Clustered Principal Components for Precomputed Radiance Transfer

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Abstract

clustered principal component analysis

Keywords

1. Introduction

p *Source radiance*

Transferred incident radiance

Exiting radiance

et al. *et al.* *et al.*

N

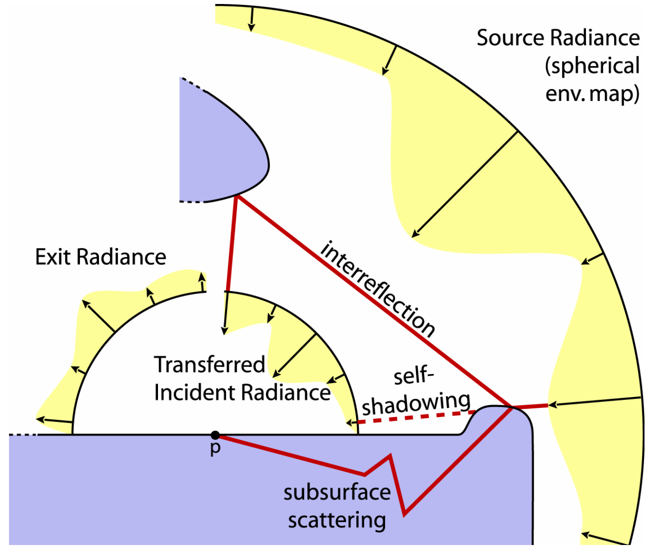


Figure 1 p

et al.

et al.

p

et al.

p

et al.

n'

clustered principal component analysis

N N

$n' \ll N \ll N$

n' N

N N

N p

n' N

et al. *et al.*

2. Related Work

et al.

et al. *et al.* *et al.*

et al.

et al.

et al.

et al.

et al.

et al. *et al.*

et al.

et al.

et al.

,

validated

s

et al.

et al.

et al.

et al.

3. Radiance Transfer Signal Representation

t_p

p *et al.* i

$y_i\ s$ p

et al.

3.1 Transferred Incident vs. Exiting Radiance Transfer

et al. *transferred incident*

exiting radiance p M_p

$M_{p,ij}$

j i

s

v $\boldsymbol{H}$ x,y,z $z \geq$ x^2 y^2 z^2

$$M_{p\ ij} = \int_{v\in \boldsymbol{H}} \int_{s\in \boldsymbol{H}} y_i\ v\ \ T_p\left(s\ \ y_j\ \ s\ \right) B\ v\ s\ \ s_z\ ds\ dv$$

T_p B

y s_z

z s T_p $y_j\ s$ $q_p\ s$

$q_p\ s$ s

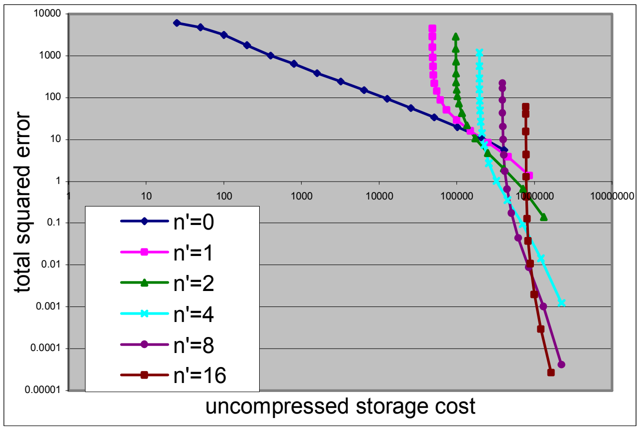


Figure 2

n'

et al.

M_p B R_p T_p T_p
et al. R_p p
z x y B

$$B_{ij} = \int_{v\in \boldsymbol{H}} \int_{s\in \boldsymbol{H}} y_i\ v\ \ y_j\ s\ \ B\ v\ s\ \ s_z\ ds\ dv$$

R_p N N
et al.

y_0

3.2 Representing Radiance over the Hemisphere

et al.

et al.

et al.

et al.

