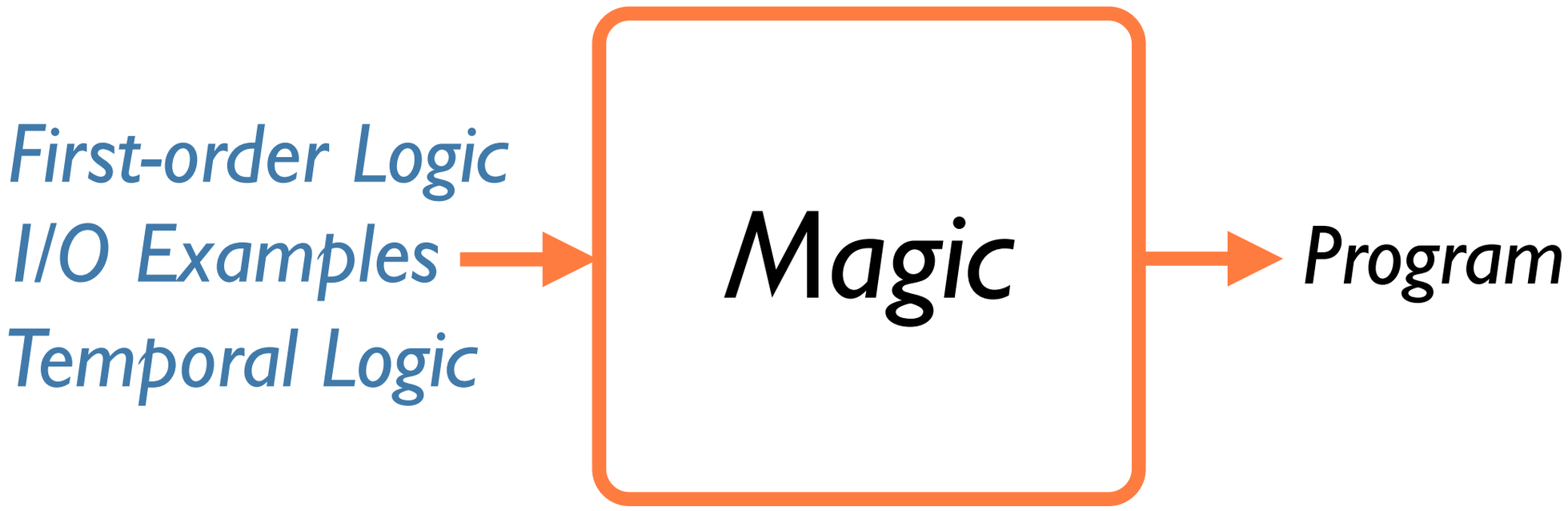

294 IEEE TRANSACTIONS ON SOFTWARE ENGINEERING, VOL. SE-5, NO. 4, JULY 1979

Synthesis: Dreams $\Longrightarrow$ Programs

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Why Synthesis from I/O?

I/O examples are *easy to specify*

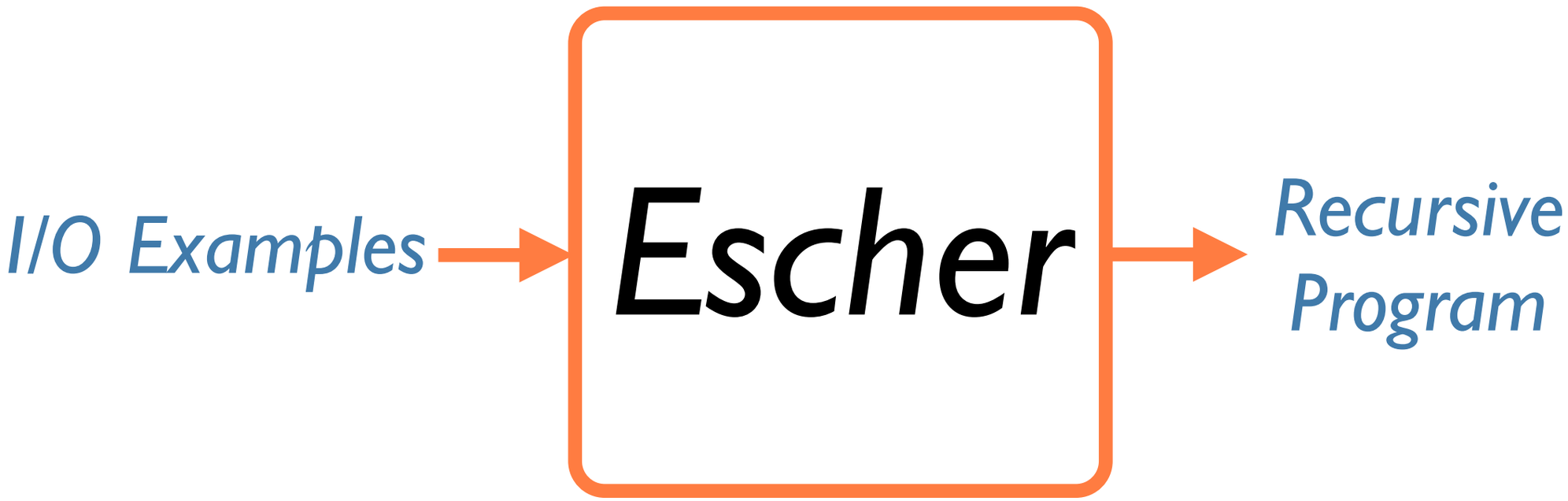
Why Synthesis from I/O?

I/O examples are *easy to specify*

Non-expert users can specify I/O behaviour

- *See, e.g., FlashFill for Excel, Smartphone scripts, etc.*
- *Desired programs are usually simple*

Contribution



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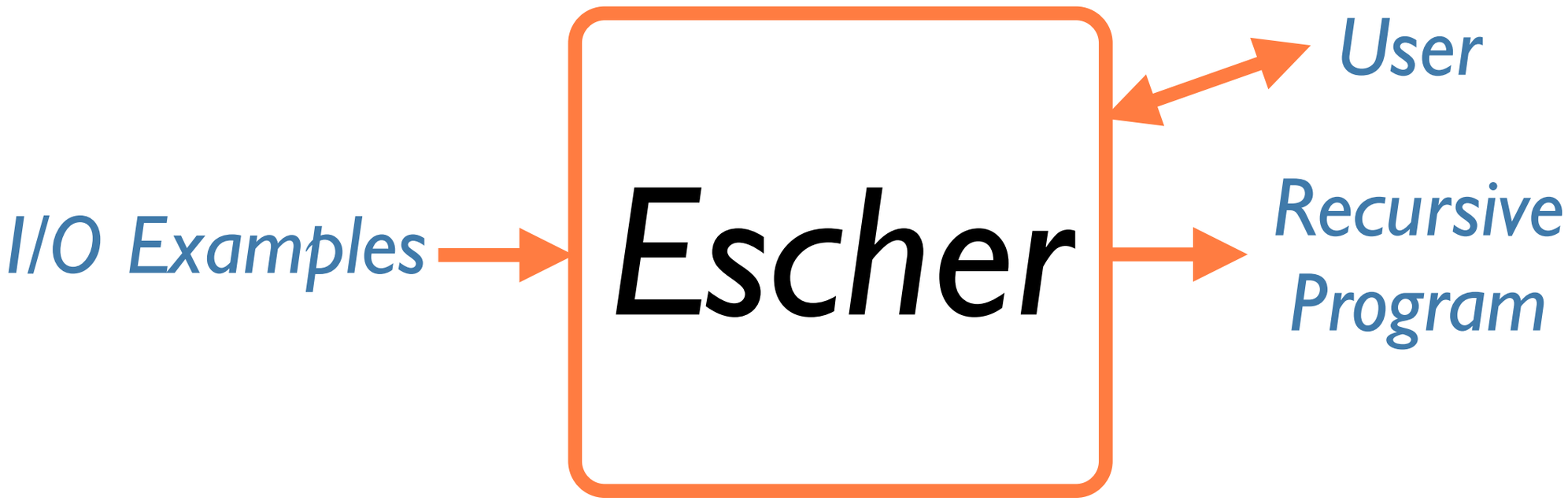

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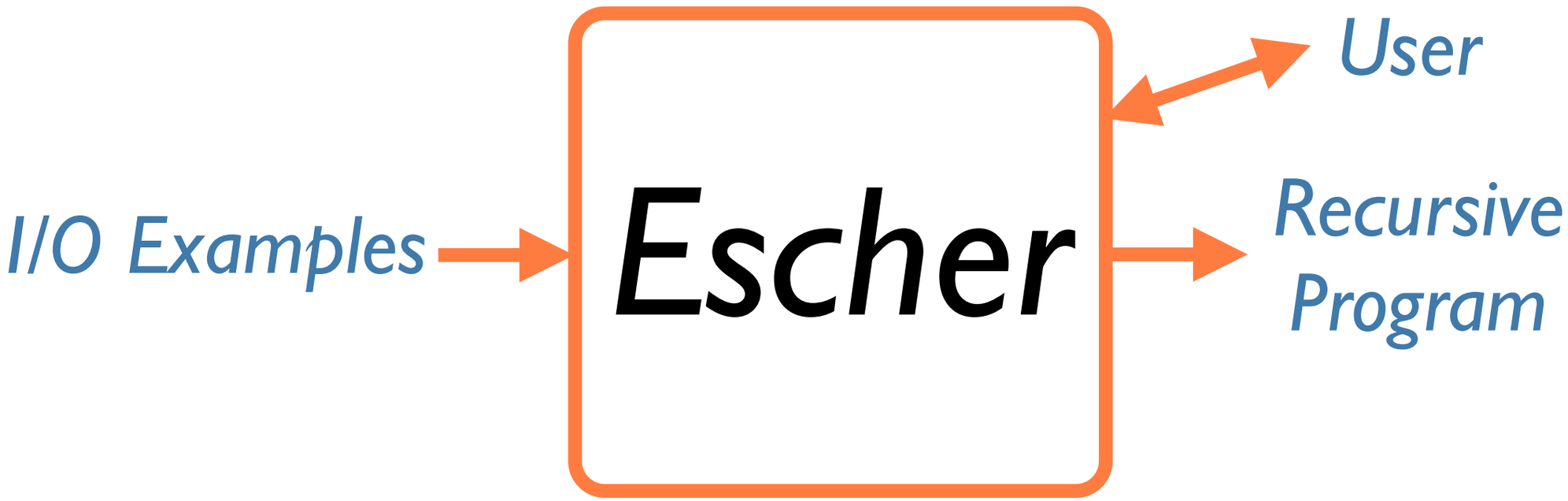

Parameterized by a set of building blocks (operations)

integers, lists, trees, etc.

