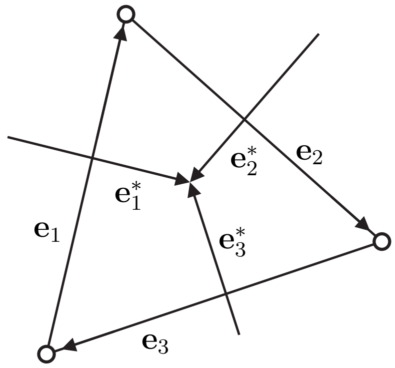

Figure 1: Computing the stress tensor on a triangle.

Stress Tensor Computation

The stress tensor on a primal triangle (projected to 2D) is computed as

$$\sigma = \left[-\mathbf{e}_2^{*\perp} + \mathbf{e}_3^{*\perp}, \mathbf{e}_1^{*\perp} - \mathbf{e}_3^{*\perp}\right] \left[\mathbf{e}_1^{\perp}, \mathbf{e}_2^{\perp}\right]^{-1}, \tag{1}$$

where $\mathbf{e}_i$ and $\mathbf{e}_i^*$ $(i = 1, 2, 3)$ are the directed primal and dual edge vectors shown in Fig. 1, and $^\perp$ denotes rotating a vector 90° counter-clockwise.

In the continuous setting, suppose the height field of a self-supporting surface is $s(x, y)$ and the Airy stress function has Hessian $\widehat{M} = \left(\begin{smallmatrix} m_{22} & -m_{12} \\ -m_{12} & m_{11} \end{smallmatrix}\right)$. Then the stress tensor is given by [1]

$$\sigma = -\frac{M\,g}{\det g}, \tag{2}$$

where $M = \left(\begin{smallmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{smallmatrix}\right)$ and $g = \begin{pmatrix} 1+s_x^2 & s_x s_y \\ s_x s_y & 1+s_y^2 \end{pmatrix}$ is the induced metric on $s(x, y)$.

References

[1] E. Vouga, M. Höbinger, J. Wallner, and H. Pottmann. Design of self-supporting surfaces. *ACM Trans. Graph. (SIGGRAPH)*, 31(4):87:1–87:11, 2012.