b

A b A

$$M_p = A^- \ B\ A^- \ R_p\ T_p$$

R_p T_p

A

3.3 Clustered PCA (CPCA) Approximation

n x_p p
x_p

n N n N

clusters

$$x_p \approx \tilde{x}_p = x \ + w_p\, x \ + w_p\, x \ + \cdots + w_p^{n'}\, x_{n'}$$

n' n x x $x_{n'}$
n' w_p w_p $\cdots$ $w_p^{n'}$ p
$n' \ll n$
x_0 *cluster mean* x_i $i \geq$
cluster PCA vectors
representative vectors

n'
n'

local linearity

4. Compressing Surface Signals with CPCA

4.1 VQ Followed by Static PCA

VQ Clustering *et al.*

Per-Cluster PCA x_0
m_k n
C $x_{p1}\ x_0\ \ x_{p2}\ x_0\ \ x_{pnk}\ x_0\ ^T$ m_k
k C $U\,D\,V^T$
U V^T D
n' V
V^T p_j
n' w_{p_j} n'
j UD x_{pj}
x_0
C n'

$$\sum_{j=}^{m_k} \left\| x_{p_j} - \tilde{x}_{p_j} \right\| \ = \sum_{i=n'+}^{n} D_i \ = \sum_{j=}^{m_k} \left\| x_{p_j} - x \ \right\| \ - \sum_{i=}^{n'} D_i$$

C
`dgesvd`
$m_k \geq n,$

n n $C^T C$ C

C

V^T m_k

$n,$ m_k m_k CC^T

C C

U V^T

$U^T D$ C `dsyevx`

$C^T C$ CC^T

n'

`dgesvd` *all*

Experimental Results

n'

m_p n' m_c n' n

m_p m_c

4.2 Iterative PCA

$\|x_p - x\|$

$\|x_p - \tilde{x}_p\|$

$$\|x_p - \tilde{x}_p\| = \|x_p - x\| - \sum_{i=}^{n'} \left(x_p - x \bullet x_i \right)$$

x_p

n'

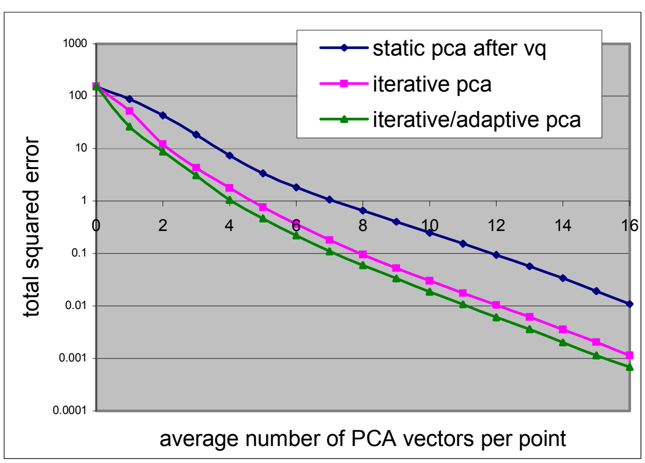


Figure 3

4.3 Per-Cluster Adaptation of Number of PCA Vectors

D_i^2

i

D_i^2/m_k

i k m_k

n'

D_i^2/m_k i

k Σm_k

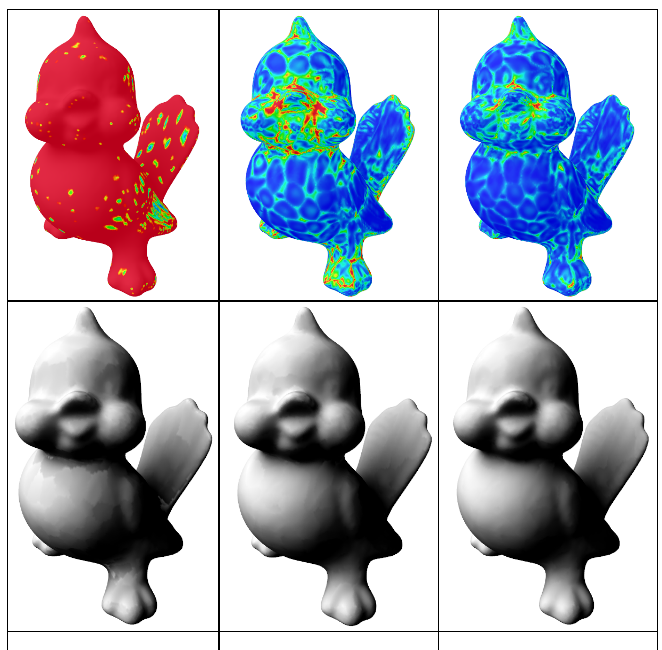


Figure 4

n'

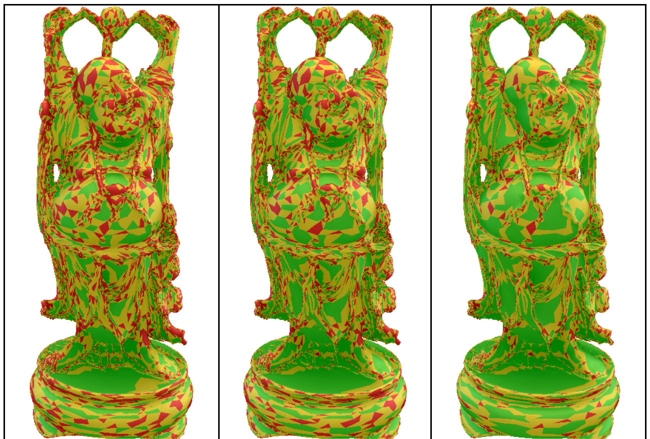


Figure 5 n'

5. Cluster Coherence Optimization

overdraw

superclustering

.