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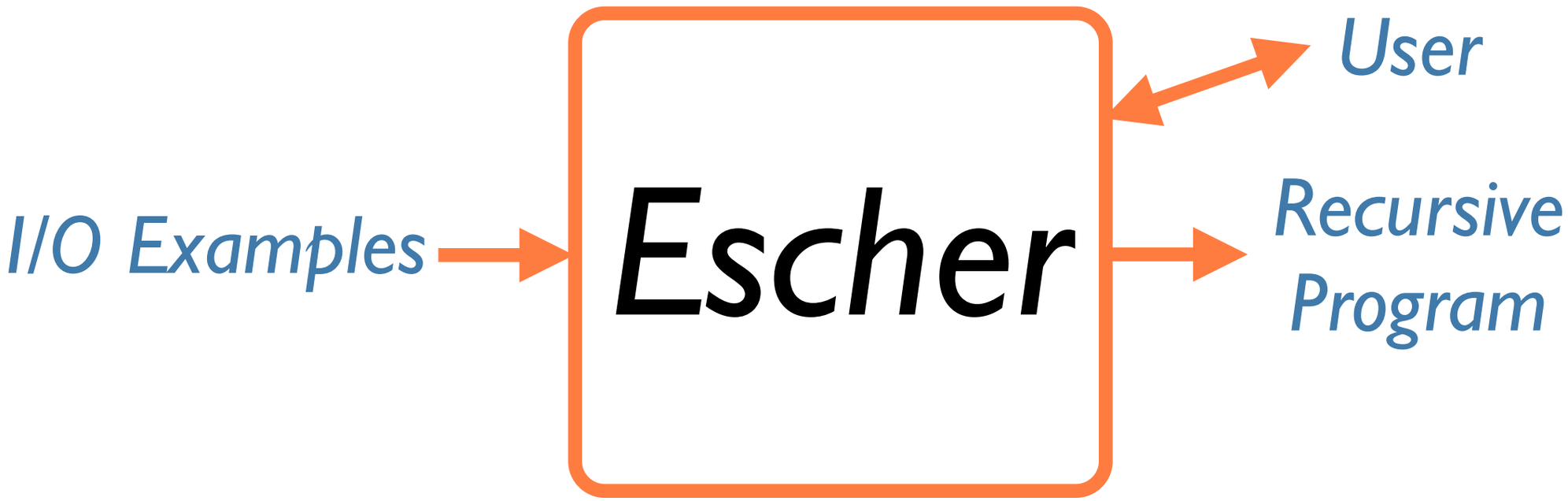

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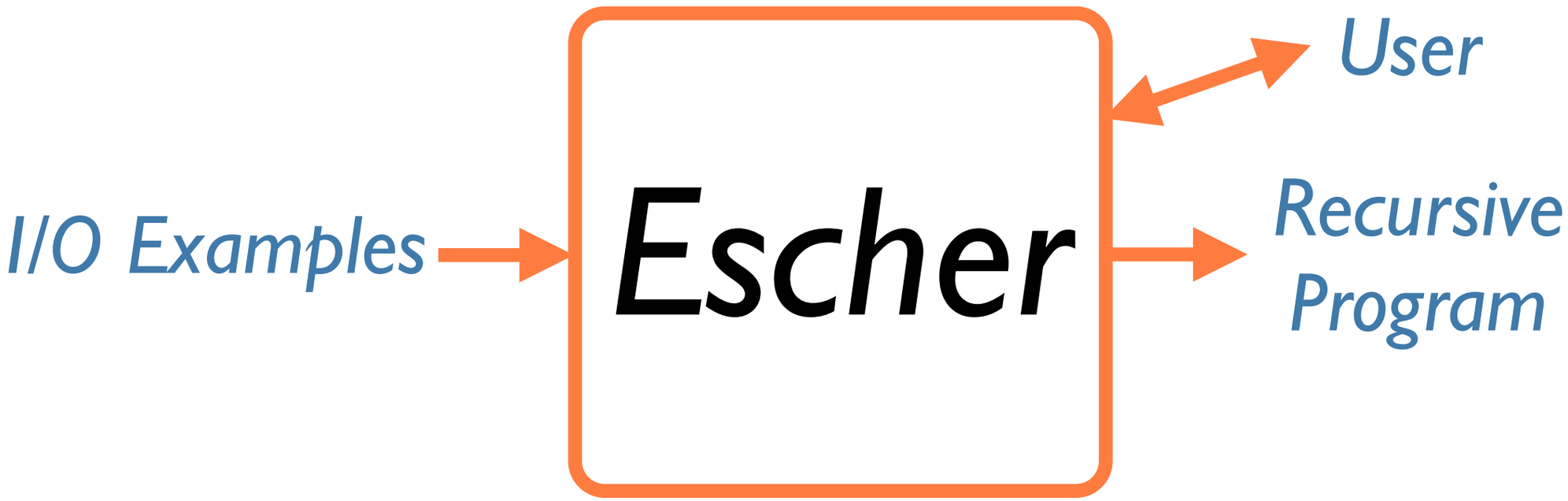
Novel search-based synthesis technique

Contribution



Parameterized by a set of building blocks (operations)

integers, lists, trees, etc.

Novel search-based synthesis technique

No templates required

High Level View

Forward Search

High Level View

Forward Search

$$I_1 \cdots I_n$$

High Level View

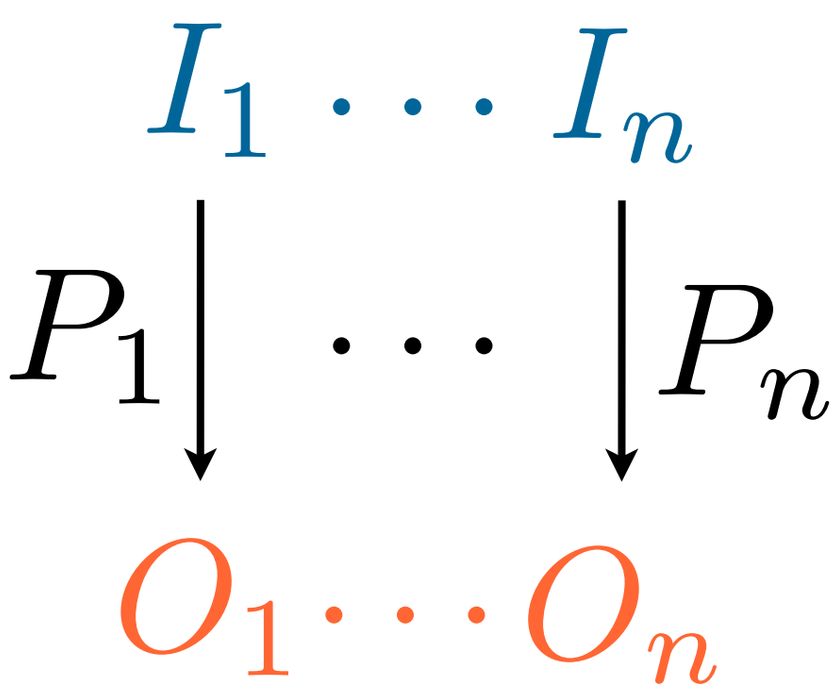
Forward Search

$$I_1 \cdots I_n$$

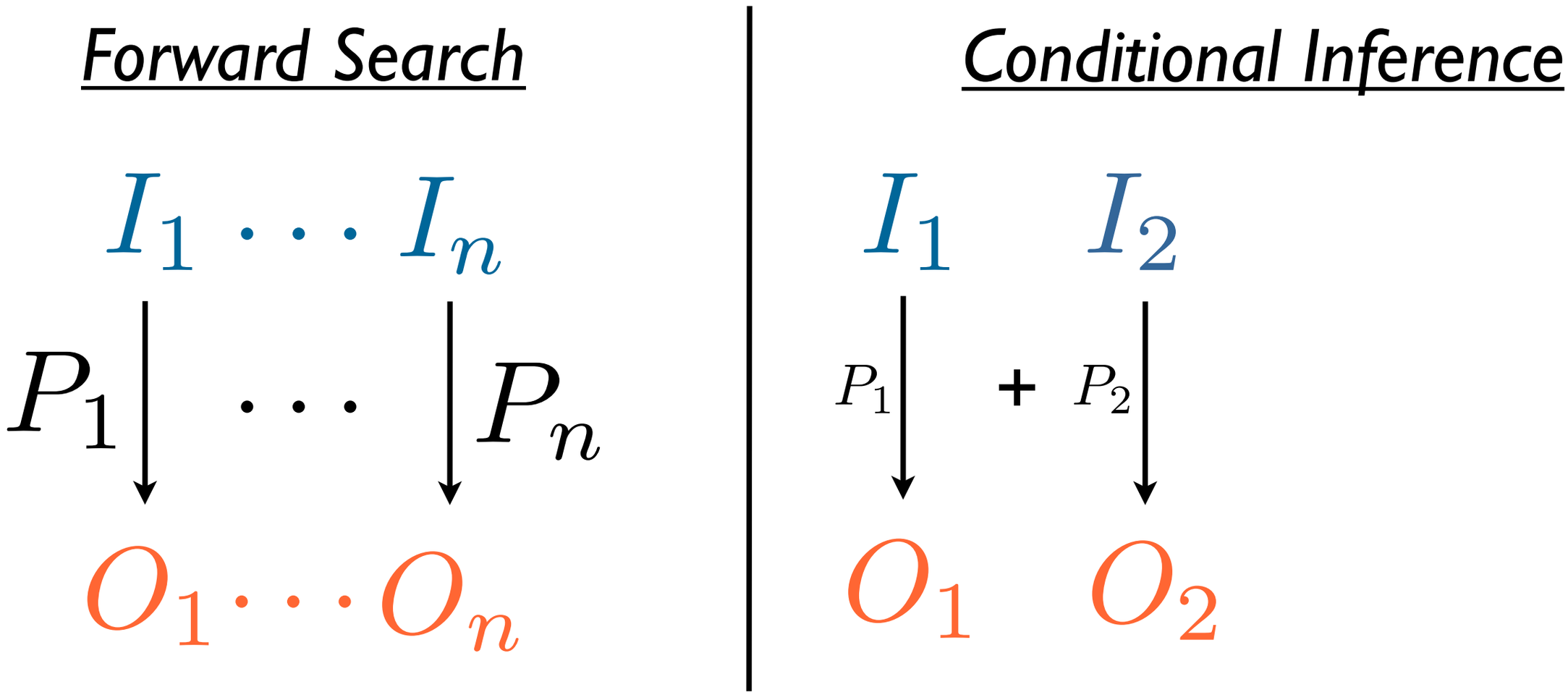
$$O_1 \cdots O_n$$

High Level View

Forward Search



High Level View



High Level View

Forward Search

$$I_1 \cdots I_n$$

$$P_1 \Big| \quad \cdots \quad \Big| P_n$$

$$O_1 \cdots O_n$$

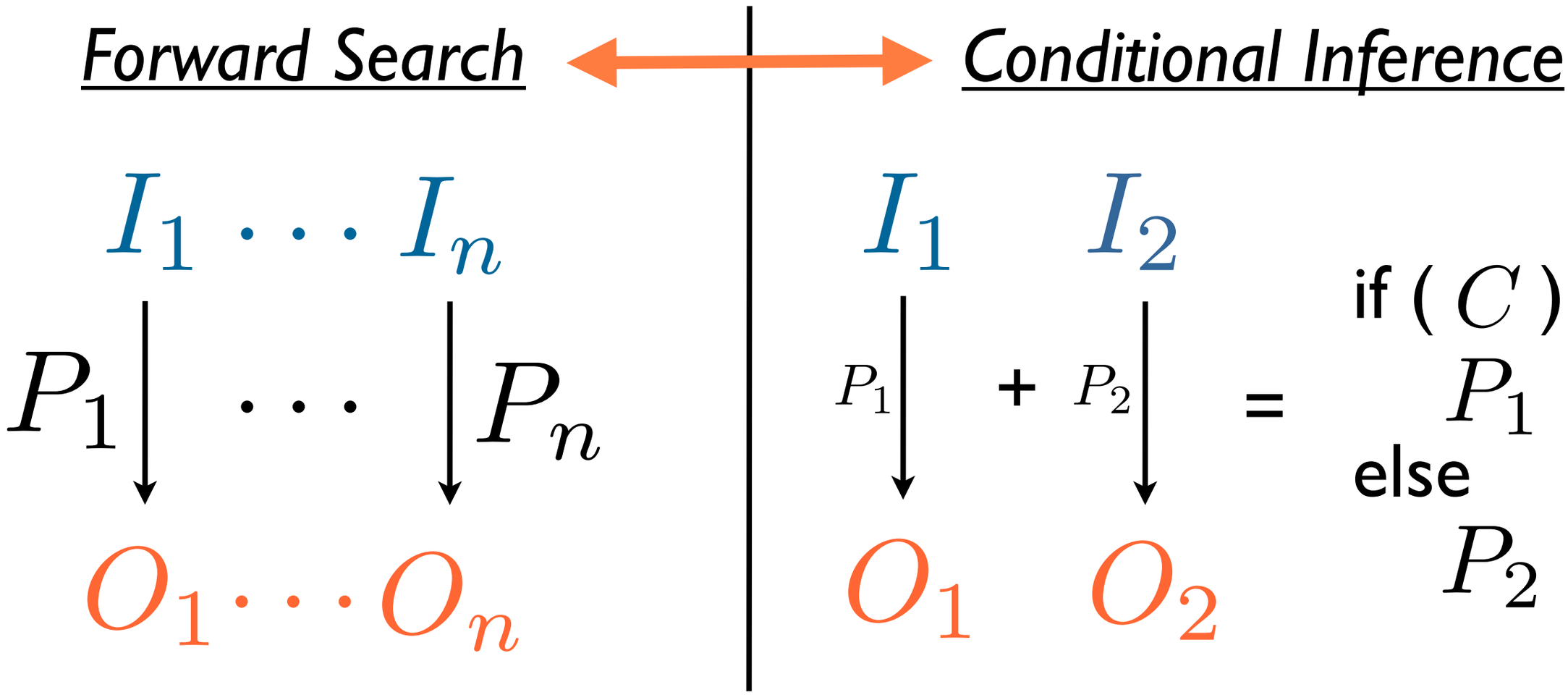
Conditional Inference

$$I_1 \quad I_2$$

$$P_1 \Big| \quad + \quad P_2 \Big| \quad = \quad \begin{array}{l} \text{if } (C) \\ \quad P_1 \\ \text{else} \\ \quad P_2 \end{array}$$

$$O_1 \quad O_2$$

High Level View



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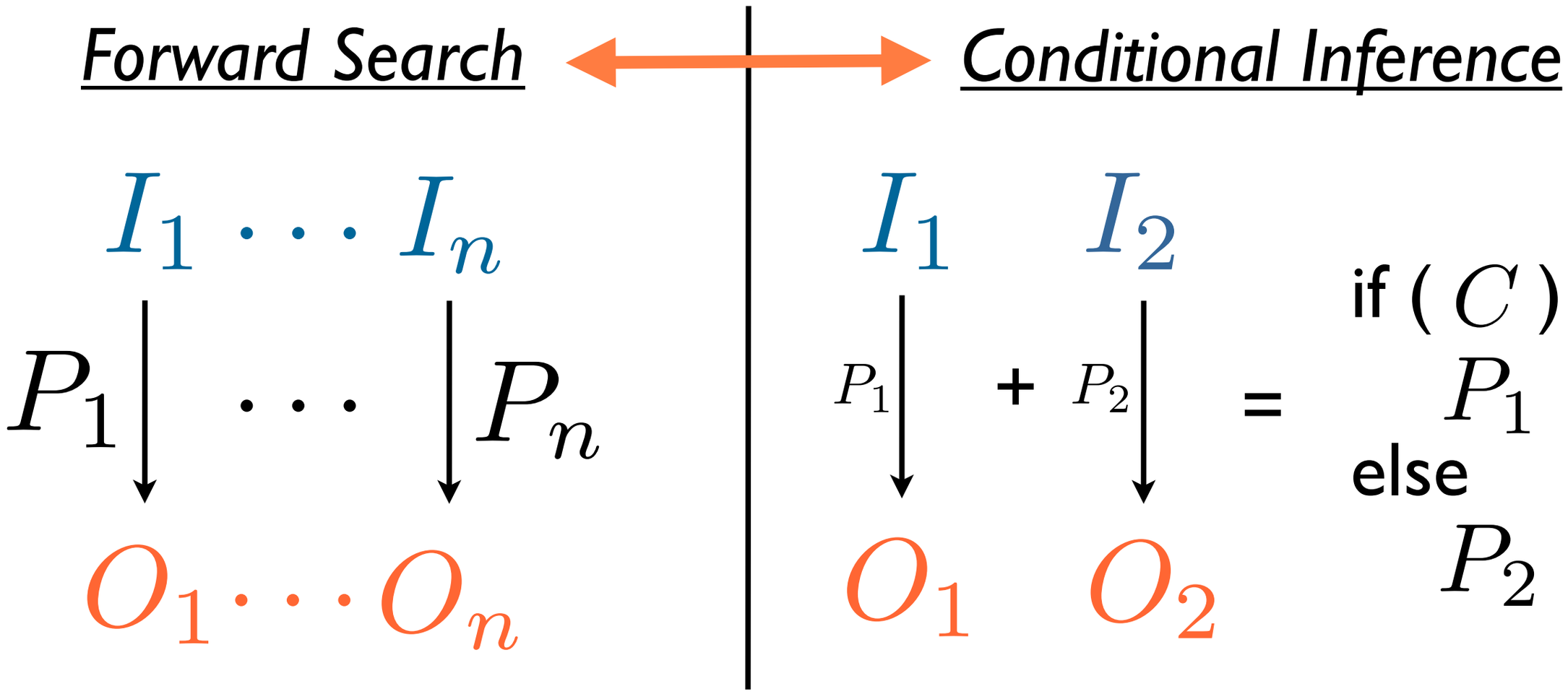


Recursive call synthesis

Reuse I/O as recursive call specification or query user

Example

Synthesize list length from examples:

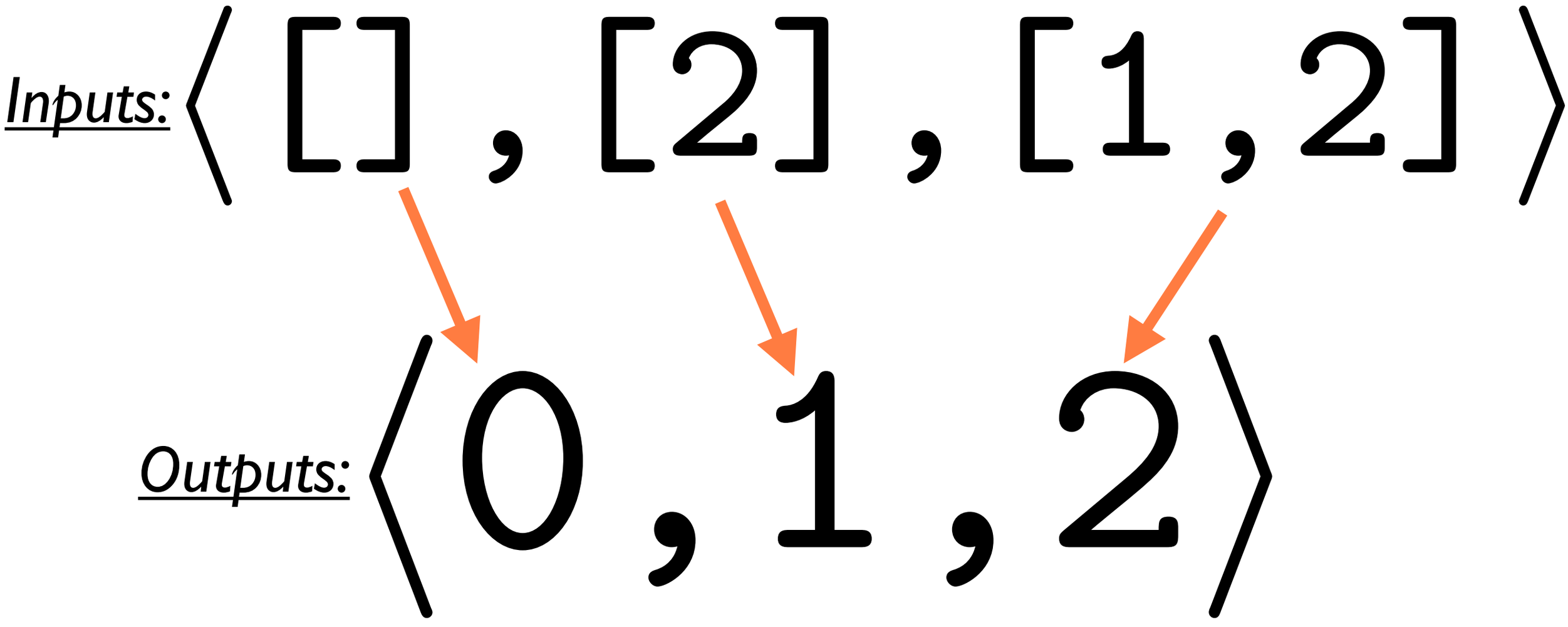
Example

Synthesize list length from examples:

Inputs: ⟨ [] , [2] , [1, 2] ⟩

Example

Synthesize list length from examples:



Ex: Forward Search

Components: inc, isEmpty, tail, zero

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Programs of size 1:

Ex: Forward Search

Components: inc, isEmpty, tail, zero

Programs of size 1:

$$\mathtt{P}_1 = \mathtt{i} \rightarrow \langle \mathtt{[]}, \mathtt{[2]}, \mathtt{[1,2]} \rangle$$

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$$\mathrm{P}_1 = \mathtt{i} \rightarrow \langle [], [2], [1,2] \rangle$$

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matches output for first input