6. Rendering Using CPCA-Encoded Transfer

p *et al.*

$$y^T v_p \left(B R_p T_p\right) l = y^T v_p \left(M_p\right) l$$

l

T_p

R_p p B y y^T p v_p y l N B R T N N N

l *et al.*

l

et al. T'_p R_p T_p f_p T'_p l B' v_p y v_p B N v_p B' *et al.*

f_p

M_p

$$\tilde{M}_p = M + w_p M + w_p M \cdots + w_p^{n'} M_{n'}$$

$\tilde{M}_p$ l

e_p

$$e_p = \tilde{M}_p l = (M\ l) + w_p (M\ l) + w_p (M\ l) + \cdots + w_p^{n'} (M_{n'} l)$$

n' N- w_p^i n' N

N n'

v_p y v_p e_p y v_p *et al.* y^T v_p B

$t_p \bullet l$ t_p N l t_p

n' $t_i \bullet l$

$$\tilde{t}_p \bullet l = (t \bullet l) + w_p (t \bullet l) + w_p (t \bullet l) + \cdots + w_p^{n'} (t_{n'} \bullet l)$$

n' N

N

N N

n'

et al.

6.1 Non-Square Matrices

M_p N_r N_l N_l

N_r

N_l

N_r

N_l

M_p

M_p

N_r

6.2 Implementation Notes

α_p

p

Draw the mesh into the zbuffer only rgb

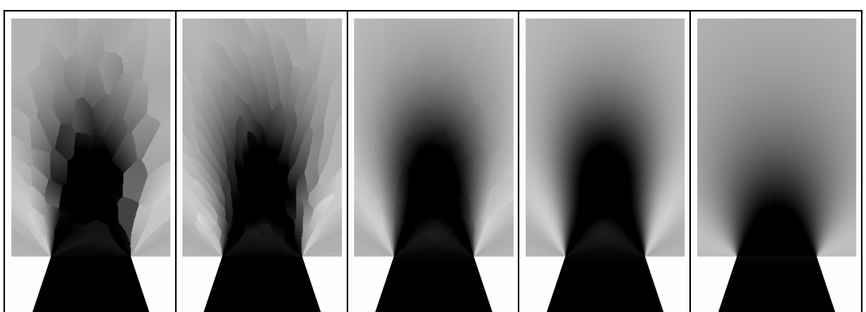
Set the blending mode to add

Foreach cluster

- Compute n' per-cluster constants $M_i\, l$ or $t_i \bullet l$ on CPU
- Load per-cluster constants to graphics hardware
- DrawCluster

w_p^i

α_p w_p^i



n'	n'	n'	n'	

Figure 6 N n' m_c N *et al.* n'

p v_p v_p e_p

y v_p

e_p

e_p

n'

$t_i \bullet l$

N_l N_r

$M_i\, l$

m_s n' N_r m_s

N_r

m_s n'

rgb

7. Results

n'

m_c

n'

m_c

et al. *et al.*

n'

n'

et al.

n'

et al.

m_c

8. Conclusions and Future Work

			m_p	Nr N_l	m_c	n'					

Table 1

m_c

Acknowledgments

References

9. Appendix: Hemispherical SH Projection

9.1 Least-Squares Optimal Projection

$f(s)$ s (x,y,z) $s \in \boldsymbol{H}$
f $y(s)$ $\boldsymbol{H}$

$$f(s) \approx \Sigma_i c_i y_i(s)$$

$\boldsymbol{H}$
c *least-squares optimal hemispherical projection* f

c_i

$$E = \int_{\boldsymbol{H}} (f(s) - \Sigma_i c_i y_i(s))^2 ds$$

c_i

$\partial E / \partial c_k$

$$\partial E / \partial c_k = \int_{\boldsymbol{H}} (f(s) - \Sigma_i c_i y_i(s)) y_k(s) ds$$

$$\Rightarrow \Sigma_i c_i \int_{\boldsymbol{H}} y_i(s) y_k(s) ds = \int_H f(s) y_k(s) ds$$