Recall outputs $\langle 0, 1, 2 \rangle$

Ex: Conditional Inference

$$\mathrm{P}_2 = \texttt{zero} \rightarrow \langle 0,0,0 \rangle$$

Outputs: $\langle 0,1,2 \rangle$

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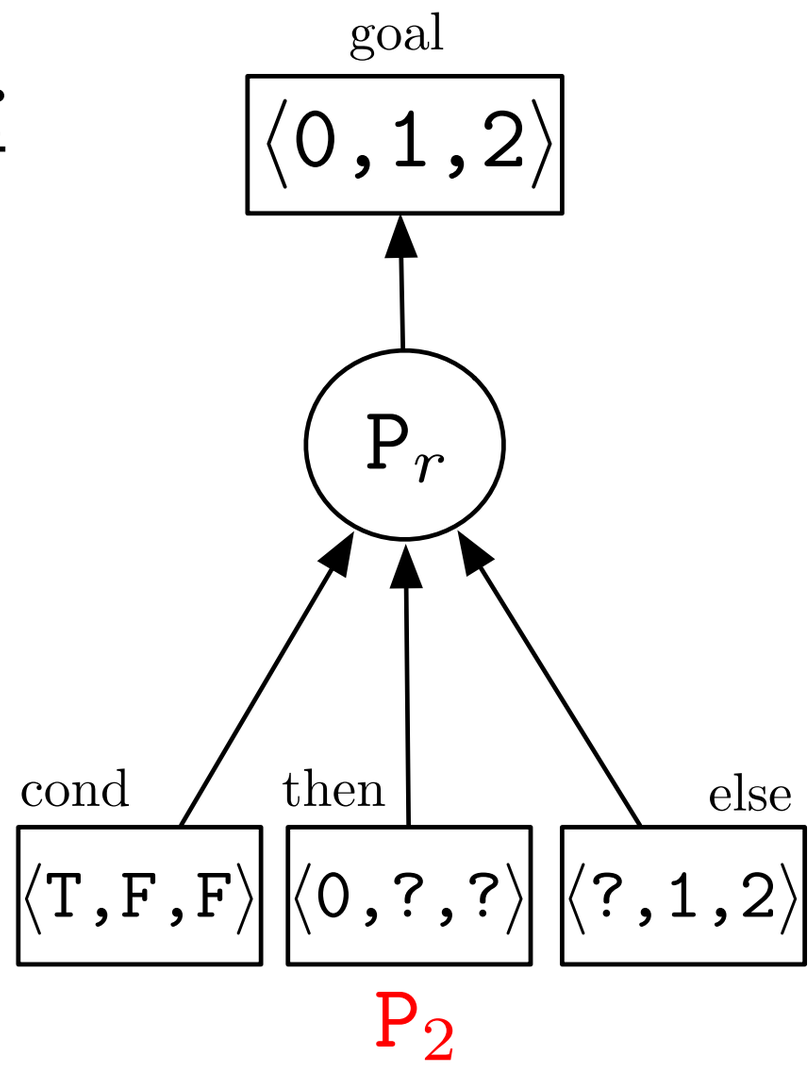
Goal Graph (GG):



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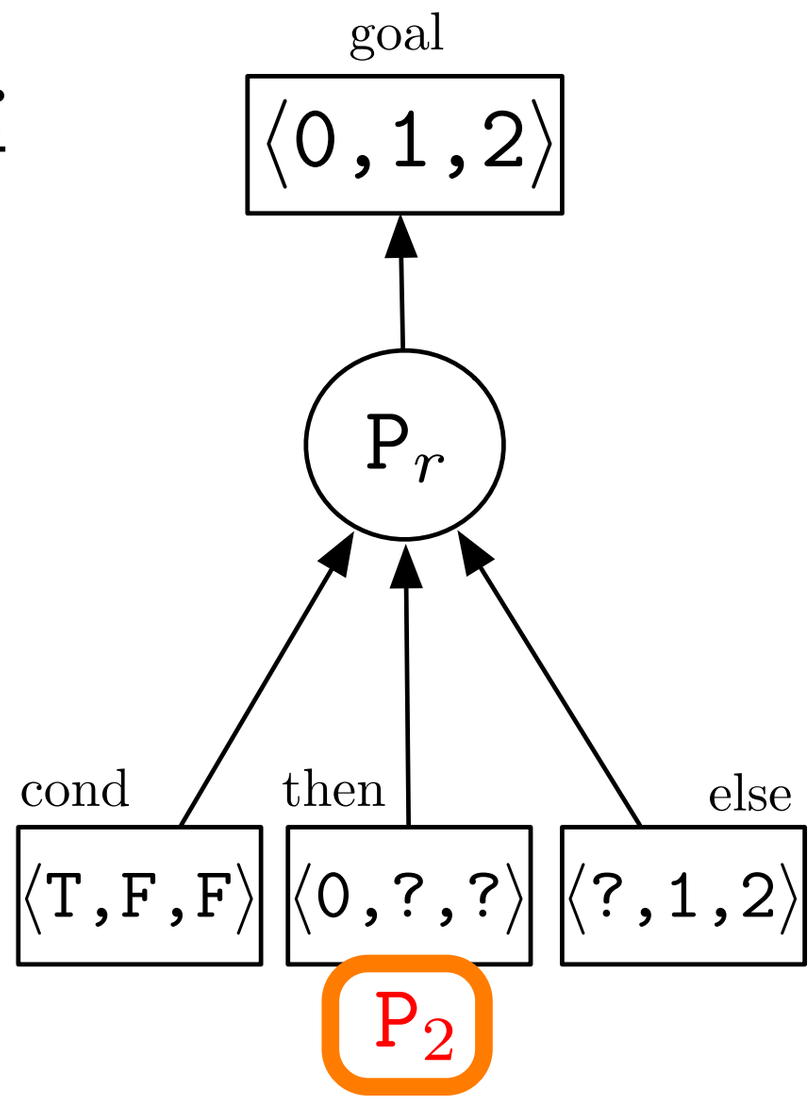

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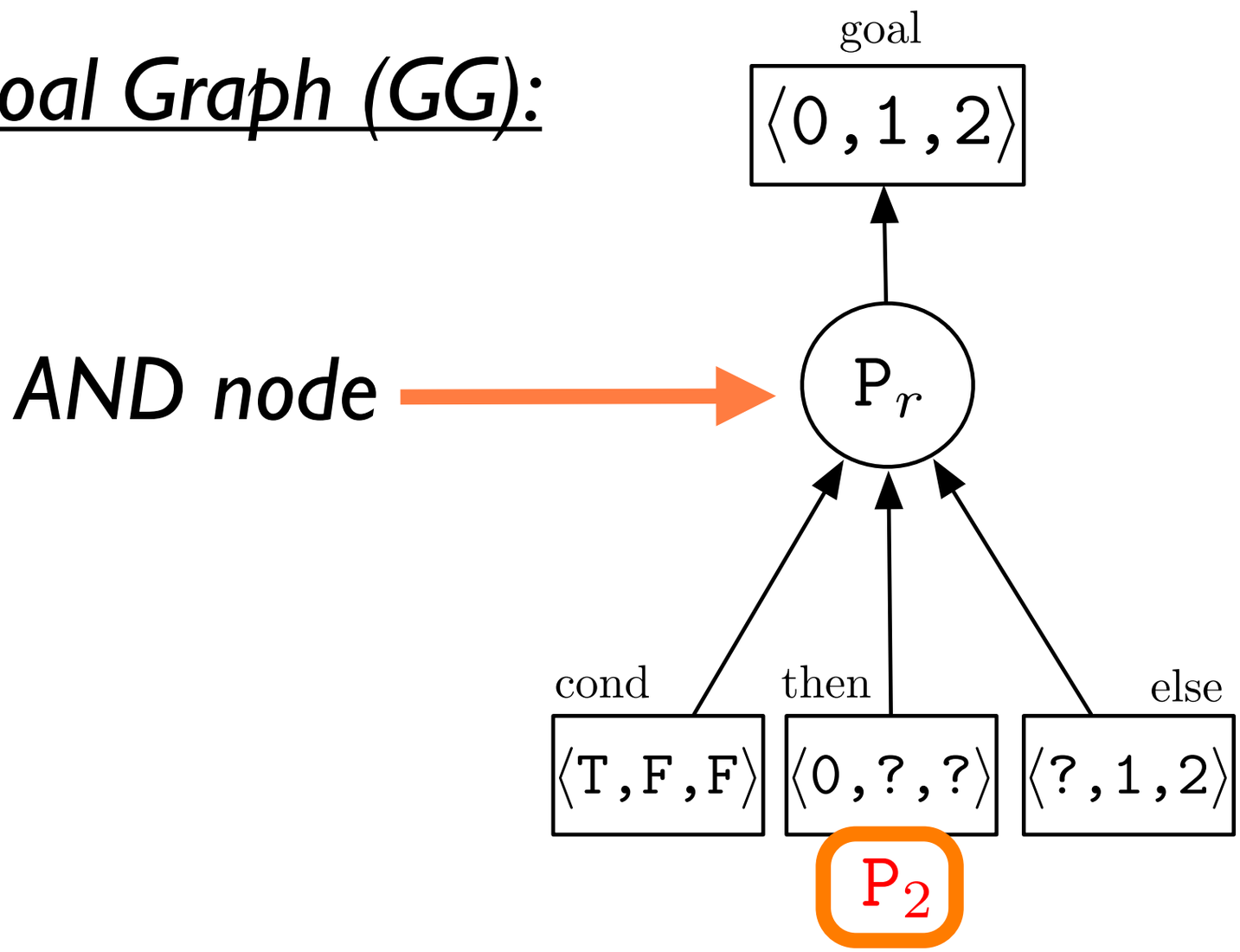

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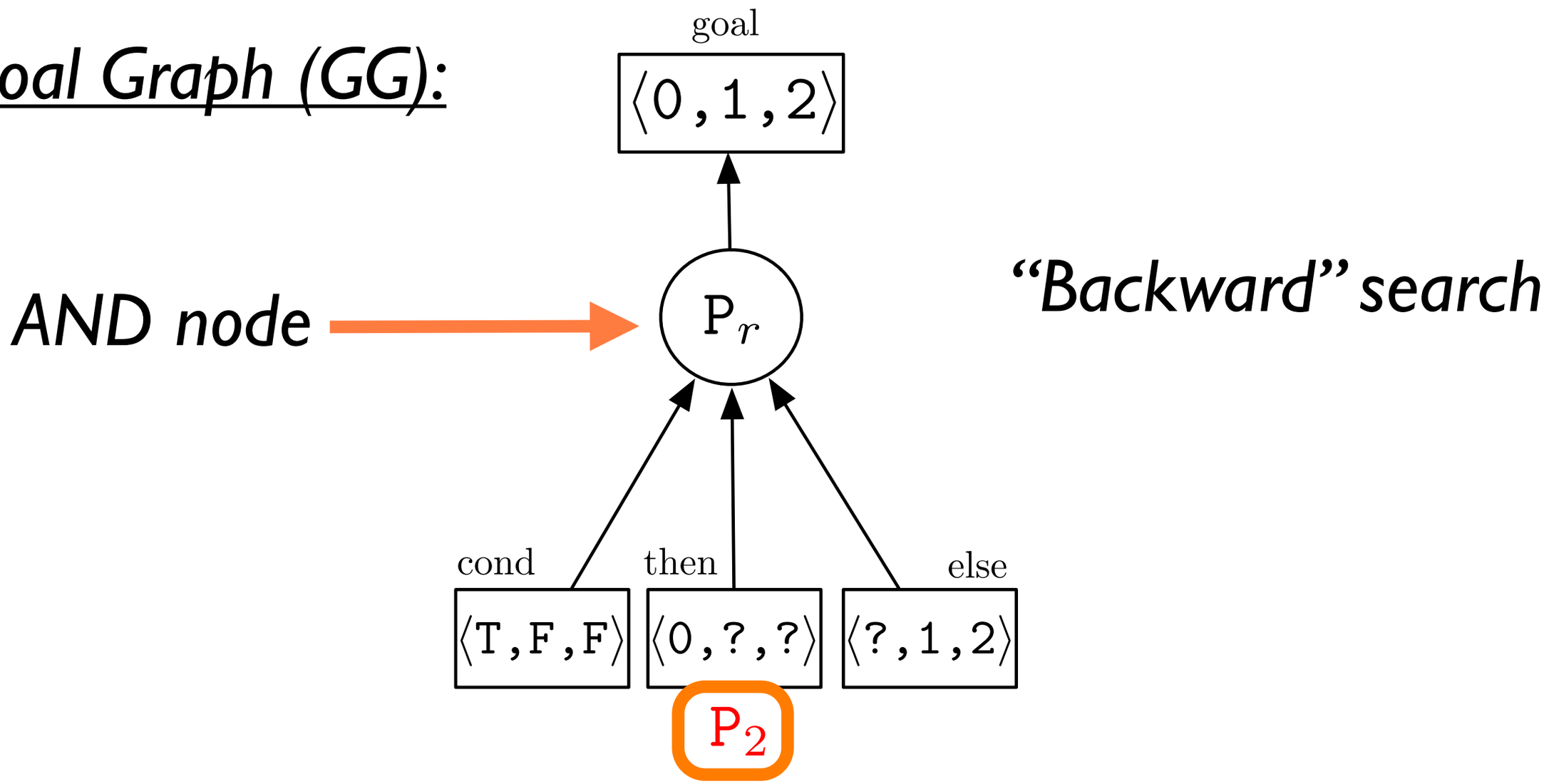

AND node

"Backward" search

Ex: Forward Search 2

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Satisfies one of our goals

Ex: Conditional Inference 2

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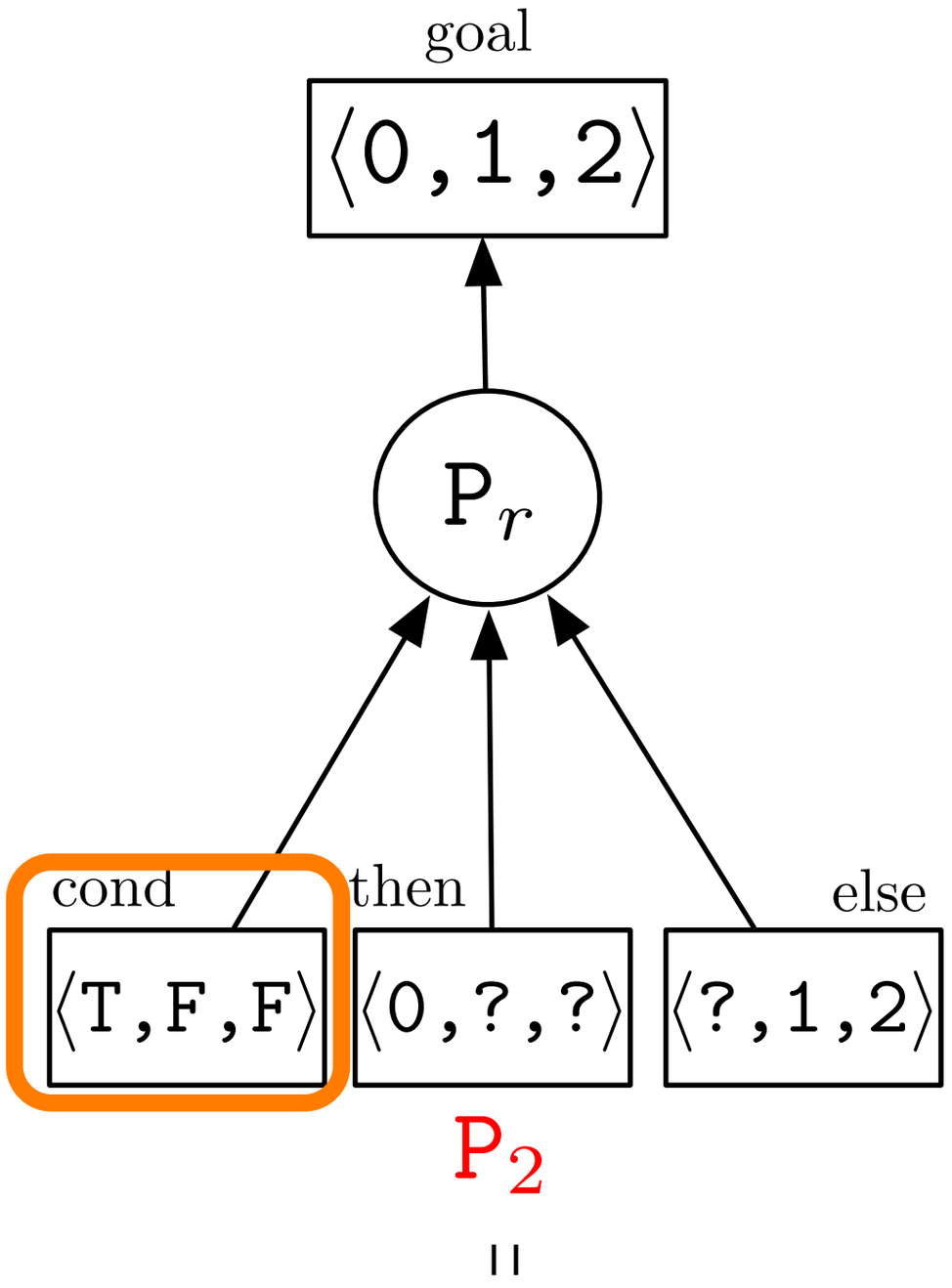

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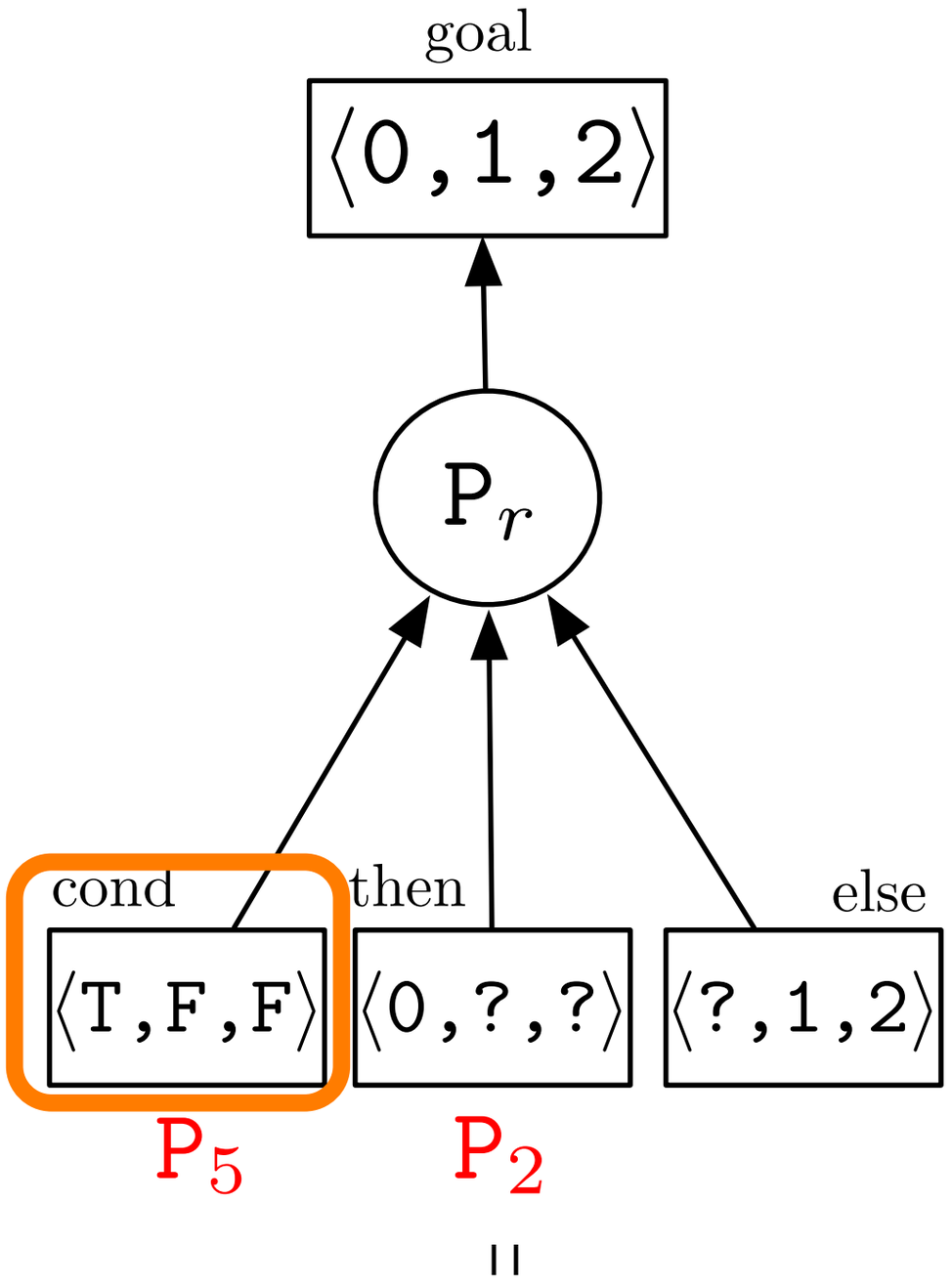