$Ac = b$ $c = A^{-1} b$ A

$$A_{ik} = \int_{\boldsymbol{H}} y_i(s) y_k(s) ds$$

b $f(s)$

$$b_k = \int_{\boldsymbol{H}} f(s) y_k(s) ds$$

b
f
zero-hemisphere hemispherical projection
A $f(s)$
A

$y(s)$ $A^{-1} y(s)$
A

l m
z

9.2 Error Analysis of Various Projections

$$\int_{\boldsymbol{H}} (\Sigma_i c_i y_i(s))^2 ds$$

c $c^T A c$
b
f $c = A^{-1} b$
f

$\boldsymbol{H}$

$$E_1 = (c-b)^T A (c-b) = c^T [(A - I)^T A (A - I)] c = c^T Q_1 c$$

I E_1

c

Q_1

et al. f
z

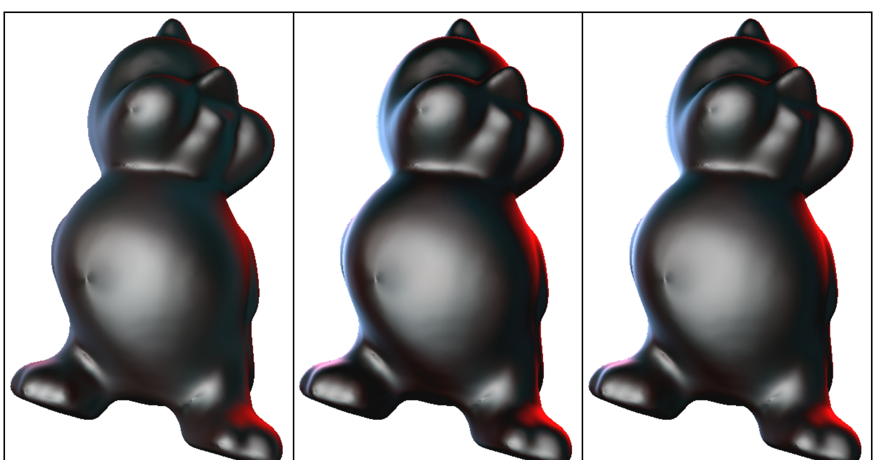


Figure 7

et al.

$$f_{odd}\ x\ y\ z\ = \begin{cases} f\ x\ y\ z & z \geq \\ -f\ x\ y\ -z & \end{cases}$$

f

odd reflection hemispherical projection f

c $z \geq$

c_i *odd*

$y_i\ x\ y\ z$ $y_i\ x\ y\ z$ *even*

$y_i\ x\ y\ z$ $y_i\ x\ y\ z$

z

c E

$$E\ = c^T\ \ D^*A - I)^T A\ \ D^*A - I\ \ c\ = c^T Q\ c$$

D^*

A

Q

c

Q

f

$$f_{even}\ x\ y\ z\ = \begin{cases} f\ x\ y\ z & z \geq \\ f\ x\ y\ -z & \end{cases}$$

E_2

D^*

Figure 8:

et al.

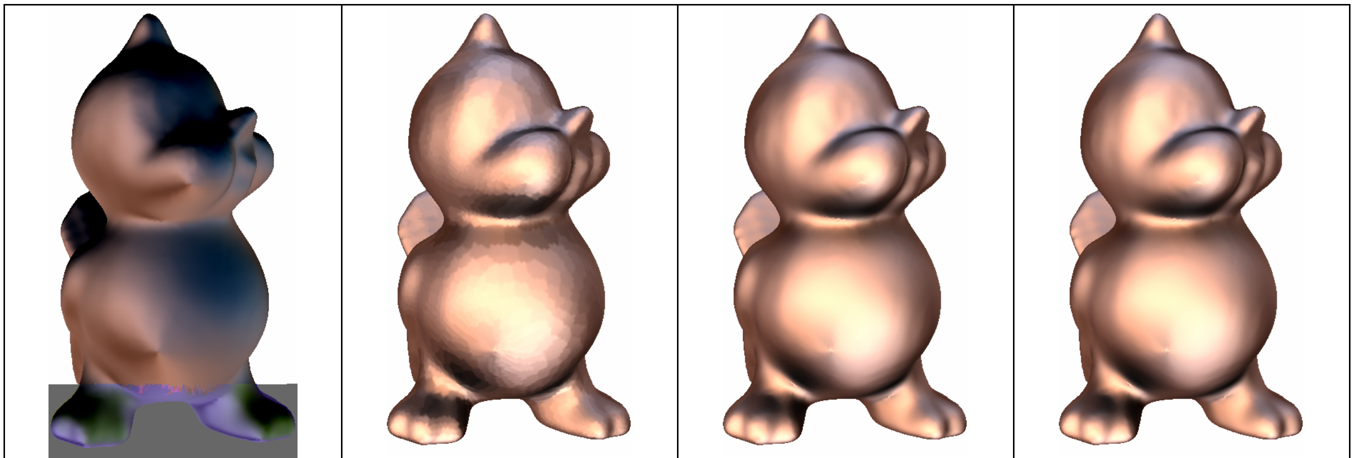
et al.