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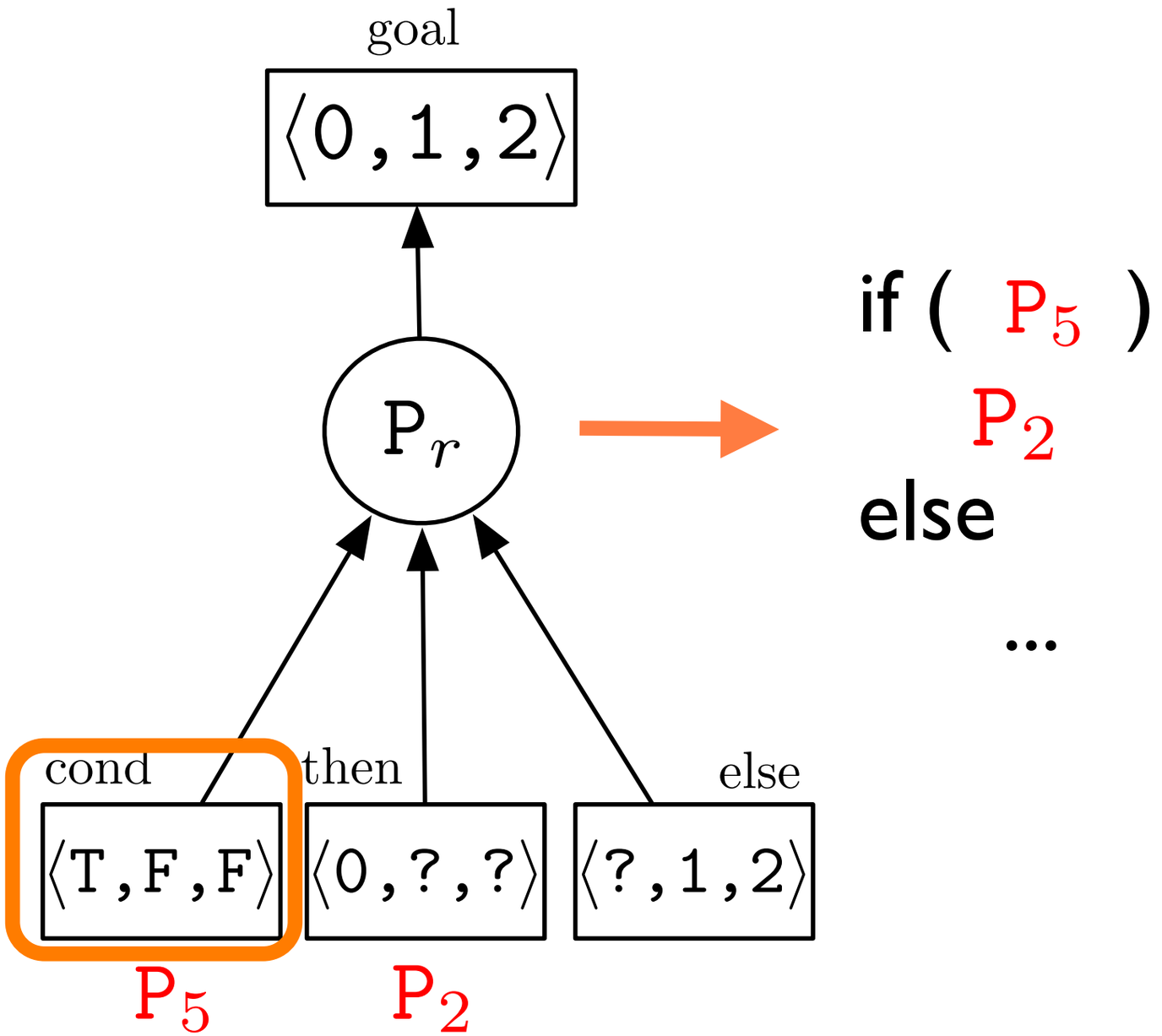

Ex: Forward Search 3

Programs of size 2

$$\mathtt{P}_3 = \mathtt{tail}(\mathtt{P}_1) \rightarrow \langle \mathtt{err}, [], [2] \rangle$$

...

Ex: Forward Search 3

Programs of size 2

$$\mathtt{P}_3 = \mathtt{tail}(\mathtt{P}_1) \rightarrow \langle \mathtt{err}, [], [2] \rangle$$

...

Programs of size 3

$$\mathtt{P}_6 = \mathtt{length}(\mathtt{P}_3) \rightarrow \langle \mathtt{err}, 0, 1 \rangle$$

Ex: Forward Search 3

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Use I/O values to simulate recursive call

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Use I/O values to simulate recursive call

Only allow calls satisfying well-founded relation

Ex: Forward Search 3

Programs of size 2

$P_3 =$ ⟩

...

Programs o

P_6 = 0, 1⟩

But what if there isn't such an I/O pair?

Query the user for new examples!

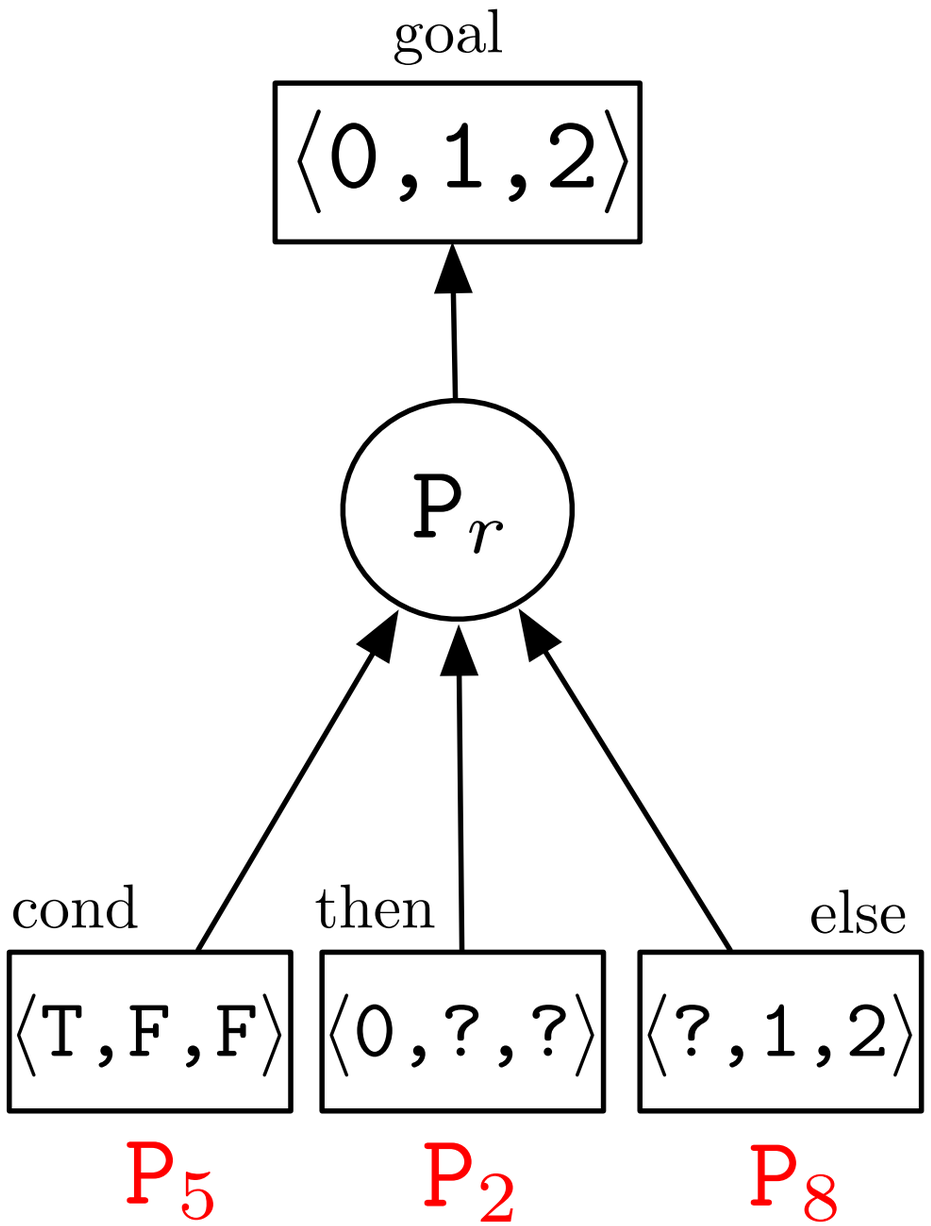
(too long for 15 min talk)

Use I/O

Only allow

Ex: Conditional Inference

One iteration later:



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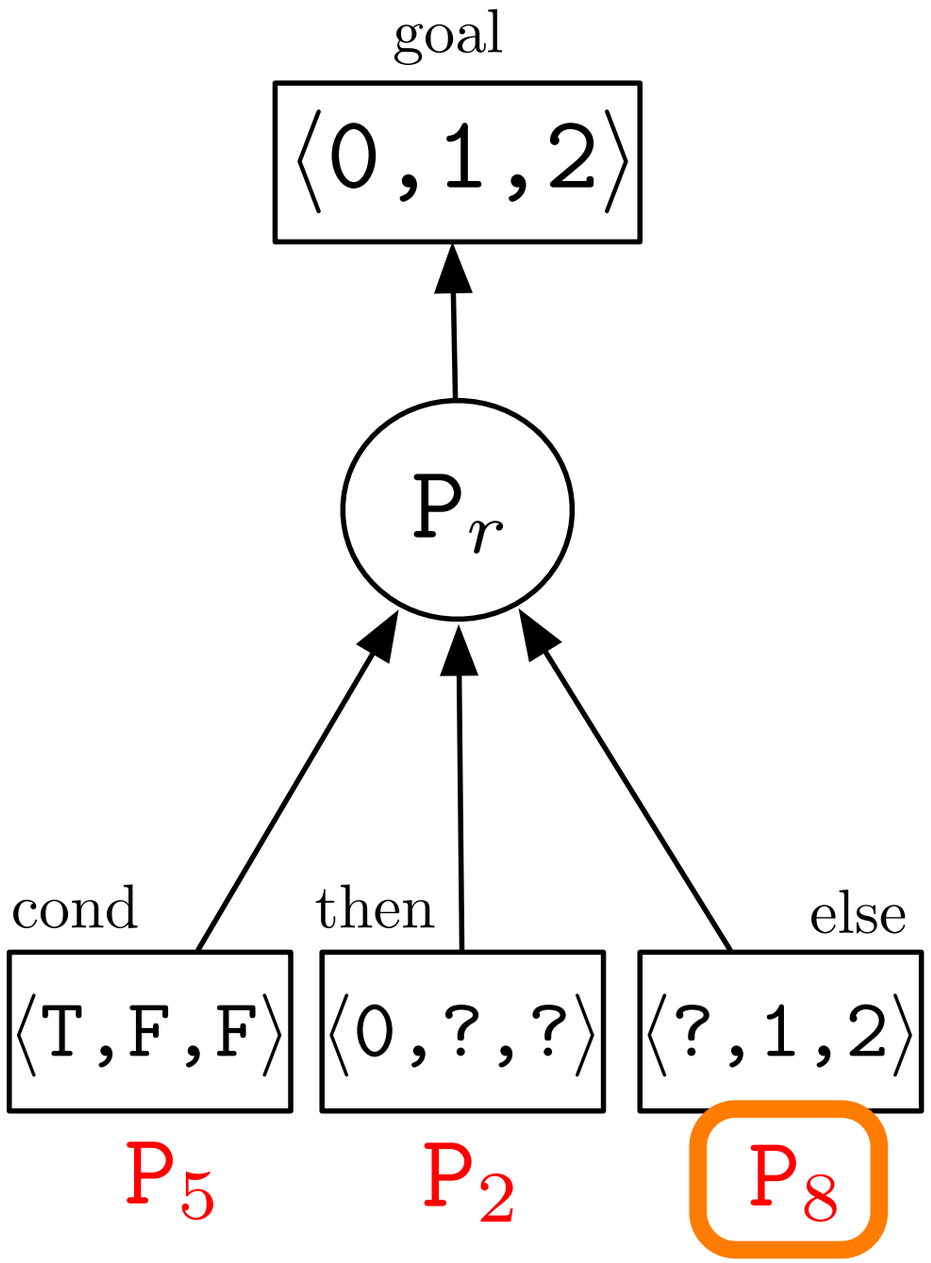

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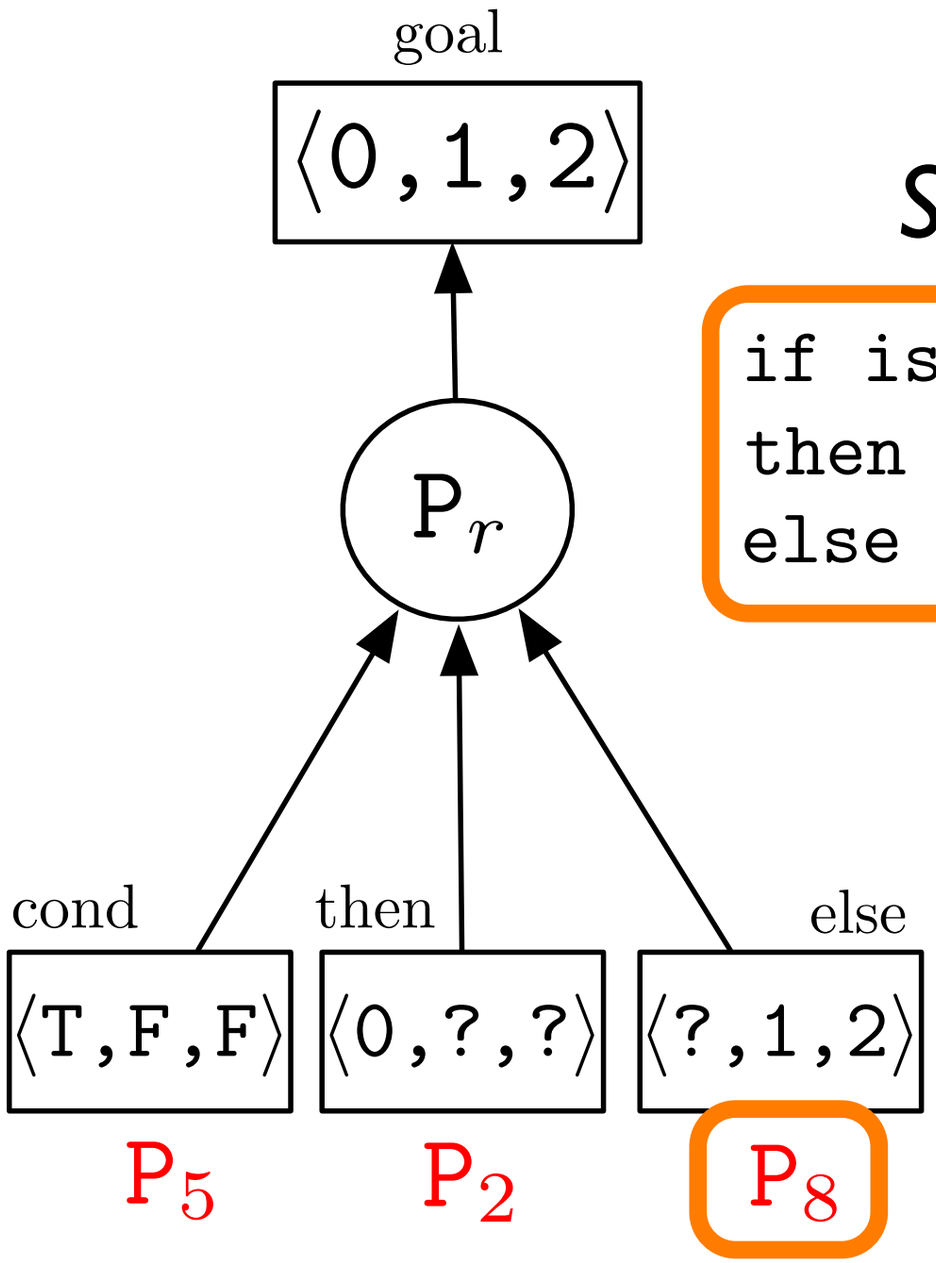

Synthesized program:

```
if isEmpty(i) P5
then 0 P2
else inc(length(tail(i))) P8
```

Escher: Recap

Forward Search

- *Apply component to all synthesized programs*
- *Heuristic-based search*

Escher: Recap

Forward Search

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Conditional Inference

- *Uses goal graph to synthesize conditionals*
- *Conditionals synthesized on demand*

Escher: Recap

Forward Search

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Conditional Inference

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Recursive call synthesis

- *Use I/O specification*
- *Query user for more output values if needed*

Implementation

Prototype implementation of Escher

- *Pluggable components*

Experimented with:

- *integer, list, and tree manipulating programs*

Experiments

(tree manipulating programs)

Time in seconds

	Escher	w/o ObsEquiv	w/o GoalGraph	w/o OE+GG
collect_leaves	**0.04**	0.09	68.9	81.8
count_leaves	**0.06**	0.2	9.3	12.3
hbal_tree	**1.5**	MEM	TIME	MEM
nodes_at_level	**10.74**	MEM	TIME	MEM

Experiments

(#components sensitivity: mult*)*

Escher: 0.7s

with all components

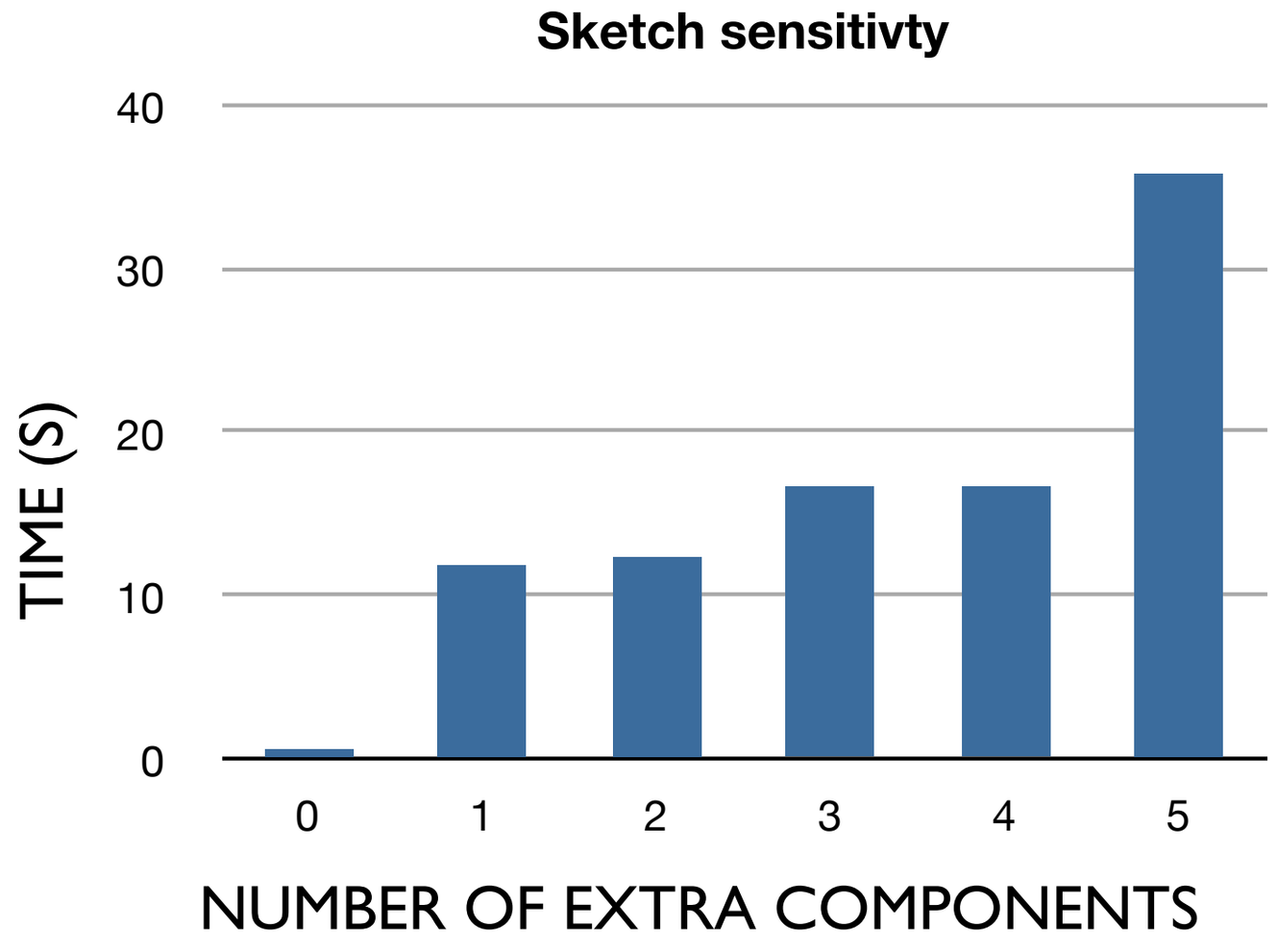
Experiments

(#components sensitivity: mult*)*

SAT-based synthesis [ASPLOS'06]