z

f

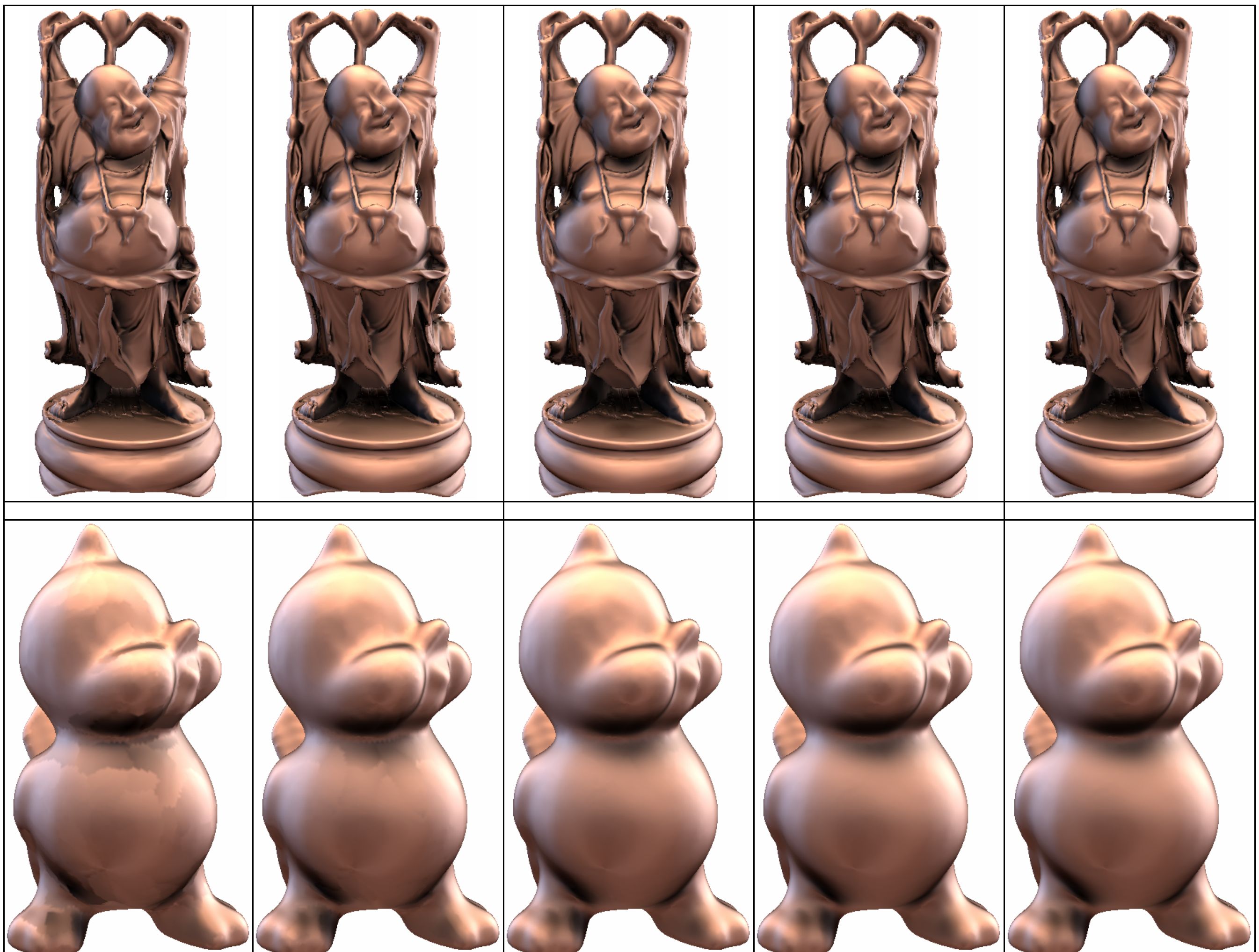
$\rightarrow$ $f\ x\ y$

Figure 9:



PCA m_c n'	VQ m_c n'	CPCA m_c n'	Uncompressed

Figure 10



CPCA, n'=2	CPCA, n'=4	CPCA, n'=8	CPCA, n'=12	uncompressed

Figure 11: n' m_c