Escher: 0.7s

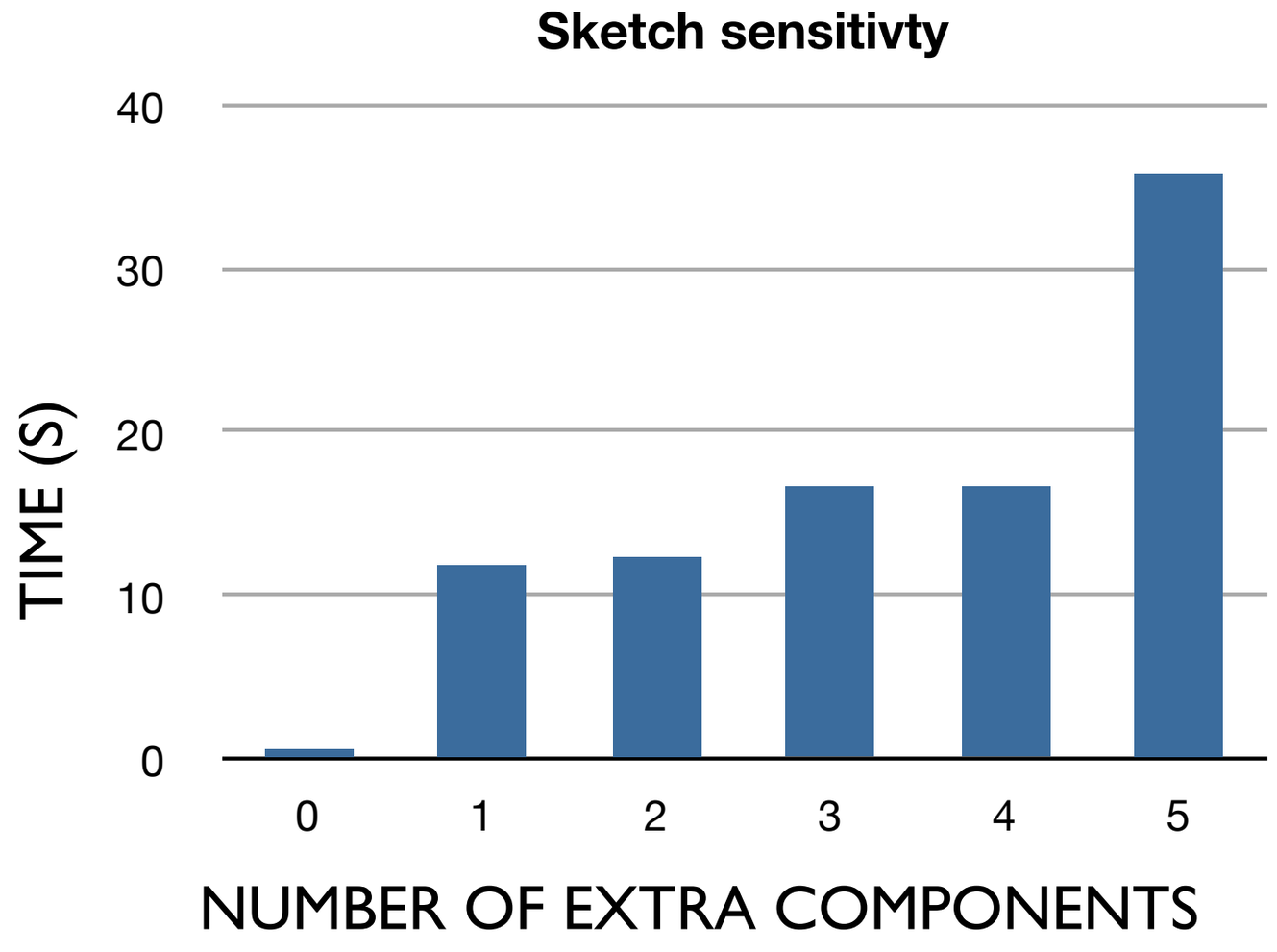
with all components



Experiments

(#components sensitivity: mult*)*

SAT-based synthesis [ASPLOS'06]

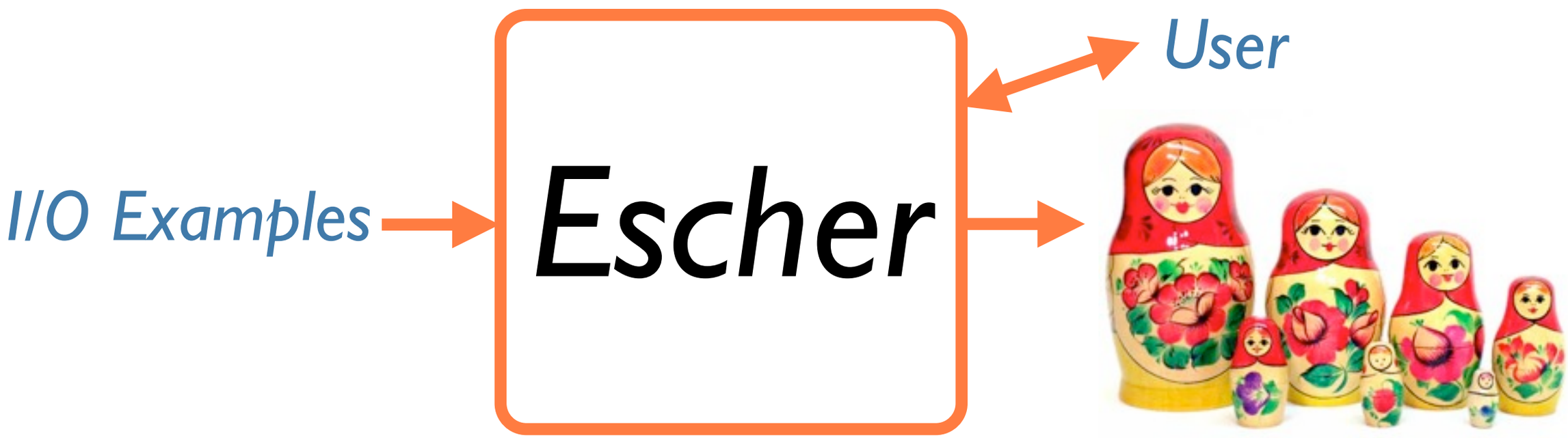


Escher: 0.7s

with all components

Had to supply Sketch with high-level conditional

Conclusion



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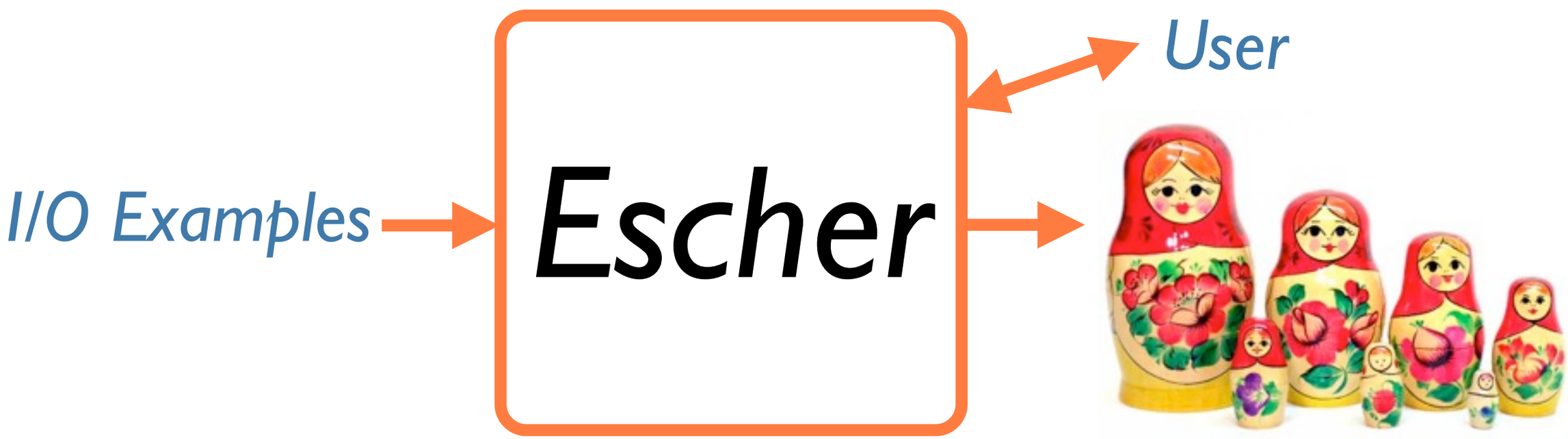
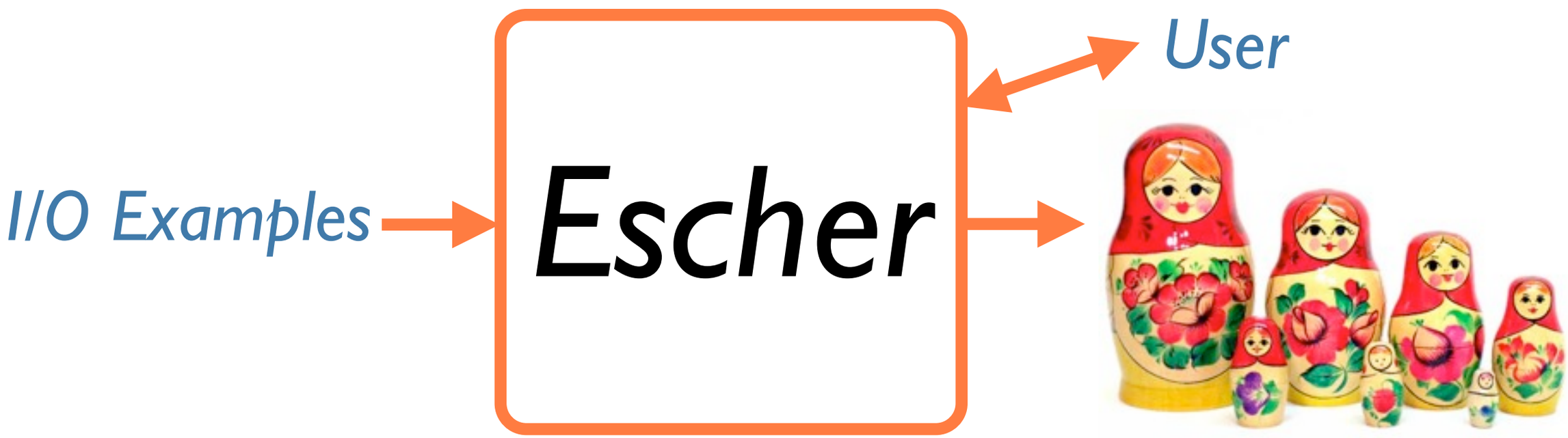


Interactive

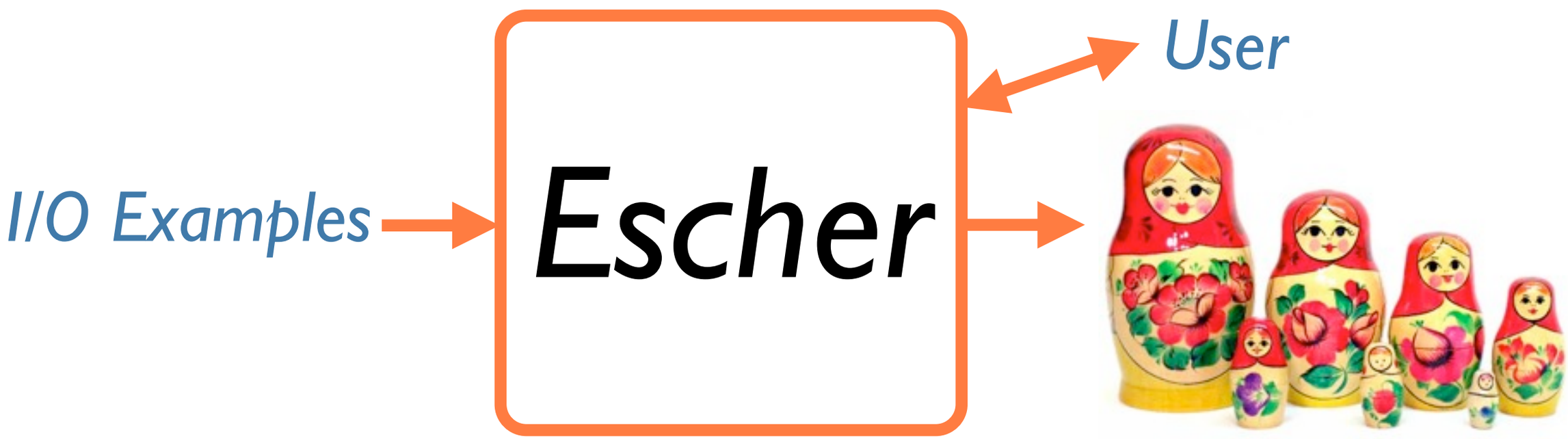
Conclusion



Interactive

Highly-customizable

Conclusion



Interactive

Highly-customizable

Novel synthesis techniques

Questions?

How can we synthesize loops?

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Integration with SMT-based search

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How can we synthesize loops?

Integration with SMT-based search

Applications of Escher in eduction

- e.g., for interacting with students

Questions?

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Theoretical under-pinnings

- e.g., see *Madhusudan [CSL'11]*

Thank You